C1 Differentiation and Integration 2

1) $y = kx^3 - x^2 + x - 5$

$\frac{dy}{dx} = 3kx^2 - 2x + 1$

b) $2y - 7x + 1 = 0$

$2y = 7x - 1$

$y = \frac{7}{2}x - \frac{1}{2}$

$m = 7/2 \quad \therefore \frac{dy}{dx} = 7/2$

$3kx^2 - 2x + 1 = \frac{7}{2}$

$3k\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) + 1 = \frac{7}{2}$

$\frac{3k}{4} + 1 + 1 = \frac{7}{2}$

$\frac{3k}{4} = \frac{3}{2}$

$k = 2$

c) $y = 2x^3 - x^2 + x - 5$

$y = 2\left(\frac{-1}{2}\right)^3 - \left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right) - 5$

$y = -\frac{2}{8} - \frac{1}{4} - \frac{1}{2} - 5$

$y = -6$

2) $y = 9 - 4x - 8x^{-1}$

$\frac{dy}{dx} = -4 + 8x^{-2}$

when $x = 2$

$\frac{dy}{dx} = -4 + \frac{8}{2^2}$

$= -2$

$y = -2x + c$

$-3 = -2(2) + c$

$-3 = -4 + c$

$1 = c$

$y = -2x + 1$

$y = 9 - 4(2) - \frac{8}{2}$

$= 9 - 8 - 4$

$= -3$

b/ $(2, -3)$ $m = \frac{1}{2}$

$y = \frac{1}{2}x + c$

$-3 = \frac{1}{2}(2) + c$

$-3 = 1 + c$

$c = -4$

$y = \frac{1}{2}x - 4$

c/ line meets x axis when $y = 0$

A: $0 = 1 - 2x$

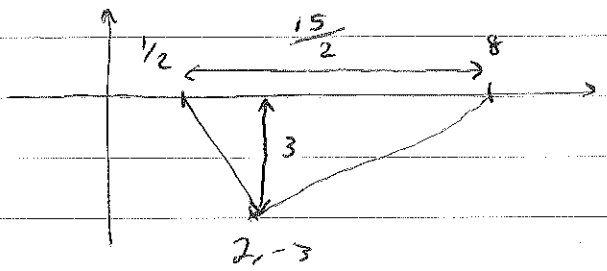
$2x = 1$

$x = \frac{1}{2}$

B: $0 = \frac{1}{2}x - 4$

$4 = \frac{1}{2}x$

$x = 8$



Area $= \frac{1}{2} \cdot$ base $\cdot$ height

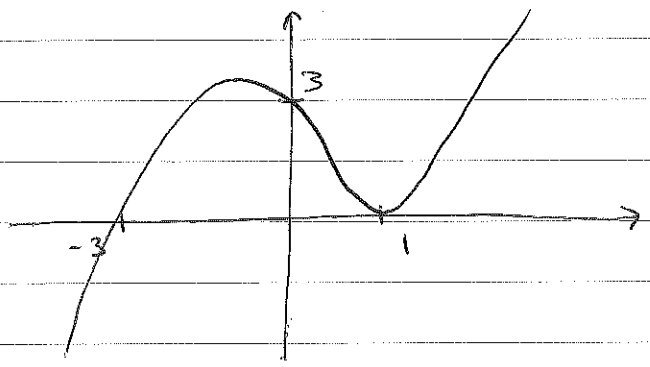
$= \frac{1}{2} \cdot \frac{15}{2} \cdot 3$

