C3 Functions

1a) $\dfrac{3x^2 - x - 2}{x^2 - 1}$

$\dfrac{(3x+2)(x-1)}{(x+1)(x-1)}$

$\dfrac{3x+2}{x+1}$

b) $\dfrac{3x^2 - x - 2}{x^2 - 1} - \dfrac{1}{x(x+1)}$

$\dfrac{3x+2}{x+1} - \dfrac{1}{x(x+1)}$

$\dfrac{x(3x+2)}{x(x+1)} - \dfrac{1}{x(x+1)}$

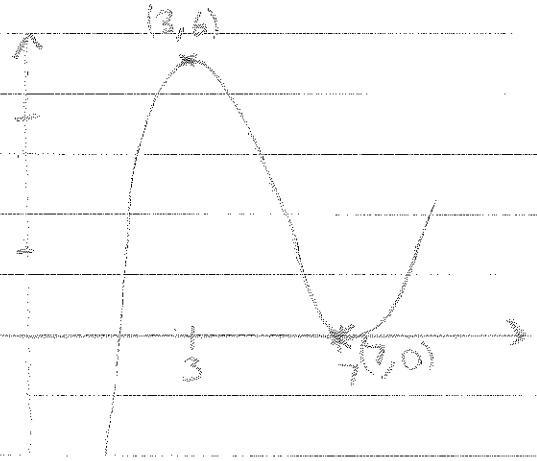
$\dfrac{x(3x+2) - 1}{x(x+1)}$

$\dfrac{3x^2 + 2x - 1}{x(x+1)}$

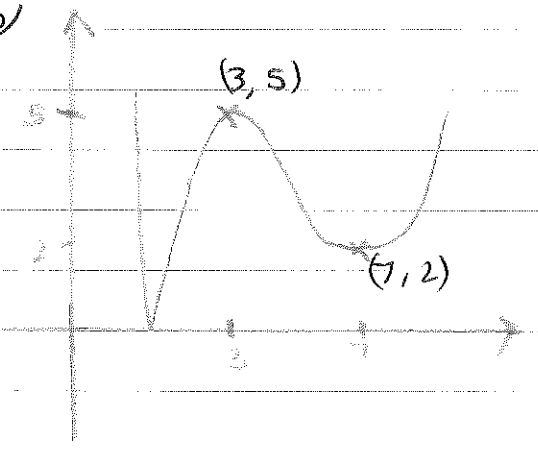
$\dfrac{(3x-1)(x+1)}{x(x+1)}$

$\dfrac{3x-1}{x}$

2a)



2b)



