C4 VECTORS

1a) l_1 and l_2 are perpendicular $\therefore$ $a.b = 0$

$\cancel{-5q + 11}$

$-2q + 2 - 8 = 0$

$-2q - 6 = 0$

$-2q = 6$

$\underline{\underline{q = -3}}$

b) $11 - 2\lambda = -5 - 3\mu$ ①

$2 + \lambda = 11 + 2\mu$ ②

$17 - 4\lambda = p + 2\mu$

$11 - 2\lambda = -5 - 3\mu$

$4 + 2\lambda = 22 + 4\mu$ ② × 2

$15 = 17 + \mu$

$\underline{-2 = \mu}$

$2 + \lambda = 11 + 2(-2)$

$2 + \lambda = 11 - 4$

$\underline{\lambda = 5}$

$17 - 4(5) = p + 2(-2)$

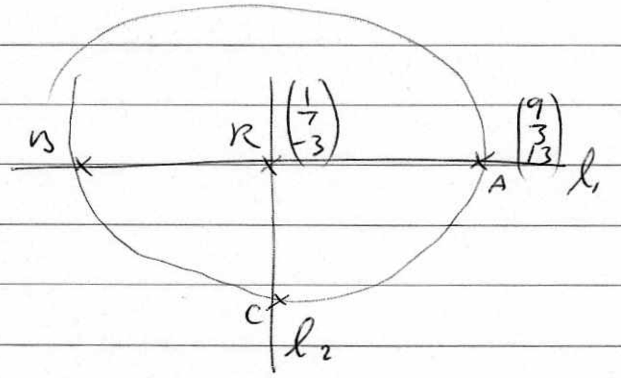
$17 - 20 = p - 4$

$-3 = p - 4$

$\underline{\underline{p = 1}}$

c) $r = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + 5\begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$

$= \underline{\underline{\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}}}$



B: $\begin{pmatrix}-7\\11\\-19\end{pmatrix}$

2a $\begin{pmatrix}8\\13\\-2\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}$

b) $\begin{pmatrix}10-9\\14-9\\-4-6\end{pmatrix} = \begin{pmatrix}1\\5\\-10\end{pmatrix}$ $\sqrt{1^2+5^2+10^2} = \underline{\underline{\sqrt{126} = 11.2}}$

c) $a \cdot b = |a|\,|b| \cos\theta$

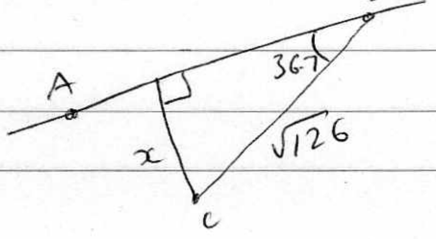
$2(1) + 1(5) - 2(-10) = \sqrt{2^2+1^2+2^2} \cdot \sqrt{1^2+5^2+10^2} \cos\theta$

$27 = \sqrt{9}\,\sqrt{126} \cos\theta$

$\cos\theta = \dfrac{27}{\sqrt{9}\sqrt{126}}$

$\underline{\underline{\theta = 36.7°}}$ (1dp)

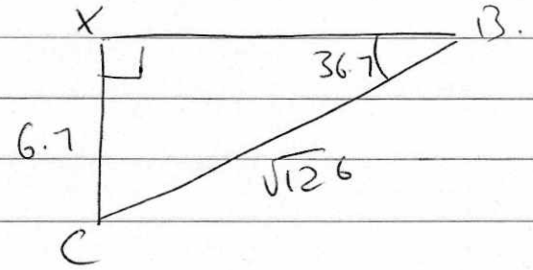
d) shortest distance = perpendicular distance



$\sin(36.9) = \dfrac{x}{\sqrt{126}}$

$\sqrt{126}\sin(36.9) = x$

$x = 6.7$ units (1dp)



Area = $\frac{1}{2} ab \sin C$

$= \frac{1}{2}(6.7)(\sqrt{126}) \sin(90 - 36.7)$

$= 30.2$ units^2 (3sf)

