1) No real roots $\therefore b^2 - 4ac < 0$

$$x^2 + kx + 2 = 0$$
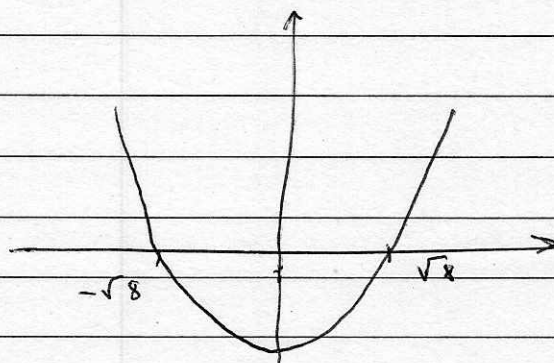
$a = 1 \quad b = k \quad c = 2$

$$k^2 - 4(1)(2) < 0$$
$$k^2 - 8 < 0$$
$$k^2 < 8$$

$$k = \sqrt{8} \qquad k = -\sqrt{8}$$

$$-\sqrt{8} < k < \sqrt{8}$$

2) Equal roots $\therefore b^2 - 4ac = 0$

$$kx^2 + 5x + k = 0$$

$a = k \quad b = 5 \quad c = k$

$$(5)^2 - 4(k)(k) = 0$$
$$25 - 4k^2 = 0$$
$$25 = 4k^2$$
$$\frac{25}{4} = k^2$$
$$k = \pm \frac{5}{2}$$