3a) $f: x \longmapsto 3x + \ln x$

$g: x \longmapsto e^{x^2}$

a) $g(x) \geq 1$

b) $fg: x \longmapsto 3(e^{x^2}) + \ln(3e^{x^2})$

$3e^{x^2} + x^2 \ln e$

$3e^{x^2} + x^2$ $\quad [\ln e = 1]$

c) $fg(x) \geq 3$

4a) $f: x \longmapsto \dfrac{2(x-1)}{x^2-2x-3} - \dfrac{1}{x-3}$

$\dfrac{2(x-1)}{(x+1)(x-3)} - \dfrac{1}{x-3}$

$\dfrac{2(x-1)}{(x+1)(x-3)} - \dfrac{1(x+1)}{(x+1)(x-3)}$

$\dfrac{2(x-1) - 1(x+1)}{(x+1)(x-3)}$

$\dfrac{2x-2-x-1}{(x+1)(x-3)}$

$\dfrac{1(x-3)}{(x+1)(x-3)}$

$\dfrac{1}{x+1}$

b) $0 < f(x) < 1/4$

c) $y = \dfrac{1}{x+1}$ $\qquad xy = 1 - y$

$x = \dfrac{1-y}{y}$

$y(x+1) = 1$

$xy + y = 1$ $\qquad f^{-1}(x) = \dfrac{1-x}{x}$ $\qquad 0 < x < 1/4$

d) $f(x) = \frac{1}{x+1}$

$g(x) = 2x^2 - 3$

$fg(x) = \frac{1}{8}$

$$\frac{1}{2x^2 - 3 + 1} = \frac{1}{8}$$

$$\frac{1}{2x^2 - 2} = \frac{1}{8}$$

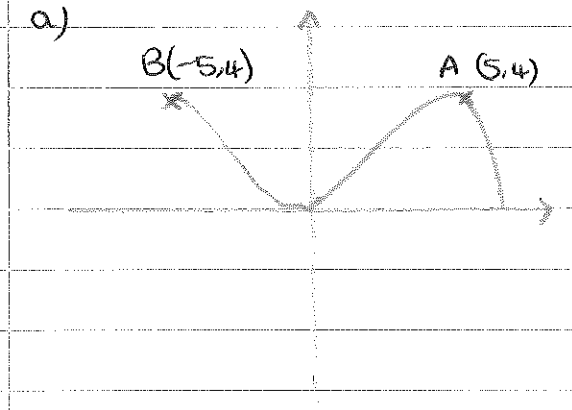
$$2x^2 - 2 = 8$$

$$2x^2 = 10$$

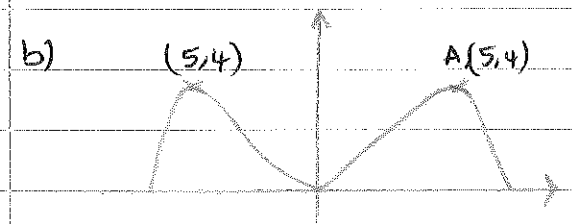
$$x^2 = 5$$

$$x = \pm\sqrt{5}$$

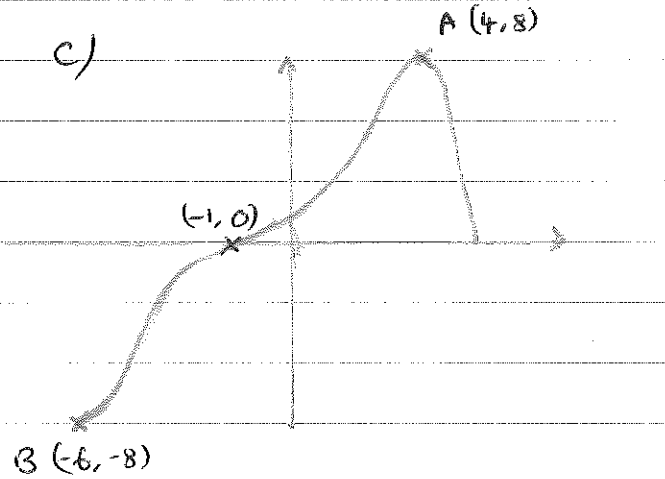
5/ a)



b)



c)



