

C1. Differentiation and Integration 1

$$\begin{aligned} 1a) \quad f(x) &= \frac{(3-4\sqrt{x})^2}{\sqrt{x}} \\ &= \frac{(3-4x^{1/2})(3-4x^{1/2})}{x^{1/2}} \\ &= \frac{9 - 24x^{1/2} + 16x}{x^{1/2}} \\ &= 9x^{-1/2} - 24 + 16x^{1/2} \quad A=16 \quad B=-24 \end{aligned}$$

$$b) \quad f'(x) = -\frac{9}{2}x^{-3/2} + 8x^{-1/2}$$

$$\begin{aligned} c) \quad f'(9) &= -\frac{9}{2}(9)^{-3/2} + 8(9)^{-1/2} \\ &= -\frac{9}{2}\left(\frac{1}{27}\right) + 8\left(\frac{1}{3}\right) \\ &= -\frac{1}{6} + \frac{8}{3} \\ &= \frac{15}{6} = \underline{\underline{\frac{5}{2}}} \end{aligned}$$

$$2) \quad \int 12x^5 - 8x^3 + 3 \, dx$$

$$\frac{12x^6}{6} - \frac{8x^4}{4} + 3x + C$$

$$2x^6 - 2x^4 + 3x + C$$

$$3a) \quad y = 2x^3 + 3x^{-2}$$

$$\frac{dy}{dx} = 6x^2 - 6x^{-3}$$

$$b) \quad \int y \, dx = \frac{2x^4}{4} + \frac{3x^{-1}}{-1} + C$$

$$= \frac{1}{2}x^4 - 3x^{-1} + C$$

$$4/ \quad f'(x) = 3x^2 - 3x^{1/2} - 7 \quad (4, 22)$$

$$f(x) = \frac{3x^3}{3} - \frac{3x^{3/2}}{3/2} - 7x + C$$
$$= x^3 - 2x^{3/2} - 7x + C$$