$= \frac{45}{4}$ units^2

3a/ $y = (x+3)(x-1)^2$

when $y = 0$ $x = -3$ $x = 1$

when $x = 0$ $y = 3$



b/ $y = (x+3)(x-1)(x-1)$

$y = (x+3)(x^2-2x+1)$

$y = x^3 - 2x^2 + x + 3x^2 - 6x + 3$

$= x^3 + x^2 - 5x + 3$

$\underline{k = 3}$

c/ $\frac{dy}{dx} = 3x^2 + 2x - 5$

$3 = 3x^2 + 2x - 5$

$0 = 3x^2 + 2x - 8$

$0 = (3x-4)(x+2)$

$\underline{x = 4/3} \quad \underline{x = -2}$

4 $f'(x) = 4x - 6x^{1/2} + 8x^{-2}$ $(4, 1)$

$f(x) = \frac{4x^2}{2} - \frac{6x^{3/2}}{3/2} + \frac{8x^{-1}}{-1} + C$

$f(x) = 2x^2 - 4x^{3/2} - 8x^{-1} + C$

$1 = 2(4)^2 - 4(4)^{3/2} - 8(4)^{-1} + C$

$1 = 32 - 32 - 2 + C$

$c = 3$

$f(x) = 2x^2 - 4x^{3/2} - 8x^{-1} + 3$

b/ $f'(x) = 4x - 6\sqrt{x} + \frac{8}{x^2}$

$f'(4) = 4(4) - 6\sqrt{4} + \frac{8}{4^2}$

$= 16 - 12 + 1/2$

$= 9/2$

gradient of normal $= -2/9$

$y = -2/9\, x + C$

$1 = -2/9(4) + C$

$1 = -8/9 + C$

$c = 17/9$

$\underline{y = -2/9\, x + 17/9}$

5 $f'(x) = 6x^2 - 10x - 12$ $(5, 65)$

$f(x) = \frac{6x^3}{3} - \frac{10x^2}{2} - 12x + C$

$f(x) = 2x^3 - 5x^2 - 12x + C$

$65 = 2(5)^3 - 5(5)^2 - 12(5) + C$

$65 = 250 - 125 - 60 + C$

$65 = 65 + C$

$C = 0$

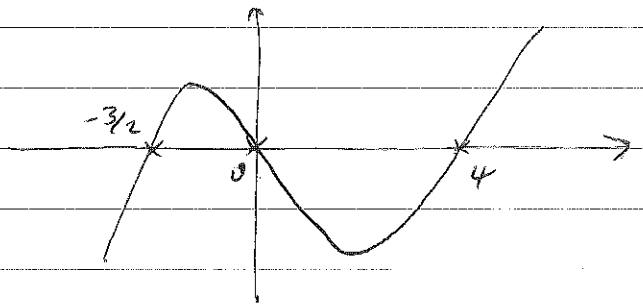
$f(x) = 2x^3 - 5x^2 - 12x$

b/ $f(x) = x(2x^2 - 5x - 12)$

$= x(2x+3)(x-4)$

c/ when $y=0$ $x=0$ $x=-3/2$ $x=4$

when $x=0$ $y=0$



6 $y = x^2(x-6) + 4/x$

when $x=1$

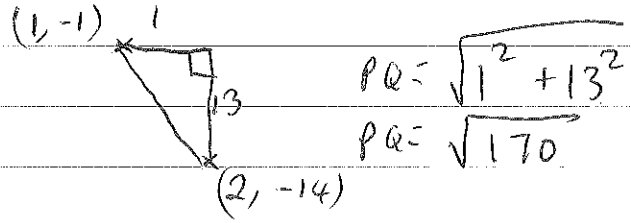
$y = (1)^2(1-6) + \frac{4}{1}$

$= -1$

when $x=2$

$y = (2)^2(2-6) + 4/2$

$= -14$



$PQ = \sqrt{1^2 + 13^2}$

$PQ = \sqrt{170}$

b/ $y = x^2(x-6) + 4x^{-1}$

$\frac{dy}{dx} = x^3 - 6x^2 + 4x^{-1}$

$\frac{dy}{dx} = 3x^2 - 12x - 4x^{-2}$

when $x = 1$

$\frac{dy}{dx} = 3(1)^2 - 12(1) - 4(1)^{-2}$

$= 3 - 12 - 4$

$= -13$

when $x = 2$

$\frac{dy}{dx} = 3(2)^2 - 12(2) - 4(2)^{-2}$

$= 12 - 24 - 1$

$= -13$

c/ gradient of normal $= 1/13$ $\quad (1, -1)$

$y = 1/13\, x + c$

$-1 = 1/13\,(1) + c$

$c = -14/13$

$\therefore \quad y = 1/13\, x + -14/13$

$13y = x - 14$

$x - 13y - 14 = 0$

7 $f'(x) = 3x^2 - 6 - 8x^{-2}$ $\quad (2, 1)$

$f(x) = \frac{3x^3}{3} - 6x - \frac{8x^{-1}}{-1} + c$

$f(x) = x^3 - 6x + 8x^{-1} + c$

$1 = (2)^3 - 6(2) + 8(2)^{-1} + c$

$1 = 8 - 12 + 4 + c$