6a) $f(x) = 1 - 2x^3$

$y = 1 - 2x^3$

$y + 2x^3 = 1$

$2x^3 = 1 - y$

$x^3 = \frac{1-y}{2}$

$x = \sqrt[3]{\frac{1-y}{2}}$

$f^{-1}(x) = \sqrt[3]{\frac{1-x}{2}}$

b) $g(x) = \frac{3}{x} - 4$

$gf(x) = \frac{3}{(1-2x^3)} - 4$

$= \frac{3}{1-2x^3} - \frac{4(1-2x^3)}{1-2x^3}$

$= \frac{3 - 4 + 8x^3}{1-2x^3}$

$= \frac{8x^3 - 1}{1-2x^3}$

c) $\frac{8x^3 - 1}{1-2x^3} = 0$

$8x^3 - 1 = 0$

$8x^3 = 1$

$x^3 = 1/8$

$x = \sqrt[3]{1/8}$

$= 1/2$

7 a) $g(4) = \frac{2}{4-3} = 2$

$f(a) = \ln(2(2)-1)$

$= \ln 3$

b) $f(x) = \ln(2x-1)$

$y = \ln(2x-1)$

$e^y = 2x-1$

$e^y + 1 = 2x$

$x = \frac{e^y+1}{2}$

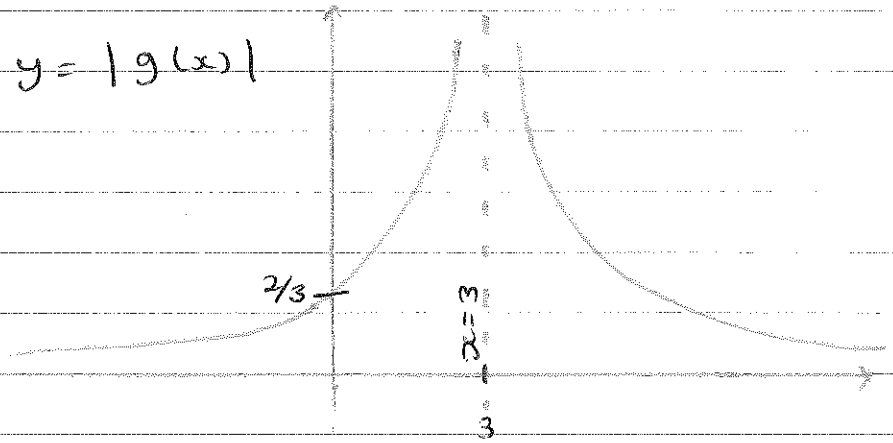
$f^{-1}(x) = \frac{e^x+1}{2}$ $\quad x \in \mathbb{R}$

c) $g(x) = \frac{2}{x-3}$

crosses y when $x=0$ $\quad y = -2/3$

crosses x when $y=0$ $\quad$ N/A.

$y = |g(x)|$



d) $\frac{2}{x-3} = 3$ $\qquad$ $\frac{2}{x-3} = -3$

$2 = 3(x-3)$ $\qquad$ $2 = -3(x-3)$

$2 = 3x-9$ $\qquad$ $2 = -3x+9$

$11 = 3x$ $\qquad$ $-7 = -3x$

$x = 11/3$ $\qquad$ $x = 7/3$

8a) $f(x) = 1 - \frac{3}{x+2} + \frac{3}{(x+2)^2}$

$= \frac{1(x+2)^2}{(x+2)^2} - \frac{3(x+2)}{(x+2)^2} + \frac{3}{(x+2)^2}$

$= \frac{(x+2)^2 - 3(x+2) + 3}{(x+2)^2}$

$= \frac{x^2 + 4x + 4 - 3x - 6 + 3}{(x+2)^2}$

$= \frac{x^2 + x + 1}{(x+2)^2}$

b) Discriminant or $\frac{dy}{dx}$ / complete the square to find minimum

$a = 1 \quad b = 1 \quad c = 1$

$b^2 - 4ac$

$1 - 4$

-3

negative ∴ no solutions, positive x^2 graph therefore must be > 0 for all values of x