3a) $-9 + 2\lambda = 3 + 3\mu$

$\lambda = 1 - \mu$

$10 - \lambda = 17 + 5\mu$

$-9 + 2(1-\mu) = 3 + 3\mu$

$-9 + 2 - 2\mu = 3 + 3\mu$

$-7 - 2\mu = 3 + 3\mu$

$-10 = 5\mu$

$\underline{\underline{\mu = -2}}$

$\lambda = 1 - (-2)$

$\lambda = \underline{\underline{3}}$

$r = -9i + 10k + 3(2i + j - k)$

$= -9i + 10k + 6i + 3j - 3k$

$= \underline{\underline{-3i + 3j + 7k}}$

b) perpendicular $\therefore$ $a.b = 0$

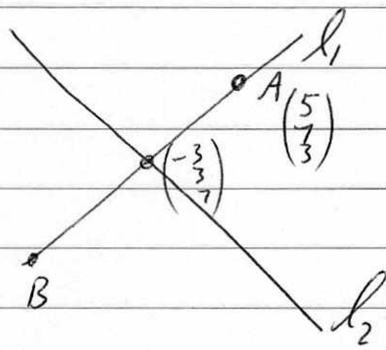
$2(3) + 1(-1) - 1(5) = 0$

c/ $\begin{pmatrix} -9 \\ 0 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}$

$$-9 + 2\lambda = 5$$
$$2\lambda = 14$$
$$\underline{\underline{\lambda = 7}}$$

$$\begin{pmatrix} -9 \\ 0 \\ 10 \end{pmatrix} + 7 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}}}$$

d/



B: $\underline{\underline{\begin{pmatrix} -11 \\ -1 \\ 11 \end{pmatrix}}}$

4/ $\vec{AB} = \begin{pmatrix} 3-2 \\ 4-6 \\ 1--1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}}}$

b/ $\underline{\underline{\begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}}}$

c/ $a.b = |a|\,|b| \cos\theta$

$1(1) + 1(2) = \sqrt{9}\,\sqrt{2} \cos\theta$

$3 = \sqrt{18} \cos\theta$

$$\cos\theta = \frac{3}{\sqrt{18}}$$

$$\underline{\underline{\theta = 45^\circ}}$$

d) ~~$\lambda\ \theta$~~ $\lambda \begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}2\\6\\-1\end{pmatrix} + \mu\begin{pmatrix}1\\-2\\2\end{pmatrix}$

$$\lambda = 2 + \mu$$
$$0 = 6 - 2\mu$$
$$\lambda = -1 + 2\mu$$

$$\underline{\mu = 3} \quad \underline{\lambda = 5}$$

$C: \underline{\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}}$ $\quad \underline{\underline{5i + 5k}}$

5a) $1 + \lambda = 1 + 2\mu$ ①
$\lambda = 3 + \mu$ ②
$-1 = 6 - \mu$ ③

From ③ $\mu = 7$
② $\lambda = 10$
① $1 + 10 = 1 + 14$
$\underline{11 = 15}$

$\therefore$ lines do not intersect.

b) A: $\begin{pmatrix}1\\0\\-1\end{pmatrix} + 1\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}2\\1\\-1\end{pmatrix}$

B: $\begin{pmatrix}1\\3\\6\end{pmatrix} + 2\begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}5\\5\\4\end{pmatrix}$

$$\vec{AB} = \begin{pmatrix} 5-2 \\ 5-1 \\ 4-1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$$

$$a \cdot b = |a|\,|b| \cos\theta$$

$$3(1) + 4(1) + 5(0) = \sqrt{2}\,\sqrt{50} \cos\theta$$

$$7 = \sqrt{100} \cos\theta$$

$$\cos\theta = \underline{\underline{\frac{7}{\sqrt{100}}}} = \underline{\underline{\frac{7}{10}}}$$

$$\cancel{\theta =}$$

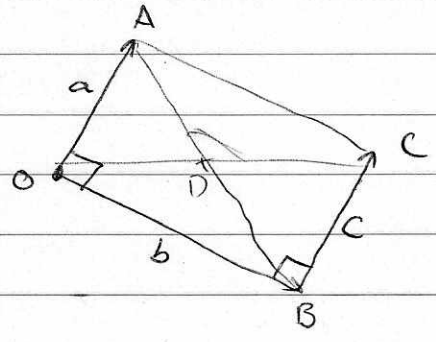
6a/ C: $\begin{pmatrix} 2+1 \\ 2+1 \\ 1-4 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -3 \end{pmatrix}$

$$\underline{\underline{3i + 3j - 3k}}$$

b/ $\vec{OA} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ $\qquad \vec{OB} = \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix}$