$$22 = (4)^3 - 2(4)^{3/2} - 7(4) + C$$

$$22 = 64 - 16 - 28 + C$$

$$C = 2$$

$$\therefore f(x) = x^3 - 2x^{3/2} - 7x + 2$$

$$5/ \quad \frac{2x^2 - x^{3/2}}{x^{1/2}}$$

$$2x^{3/2} - x \quad p = 3/2 \quad q = 1$$

$$6/ \quad y = 5x^4 - 3 + 2x^{3/2} - x$$

$$\frac{dy}{dx} = 20x^3 + 3x^{1/2} - 1$$

$$7/ \quad \int 3x^2 + 4x^5 - 7 \, dx$$

~~6x~~

$$\frac{3x^3}{3} + \frac{4x^6}{6} - 7x + C$$

$$x^3 + \frac{2}{3}x^6 - 7x + C$$

$$7/ \quad \int 2 + 5x^2 \, dx$$

$$2x + \frac{5x^3}{3} + C$$

$$8/ \quad f(x) = 3x + x^3$$

$$f'(x) = 3 + 3x^2$$

$$b/ \quad 3 + 3x^2 = 15$$

$$3x^2 = 12$$

$$x^2 = 4$$

$$x = 2$$

$$9/ \quad y = 4x^3 - 1 + 2x^{1/2}$$

$$\frac{dy}{dx} = 12x^2 + x^{-1/2}$$

$$10/ \quad \frac{dy}{dx} = \frac{(x^2 + 3)^2}{x^2}$$

$$= \frac{(x^2 + 3)(x^2 + 3)}{x^2}$$

$$= \frac{x^4 + 6x^2 + 9}{x^2}$$

$$= x^2 + 6 + 9x^{-2}$$

$$b/ \quad y = \frac{x^3}{3} + 6x + \frac{9x^{-1}}{-1} + C$$

$$y = \frac{1}{3}x^3 + 6x - 9x^{-1} + C$$

$$(3, 20) \quad 20 = \frac{1}{3}(3)^3 + 6(3) - \frac{9}{3} + C$$

$$20 = 9 + 18 - 3 + C$$

$$C = -4$$

$$y = \frac{1}{3}x^3 + 6x - 9x^{-1} - 4$$

$$11a/ \quad \frac{2x^{1/2} + 3}{x}$$

$$\frac{2x^{-1/2} + 3x^{-1}}{x}$$

$$b) \quad y = 5x - 7 + 2x^{-1/2} + 3x^{-1}$$

$$\frac{dy}{dx} = 5 - x^{-3/2} - 3x^{-2}$$

$$12) \quad y = 3x^2 + 4x^{1/2}$$

$$a) \quad \frac{dy}{dx} = 6x + 2x^{-1/2}$$

$$b) \quad \frac{d^2y}{dx^2} = 6 - x^{-3/2}$$

$$c) \quad \int y \, dx = \frac{3x^3}{3} + \frac{4x^{3/2}}{3/2} + C$$

$$= x^3 + \frac{8}{3}x^{3/2} + C$$

$$13a) \quad (4 + 3x^{1/2})(4 + 3x^{1/2})$$

$$16 + 12x^{1/2} + 12x^{1/2} + 9x$$

$$16 + 24x^{1/2} + 9x$$

$$b) \quad 16x + \frac{24x^{3/2}}{3/2} + \frac{9x^2}{2} + C$$

$$16x + 16x^{3/2} + \frac{9}{2}x^2 + C$$

$$14a) \quad y = x^4 + 6x^{1/2}$$

$$\frac{dy}{dx} = 4x^3 + 3x^{-1/2}$$

$$b) \quad \frac{(x+4)(x+4)}{x} = \frac{x^2 + 8x + 16}{x} = x + 8 + 16x^{-1}$$

$$\frac{dy}{dx} = 1 - 16x^{-2}$$

$$c) \quad \int (1 + 3x^{1/2} - x^{-2}) \, dx$$
~~$$x + \frac{3x^{3/2}}{3/2} - \frac{x^{-1}}{-1} + C$$

$$x + 2x^{3/2} + x^{-1} + C$$~~

15.

$$y = 5x^3 + 7x + 3$$

$$a) \frac{dy}{dx} = 15x^2 + 7$$

$$b) \frac{d^2y}{dx^2} = 30x$$

$$ii) \int 1 + 3x^{1/2} - x^{-2} dx$$

$$x + \frac{3x^{3/2}}{3/2} - \frac{x^{-1}}{-1} + C$$

$$x + 2x^{3/2} + x^{-1} + C$$

16a)

$$\frac{(3 - x^{1/2})(3 - x^{1/2})}{x^{1/2}}$$

$$\frac{9 - 6x^{1/2} + x}{x^{1/2}}$$

$$9x^{-1/2} - 6 + x^{1/2}$$

b)

$$\frac{dy}{dx} = 9x^{-1/2} - 6 + x^{1/2} \quad (1, 2/3)$$

$$y = \frac{9x^{1/2}}{1/2} - 6x + \frac{x^{3/2}}{3/2} + C \quad y = 18x^{1/2} - 6x + \frac{2}{3}x^{3/2} + C$$

$$2/3 = \frac{9(1)^{1/2}}{1/2} - 6(1) + \frac{(1)^{3/2}}{3/2} + C$$

$$\frac{2}{3} = 18 - 6 + \frac{2}{3} + C$$

$$C = -12$$

$$y = 18x^{1/2} - 6x + \frac{2}{3}x^{3/2} - 12$$

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$$y = 2x^2 - 6x^{-3}$$

$$\frac{dy}{dx} = 4x + 18x^{-4}$$

$$\int y dx = \frac{2x^3}{3} - \frac{6x^{-2}}{-2} + C = \frac{2}{3}x^3 + 3x^{-2} + C$$

18/

$$f'(x) = 3 + 5x^{3/2} + 2x^{-1/2}$$

(1, 6)

$$f(x) = 3x + \frac{5x^{5/2}}{5/2} + \frac{2x^{1/2}}{1/2} + C$$

$$f(x) = 3x + 2x^{5/2} + 4x^{1/2} + C$$

$$6 = 3(1) + 2(1)^{5/2} + 4(1)^{1/2} + C$$

$$C = -3$$

$$f(x) = 3x + 2x^{5/2} + 4x^{1/2} - 3$$

19/

$$y = 6x - 4x^{-2}$$

$$\frac{dy}{dx} = 6 + 8x^{-3}$$

$$\int y dx = \frac{6x^2}{2} - \frac{4x^{-1}}{-1} + C$$

$$= 3x^2 + 4x^{-1} + C$$