c) $\frac{\text{positive}}{\text{positive}} = \text{positive}$

any number squared > 0 ∴ positive

9a) $f(x) = x^4 - 4x - 8$

$f(-2) = (-2)^4 - 4(-2) - 8 = 16$

$f(-1) = (-1)^4 - 4(-1) - 8 = -3$

change of sign ∴ there is a root in the interval $[-2, -1]$

b) $f(x) = x^4 - 4x - 8$

$f'(x) = 4x^3 - 4$

Turning point where $f'(x) = 0$

$$4x^3 - 4 = 0$$
$$4x^3 = 4$$
$$x^3 = 1$$
$$x = 1$$

$f(1) = (1)^4 - 4(1) - 8 = -11$

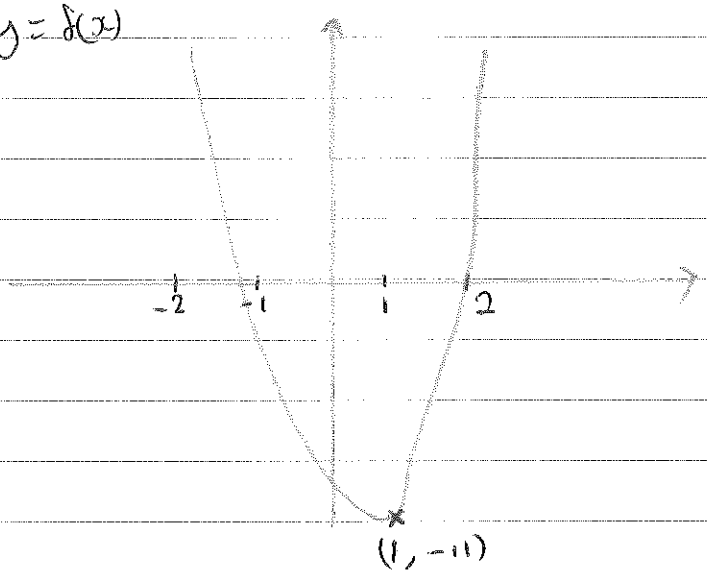
$\underline{(1, -11)}$

c)
$$\begin{array}{r} x^3 + 2x^2 + 4x + 4 \\ x - 2 \overline{)\, x^4 + 0x^3 + 0x^2 - 4x - 8} \\ \underline{x^4 - 2x^3} \\ 2x^3 \\ \underline{2x^3 - 4x^2} \\ 4x^2 - 4x \\ \underline{4x^2 - 8x} \\ 4x - 8 \\ \underline{4x - 8} \\ 0 \end{array}$$

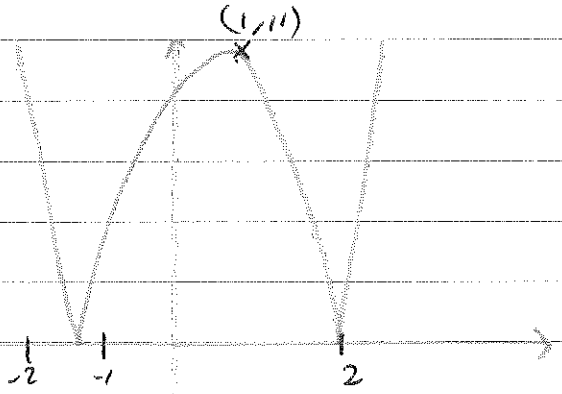
$f(x) = (x - 2)(x^3 + 2x^2 + 4x + 4)$

$a = 2 \quad b = 4 \quad c = 4$

d) $y = f(x)$

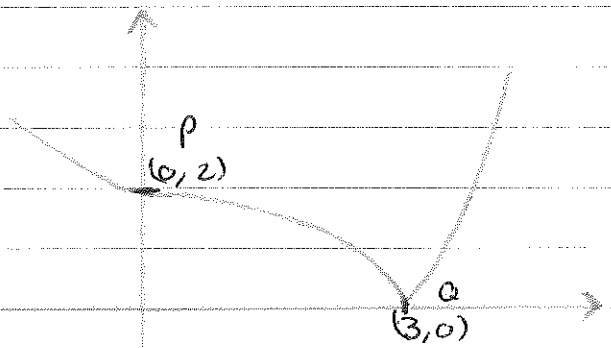


e)

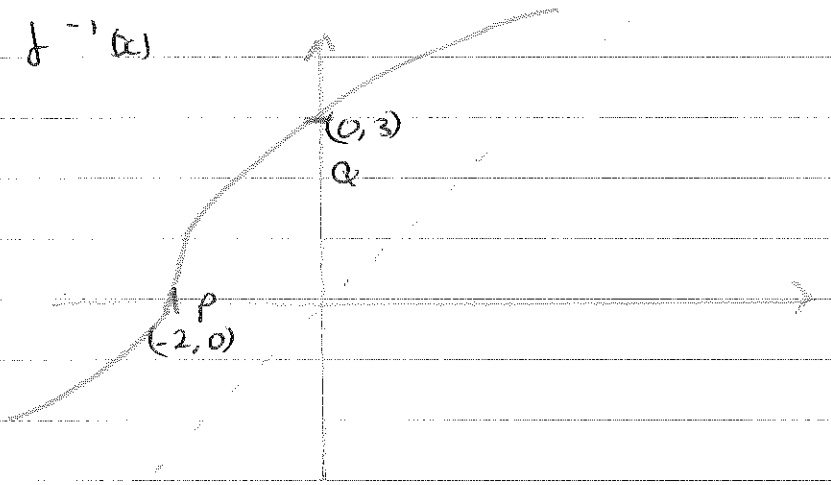


10a)