$c = 1$

$f(x) = x^3 - 6x + 8x^{-1} + 1$

b/ $f'(2) = 3(2)^2 - 6 - 8(2)^{-2}$

$= 12 - 6 - 2$

$= 4$

$y = 4x + c$

$1 = 4(2) + c$

$c = -7$

$y = 4x - 7$

8 a) $y = 4x + 3x^{3/2} - 2x^2$

$\frac{dy}{dx} = 4 + \frac{9}{2}x^{1/2} - 4x$

b) $8 = 4(4) + 3(4)^{3/2} - 2(4)^2$

$8 = 16 + 24 - 32$

$8 = 40 - 32$

$8 = 8$ Shown

c) at $x = 4$ $\frac{dy}{dx} = 4 + \frac{9}{2}(4)^{1/2} - 4(4)$

$= 4 + 9 - 16$

$= -3$

gradient of normal $= \frac{1}{3}$

$y = \frac{1}{3}x + c$ (4, 8)

$8 = \frac{1}{3}(4) + c$

$8 = \frac{4}{3} + c$

$c = \frac{20}{3}$

$y = \frac{1}{3}x + \frac{20}{3}$

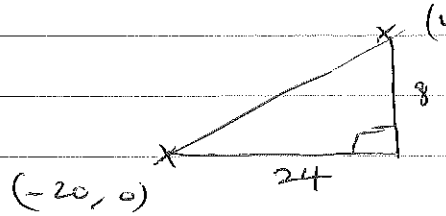
$3y = x + 20$

d) cuts x axis when $y = 0$

$0 = x + 20$

$x = -20$

(-20, 0) (4, 8)



$PQ = \sqrt{24^2 + 8^2}$

$= \sqrt{576 + 64}$

$= \sqrt{640}$

$= \sqrt{64}\sqrt{10}$

$= 8\sqrt{10}$

9 $f'(x) = 2x + 3x^{-2}$ $\left(3, \frac{15}{2}\right)$

$f(x) = \frac{2x^2}{2} + \frac{3x^{-1}}{-1} + c$

$f(x) = x^2 - 3x^{-1} + c$

$\frac{15}{2} = (3)^2 - 3(3)^{-1} + c$

$\frac{15}{2} = 9 - 1 + c$

$\frac{15}{2} = 8 + c$

$c = -1/2$

$f(x) = x^2 - 3x^{-1} - 1/2$

b/ $f(-2) = (-2)^2 - 3(-2)^{-1} - 1/2$

$= 4 + 3/2 - 1/2$

$= 5$

c/ $f'(-2) = 2(-2) + 3(-2)^{-2}$

$= -4 + 3/4$

$= -13/4$

$y = \frac{-13}{4}x + c$ $(-2, 5)$

$5 = \frac{-13}{4}(-2) + c$

$5 = \frac{26}{4} + c$

$c = -\frac{3}{2}$

$y = \frac{-13}{4}x - \frac{3}{2}$

$4y = -13x - 6$

$13x + 4y + 6 = 0$

10a/ $y = 4x^2 + 5x^{-1} - 1$

$\frac{dy}{dx} = 8x - 5x^{-2}$

when $x = 1$

$\frac{dy}{dx} = 8(1) - 5(1)^{-2}$

$= 3$

b/ $y = 3x + c$ when $x = 1$

$y = 4(1)^2 + \frac{5-1}{1}$

$= 8$

$8 = 3 + c$

$c = 5$

$y = 3x + 5$

c/ meets x axis when $y = 0$

$0 = 3x + 5$

$-5 = 3x$

$x = -5/3$

11/ $\frac{dy}{dx} = (3x-1)(3x-1)$

$= 9x^2 - 6x + 1$

a/ when $x = 1$ $\frac{dy}{dx} = 9 - 6 + 1 = 4$

gradient of normal $= -1/4$

$y = -1/4 x + c$ $(1, 4)$

$4 = -1/4 (1) + c$

$17/4 = c$

$y = -1/4 x + 17/4$

b/ $y = \frac{9x^3}{3} - \frac{6x^2}{2} + x + c$

$y = 3x^3 - 3x^2 + x + c$

$4 = 3(1)^3 - 3(1)^2 + 1 + c$

$c = 3$

$y = 3x^3 - 3x^2 + x + 3$

c/ $\frac{dy}{dx} = (3x-1)^2$ $m = -2$

$(3x-1)^2 = -2$

no solutions as you cannot sqroot a negative no.