$\vec{OA}$ and $\vec{OB}$ are perpendicular as $a \cdot b = 0$

$$2(1) + 2(1) + 1(-4) = 0$$



Length of $\vec{OA} = \sqrt{2^2 + 2^2 + 1^2} = \sqrt{9} = 3$

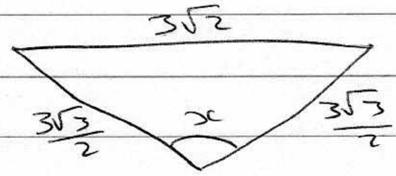
$\vec{OB} = \sqrt{1^2 + 1^2 + 4^2} = \sqrt{18} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$

Area $= 3 \times 3\sqrt{2} = \underline{\underline{9\sqrt{2}}}$ units^2

c/ $\frac{3}{2}i + \frac{3}{2}j - \frac{3}{2}k$

d/ Length of AD $= \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2}$

$= \frac{3\sqrt{3}}{2}$



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{\left(\frac{3\sqrt{3}}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2 - \left(3\sqrt{2}\right)^2}{2\left(\frac{3\sqrt{3}}{2}\right)\left(\frac{3\sqrt{3}}{2}\right)}$$

$$= -\frac{1}{3}$$

$$A = 109.4712206^\circ$$

$$= \underline{\underline{109.5^\circ \text{ (1dp)}}}$$

7a/

$$\begin{pmatrix} 6 \\ 19 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$$

$6 + \lambda = 0$

$\lambda = -6$

$$\begin{pmatrix} 6 \\ 19 \\ -1 \end{pmatrix} \cancel{+} -6 \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 0 \\ -5 \\ 11 \end{pmatrix}}}$$

$a = -5 \quad b = 11$

b/ $P: \begin{pmatrix} 6 + \lambda \\ 19 + 4\lambda \\ -1 - 2\lambda \end{pmatrix}$

perpendicular $\therefore$ $a.b = 0$

$1(6+\lambda) + 4(19+4\lambda) - 2(-1-2\lambda) = 0$

$6 + \lambda + 76 + 16\lambda + 2 + 4\lambda = 0$

$21\lambda + 84 = 0$

$21\lambda = -84$

$\lambda = -4$

$\begin{pmatrix} 6-4 \\ 19+4(-4) \\ -1-2(-4) \end{pmatrix}$

$\underline{\underline{(2, 3, 7)}}$

~~c/~~ $\cancel{\sqrt{2^2 + 3^2 + 7^2} = \sqrt{62}}$

c/ A: $(0, -5, 11)$
B: $(5, 15, 1)$
P: $(2, 3, 7)$

AP Length $= \sqrt{2^2 + 8^2 + 4^2} = 2\sqrt{21}$

PB Length $= \sqrt{3^2 + 12^2 + 6^2} = 3\sqrt{21}$

Collinear as $\vec{AP} = k(\vec{PB})$

$2i + 8j - 4k = k(3i + 12j - 6k)$

$1.5 \times \vec{AP} = \vec{PB}$ (and both pass through P)

Ratio $2\sqrt{21} : 3\sqrt{21}$

$\underline{\underline{2 : 3}}$

8/ $\begin{pmatrix} 8 \\ 12 \\ 14 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ a \end{pmatrix}$

$8 + \lambda = 4 \quad \underline{\underline{\lambda = -4}}$

$14 + -4(-1) = a$

$\underline{\underline{a = 18}}$

$$\begin{pmatrix} 8 \\ 12 \\ 14 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} b \\ 13 \\ 13 \end{pmatrix}$$

$12 + \lambda = 13$

$\lambda = 1$

$8 + 1(1) = b$

$\underline{\underline{b = 9}}$

b) $P: \begin{pmatrix} 8 + \lambda \\ 12 + \lambda \\ 14 - \lambda \end{pmatrix}$

perpendicular $\therefore$ $a.b = 0.$

$1(8 + \lambda) + 1(12 + \lambda) - 1(14 - \lambda) = 0$

$8 + \lambda + 12 + \lambda - 14 + \lambda = 0$

$3\lambda + 6 = 0$

$\underline{\underline{\lambda = -2}}$

$\underline{\underline{(6, 10, 16)}}$

c) $\sqrt{6^2 + 10^2 + 16^2} = \underline{\underline{14\sqrt{2}}}$ units