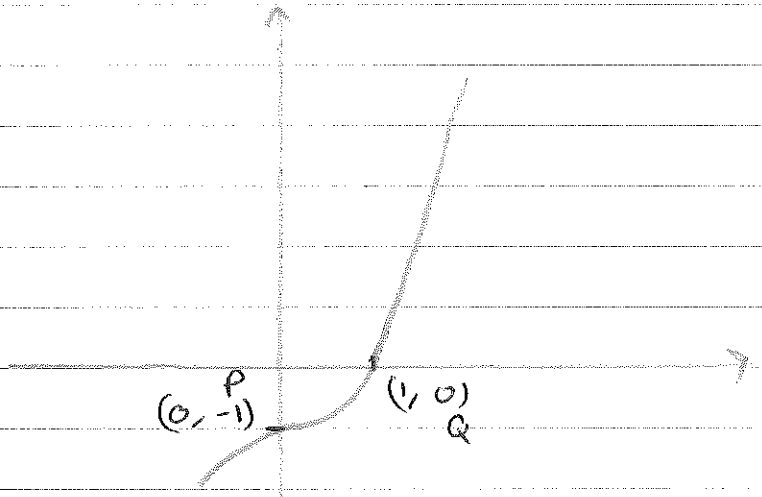
$y = |f(x)|$



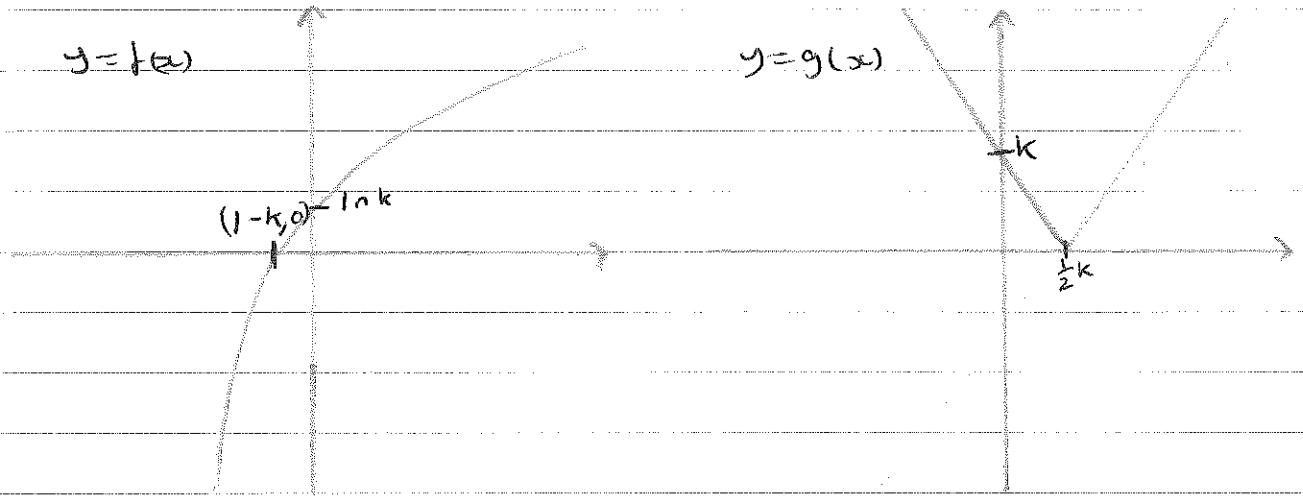
b) $y = f^{-1}(x)$



c)



11a)