12a/ P: $x = -2$ $y = (x-1)(x+2)(x-2)$

Q $x = 2$

b/ $y = (x-1)(x^2-4)$

$y = x^3 - x^2 - 4x + 4$

$\frac{dy}{dx} = 3x^2 - 2x - 4$

c/ $(-1, 6)$

when $x = -1$ $\frac{dy}{dx} = 3(-1)^2 - 2(-1) - 4$

$= 3 + 2 - 4$

$= 1$

$M = 1$

$y = x + c$

$6 = -1 + c$

$c = 7$

$y = x + 7$

d/ $\frac{dy}{dx} = 1$

$3x^2 - 2x - 4 = 1$

$3x^2 - 2x - 5 = 0$

$(3x - 5)(x + 1) = 0$

$x = \frac{5}{3}$ $x = -1$

when $x = 5/3$ $y = (\frac{5}{3} - 1)((\frac{5}{3})^2 - 4)$

$= (\frac{2}{3})(\frac{25}{9} - \frac{36}{9})$

$= (\frac{2}{3})(\frac{-11}{9})$

$= -\frac{22}{27}$

$(\frac{5}{3}, -\frac{22}{27})$

13a/ $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$

$0 = \frac{1}{3}(3)^3 - 4(3)^2 + 8(3) + 3$

$0 = 9 - 36 + 24 + 3$

$0 = 0$

b) $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$

$\frac{dy}{dx} = x^2 - 8x + 8$

when $x = 3$

$\frac{dy}{dx} = (3)^2 - 8(3) + 8$

$= 9 - 24 + 8$

$= -7$

$\therefore m = -7$

$y = -7x + c$

$0 = -7(3) + c$

$c = 21$

$y = -7x + 21$

c) $\frac{dy}{dx} = -7$

$x^2 - 8x + 8 = -7$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3 \quad x = 5$

when $x = 5$ $y = \frac{1}{3}(5)^3 - 4(5)^2 + 8(5) + 3$

$= \frac{125}{3} - 100 + 40 + 3$

$= \frac{125}{3} - \frac{300}{3} + \frac{120}{3} + \frac{9}{3}$

$= \frac{-46}{3}$

14 $y = x^3 - 2x^2 - x + 9$

$7 = (2)^3 - 2(2)^2 - (2) + 9$

$7 = 8 - 8 - 2 + 9$

$7 = 7$

b) $\frac{dy}{dx} = 3x^2 - 4x - 1$

when $x = 2$ $\frac{dy}{dx} = 3(2)^2 - 4(2) - 1 = 3$

$m = 3 \qquad (2, 7)$

$y = 3x + c$

$7 = 3(2) + c$

$c = 1$

$y = 3x + 1$

c) $\frac{dy}{dx} = -\frac{1}{3}$

$3x^2 - 4x - 1 = -\frac{1}{3}$

$3x^2 - 4x - \frac{2}{3} = 0$

$9x^2 - 12x - 2 = 0$

$a = 9 \quad b = -12 \quad c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(9)(-2)}}{2(9)}$$

$$= \frac{12 \pm \sqrt{144 + 72}}{18}$$

$$= \frac{12 \pm \sqrt{216}}{18} \qquad \sqrt{216} = \sqrt{36} \cdot \sqrt{6}$$

$$= \frac{12 \pm 6\sqrt{6}}{18}$$

$$= \frac{2 \pm \sqrt{6}}{3}$$

$$= \frac{1}{3}(2 \pm \sqrt{6})$$

x is positive $\therefore x = \frac{1}{3}(2 + \sqrt{6})$