Edexcel GCE

Core Mathematics C4

Vectors

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. With respect to a fixed origin O the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \qquad l_2 : \mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

where λ and μ are parameters and p and q are constants. Given that l_1 and l_2 are perpendicular,

(*a*) show that $q = -3$.

(2)

Given further that l_1 and l_2 intersect, find

(*b*) the value of p,

(6)

(*c*) the coordinates of the point of intersection.

(2)

The point A lies on l_1 and has position vector $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$. The point C lies on l_2.

Given that a circle, with centre C, cuts the line l_1 at the points A and B,

(*d*) find the position vector of B.

(3)

[January 2009]

2. Relative to a fixed origin O, the point A has position vector $(8\mathbf{i} + 13\mathbf{j} - 2\mathbf{k})$, the point B has position vector $(10\mathbf{i} + 14\mathbf{j} - 4\mathbf{k})$, and the point C has position vector $(9\mathbf{i} + 9\mathbf{j} + 6\mathbf{k})$.

The line l passes through the points A and B.

(*a*) Find a vector equation for the line l.

(3)

(*b*) Find $|\overrightarrow{CB}|$.

(2)

(*c*) Find the size of the acute angle between the line segment CB and the line l, giving your answer in degrees to 1 decimal place.

(3)

(*d*) Find the shortest distance from the point C to the line l.

(3)

The point X lies on l. Given that the vector $\overrightarrow{CX}$ is perpendicular to l,

(*e*) find the area of the triangle CXB, giving your answer to 3 significant figures.

(3)

[June 2009]

3. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2 : \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where λ and μ are scalar parameters.

(*a*) Show that l_1 and l_2 meet and find the position vector of their point of intersection.

(6)

(*b*) Show that l_1 and l_2 are perpendicular to each other.

(2)

The point A has position vector $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$.

(*c*) Show that A lies on l_1.

(1)

The point B is the image of A after reflection in the line l_2.

(*d*) Find the position vector of B.

(3)

[June 2008]

4. The points A and B have position vectors $2\mathbf{i} + 6\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ respectively.

The line l_1 passes through the points A and B.

(*a*) Find the vector $\overrightarrow{AB}$.

(2)

(*b*) Find a vector equation for the line l_1.

(2)

A second line l_2 passes through the origin and is parallel to the vector $\mathbf{i} + \mathbf{k}$. The line l_1 meets the line l_2 at the point C.

(*c*) Find the acute angle between l_1 and l_2.

(3)

(*d*) Find the position vector of the point C.

(4)

[January 2008]

5. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

(*a*) Show that l_1 and l_2 do not meet.

(4)

The point A is on l_1 where $\lambda = 1$, and the point B is on l_2 where $\mu = 2$.

(*b*) Find the cosine of the acute angle between AB and l_1.

(6)

[June 2007]

6. The point A has position vector $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and the point B has position vector $\mathbf{b} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$, relative to an origin O.

(*a*) Find the position vector of the point C, with position vector $\mathbf{c}$, given by $\mathbf{c} = \mathbf{a} + \mathbf{b}$.

(1)

(*b*) Show that $OACB$ is a rectangle, and find its exact area.

(6)

The diagonals of the rectangle, AB and OC, meet at the point D.

(*c*) Write down the position vector of the point D.

(1)

(*d*) Find the size of the angle ADC.

(6)

[January 2007]

7. The point A, with coordinates $(0, a, b)$ lies on the line l_1, which has equation

$$\mathbf{r} = 6\mathbf{i} + 19\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}).$$

(*a*) Find the values of a and b.

(3)

The point P lies on l_1 and is such that OP is perpendicular to l_1, where O is the origin.

(*b*) Find the position vector of point P.

(6)

Given that B has coordinates $(5, 15, 1)$,

(*c*) show that the points A, P and B are collinear and find the ratio $AP : PB$.

(4)

[June 2006]

8. The line l_1 has vector equation

$$\mathbf{r} = 8\mathbf{i} + 12\mathbf{j} + 14\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k}),$$

where λ is a parameter.

The point A has coordinates $(4, 8, a)$, where a is a constant. The point B has coordinates $(b, 13, 13)$, where b is a constant. Points A and B lie on the line l_1.

(*a*) Find the values of a and b.

(3)

Given that the point O is the origin, and that the point P lies on l_1 such that OP is perpendicular to l_1,

(*b*) find the coordinates of P.

(5)

(*b*) Hence find the distance OP, giving your answer as a simplified surd.

(2)

[January 2006]