11b) $f(x) \in \mathbb{R}$

11c) $g(k/4) = |2(k/4) - k|$

$= |\frac{1}{2}k - k|$

$= |-\frac{1}{2}k|$

$= \frac{1}{2}k$

$f(\frac{1}{2}k) = \ln(\frac{1}{2}k + k)$

$= \ln(\frac{3}{2}k)$

d) $f(x) = \ln(x+k)$

$f'(x) = \frac{1}{x+k}$

$9y = 2x + 1$

$y = \frac{2}{9}x + \frac{1}{9}$

when $x = 3$ $\quad \frac{1}{x+k} = \frac{2}{9}$

$\frac{1}{3+k} = \frac{2}{9}$

$\frac{1}{3+k} = \frac{1}{4.5}$

$k = 1.5$

12) $\frac{2x^2+3x}{(2x+3)(x-2)} - \frac{6}{x^2-x-2}$

$\frac{x(2x+3)}{(2x+3)(x-2)} - \frac{6}{(x-2)(x+1)}$

$\frac{x(x+1)}{(x-2)(x+1)} - \frac{6}{(x-2)(x+1)}$

$\frac{x^2+x-6}{(x-2)(x+1)}$

$\frac{(x-2)(x+3)}{(x-2)(x+1)} = \frac{x+3}{x+1}$

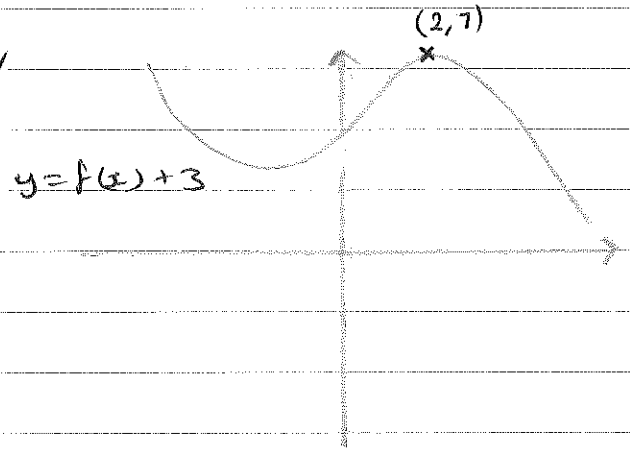
13)

$$\begin{array}{r|l} & 2x^2 - 1 \\ \hline x^2 - 1 & 2x^4 + 0x^3 - 3x^2 + x + 1 \\ & \underline{-2x^4} \qquad\quad \underline{-2x^2} \\ & \qquad -x^2 + x + 1 \\ & \qquad \underline{-x^2 \qquad +1} \\ & \qquad\qquad x \end{array}$$

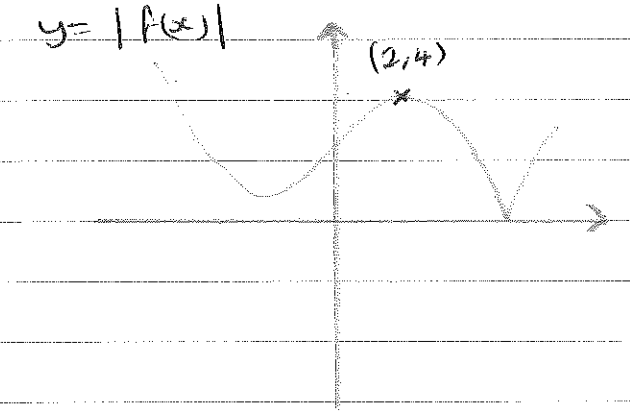
$$2x^2 - 1 + \frac{x}{x^2 - 1}$$

$a = 2 \quad b = 0 \quad c = -1 \quad d = 1 \quad e = 0$

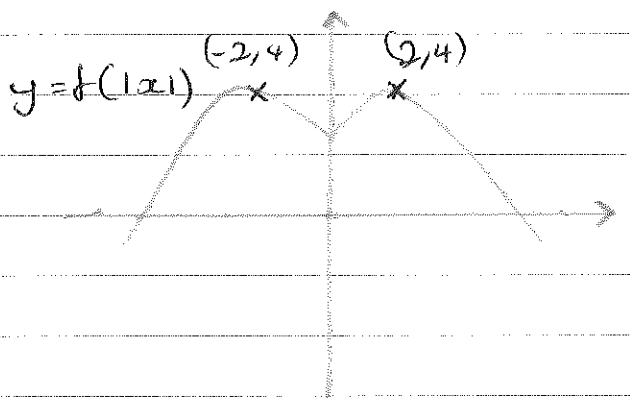
14) a/



b/ $y = |f(x)|$



c/ $y = f(|x|)$

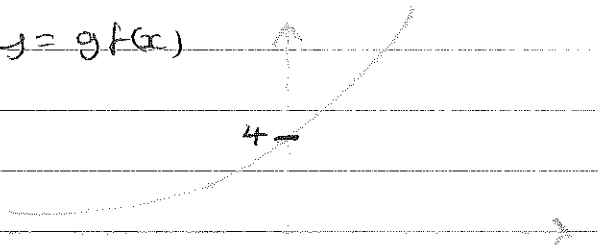


15a) $f(x) = 2x + \ln 2$

$g(x) = e^{2x}$

$$gf(x) = e^{2(2x+\ln 2)}$$
$$= e^{4x + 2\ln 2}$$
$$= e^{4x + \ln 4}$$
$$= (e^{4x})(e^{\ln 4})$$
$$= (e^{4x})(4)$$
$$= \underline{\underline{4e^{4x}}}$$

b) $y = gf(x)$



c) $gf(x) > 0$

16a) $$f(x) = \frac{5x+1}{x^2+x-2} - \frac{3}{x+2}$$

$$\frac{5x+1}{(x+2)(x-1)} - \frac{3}{x+2}$$

$$\frac{5x+1}{(x+2)(x-1)} - \frac{3(x-1)}{(x+2)(x-1)}$$

$$\frac{5x+1-3x+3}{(x+2)(x-1)}$$

$$\frac{2x+4}{(x+2)(x-1)}$$

$$\frac{2(x+2)}{(x+2)(x-1)} = \frac{2}{x-1}$$

16b) $f(x) = \frac{2}{x-1}$

$y = \frac{2}{x-1}$

$y(x-1) = 2$

$xy - y = 2$

$xy = 2 + y$

$x = \frac{2+y}{y}$

$f^{-1}(x) = \frac{2+x}{x}$

c) $f(x) = \frac{2}{x-1}$

$g(x) = x^2 + 5$

$fg(x) = \frac{1}{4}$

$\frac{2}{x^2+5-1} = \frac{1}{4}$

$\frac{2}{x^2+4} = \frac{1}{4}$

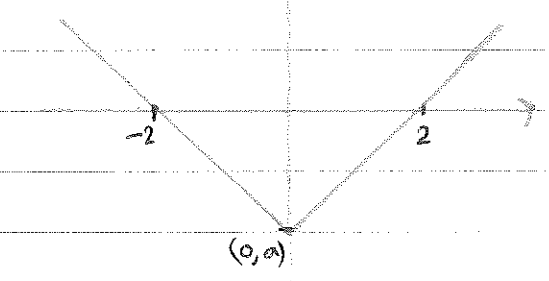
$8 = x^2 + 4$

$x^2 = 4$

$x = \pm\sqrt{2}$

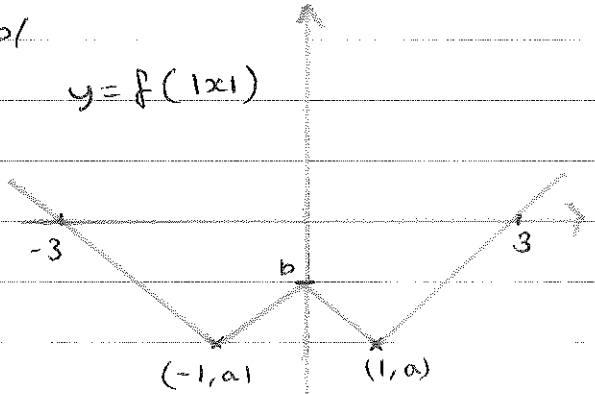
17a/

$y = f(x+1)$



b/

$y = f(|x|)$



17c) $f(x) = |x-1| - 2$

$(1, a)$ $a = |1-1| - 2$

$a = -2$

$(0, b)$ $b = |0-1| - 2$

$b = 1 - 2$

$b = -1$

d) $y = 5x$ will cross negative graph.

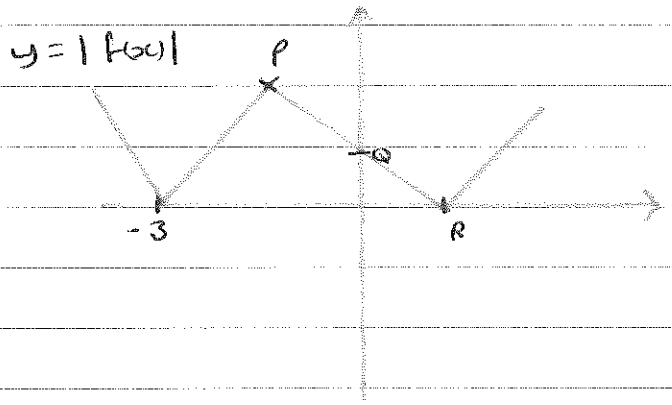
$$-(x-1) - 2 = 5x$$

$$-x + 1 - 2 = 5x$$

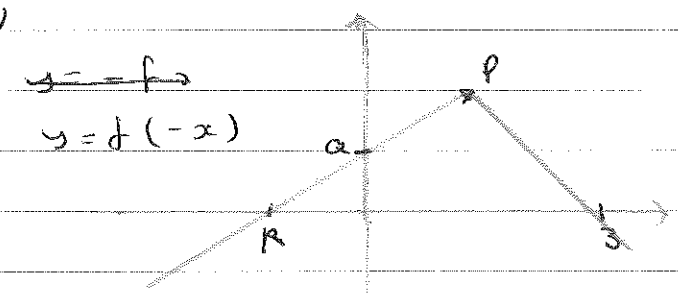
$$-1 = 6x$$

$$x = -1/6$$

18a)



b)



18c) $P = \mathbb{R}$ $(-1, 2)$

$Q =$ $(0, 1)$

$R =$ $(1, 0)$

d) $y = \frac{1}{2}x$ will cross both graphs

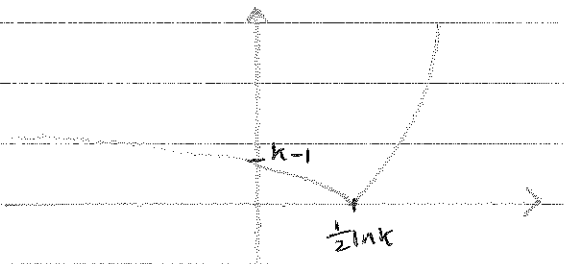
$\frac{1}{2}x = 2 - (x+1)$ $\qquad$ $\frac{1}{2}x = 2 - -(x+1)$

$\frac{1}{2}x = 2 - x - 1$ $\qquad$ $\frac{1}{2}x = 2 + x + 1$

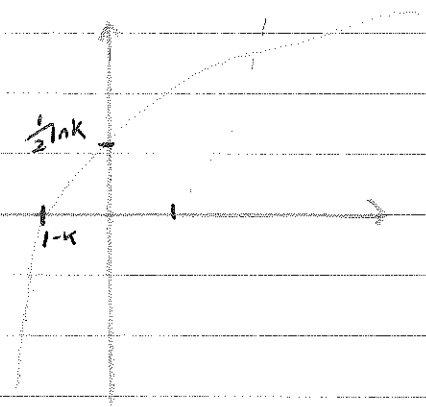
$\frac{3}{2}x = 1$ $\qquad$ $-\frac{1}{2}x = 3$

$x = 2/3$ $\qquad$ $x = -6$

19a)



b)



c) $f(x) > -k$

d) $y = e^{2x} - k$ $\qquad$ $f^{-1}(x) = \frac{1}{2}\ln(x+k)$

$y + k = e^{2x}$

$2x = \ln(y+k)$

$x = \frac{1}{2}\ln(y+k)$

e) $x > -k$