k is a positive constant $\quad \therefore k = \frac{5}{2}$

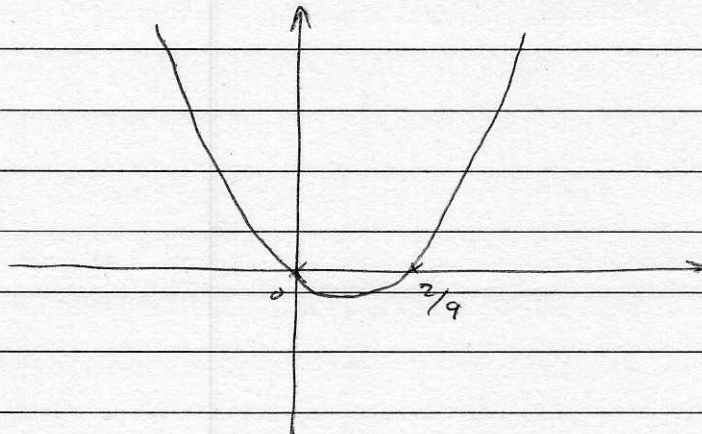3) $kx^2 + 6kx + 2 = 0$

$a = k \qquad b = 6k \qquad c = 2$

$b^2 - 4ac < 0$

$(6k)^2 - 4(k)(2) < 0$

$36k^2 - 8k < 0$

$4k(9k - 2) < 0$

$k = 0 \qquad k = \dfrac{2}{9}$

$0 < k < \dfrac{2}{9}$

4) $\quad (k+5)x^2 + 4x + (k+2) = 0$

$a = (k+5) \qquad b = 4 \qquad c = k+2$

$$b^2 - 4ac > 0$$

$$(4)^2 - 4(k+5)(k+2) > 0$$

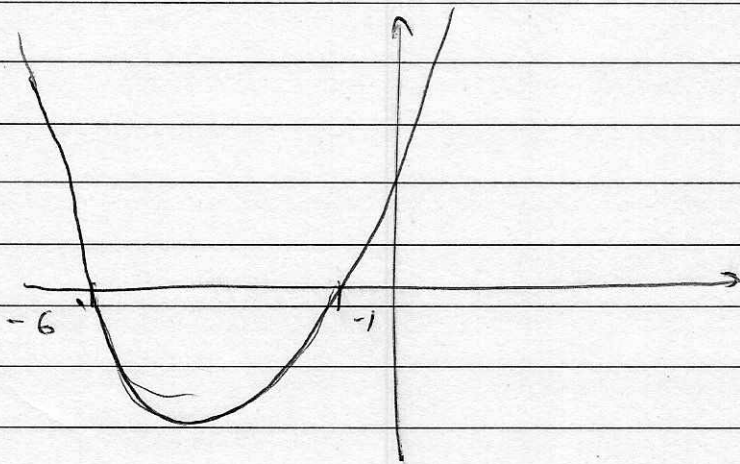$$16 - 4(k^2 + 2k + 5k + 10) > 0$$

$$16 - 4(k^2 + 7k + 10) > 0$$

$$16 - 4k^2 - 28k - 40 > 0$$

$$0 > 4k^2 + 28k + 24$$

$$0 > k^2 + 7k + 6$$

$$0 > (k+6)(k+1)$$

$$k = -6 \qquad k = -1$$

$$-6 < k < -1$$

5) $x^2 + (n+1)x + (3-3n) = 0$

$a = 1 \quad b = n+1 \quad c = 3-3n$

$b^2 - 4ac > 0$
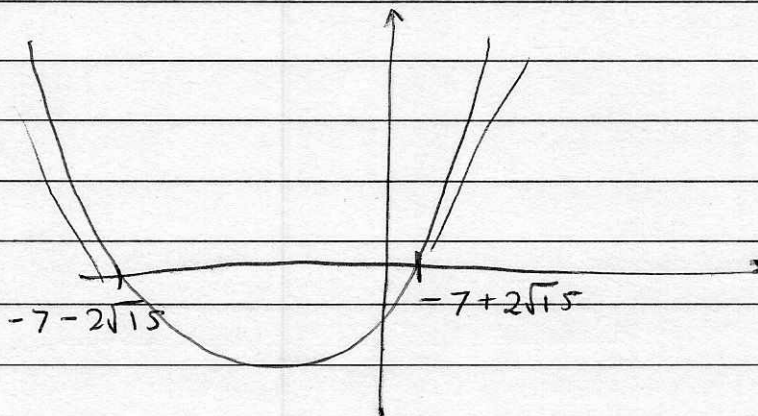
$(n+1)^2 - 4(1)(3-3n) > 0$

$(n+1)(n+1) - 4(3-3n) > 0$

$n^2 + n + n + 1 - 12 + 12n > 0$

$n^2 + 14n - 11 > 0$

$a = 1 \quad b = 14 \quad c = -11$

$n = \dfrac{-(14) \pm \sqrt{(14)^2 - 4(1)(-11)}}{2(1)}$

$n = -7 + 2\sqrt{15}$ \qquad or \qquad $n = -7 - 2\sqrt{15}$

$\underline{n < -7 - 2\sqrt{15}} \qquad \underline{n > -7 + 2\sqrt{15}}$

6) $$x^2 + (2k-3)x + (k+3) = 0$$

$$a = 1 \quad b = 2k-3 \quad c = k+3$$

$$b^2 - 4ac < 0$$

$$(2k-3)^2 - 4(1)(k+3) < 0$$
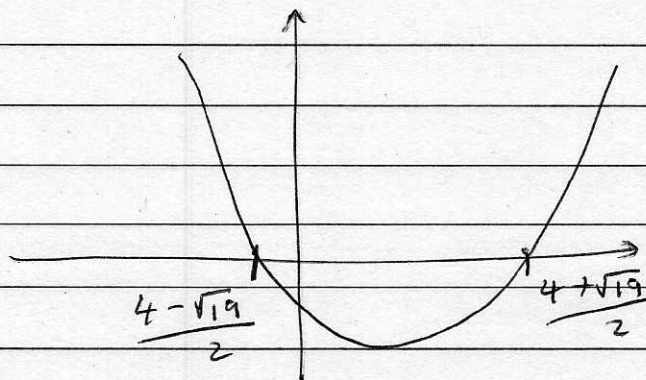
$$(2k-3)(2k-3) - 4(k+3) < 0$$

$$4k^2 - 6k - 6k + 9 - 4k - 12 < 0$$

$$4k^2 - 16k - 3 < 0$$

$$a = 4 \quad b = -16 \quad c = -3$$

$$k = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(4)(-3)}}{2(4)}$$

$$\cancel{k =} \quad k = \frac{4 + \sqrt{19}}{2} \qquad k = \frac{4 - \sqrt{19}}{2}$$

$$\frac{4 - \sqrt{19}}{2} < k < \frac{4 + \sqrt{19}}{2}$$

7a)   does not intersect   → no solutions.

$$y = px^2 - 4px - 5p \qquad y = 2x - 12$$

$$px^2 - 4px - 5p = 2x - 12$$

$$px^2 - 4px - 2x - 5p + 12 = 0$$

$$px^2 - (4p + 2)x + (12 - 5p) = 0$$

$$a = p \qquad b = -(4p + 2) \qquad c = (12 - 5p)$$

$$b^2 - 4ac \qquad < 0$$

$$(4p + 2)(4p + 2) - 4(p)(12 - 5p) < 0$$

$$16p^2 + 8p + 8p + 4 - 4p(12 - 5p) < 0$$
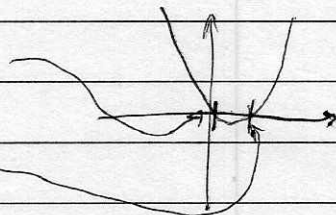
$$16p^2 + 16p + 4 - 48p + 20p^2 < 0$$

$$36p^2 - 32p + 4 \qquad\qquad < 0$$

$$9p^2 - 8p + 1 \qquad\qquad < 0$$

b)   ~~f⁴~~ $p = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(9)(1)}}{2(9)}$

$$p = \frac{4 + \sqrt{7}}{9} \qquad\qquad p = \frac{4 - \sqrt{7}}{9}$$

$$\frac{4 - \sqrt{7}}{9} < p < \frac{4 + \sqrt{7}}{9}$$

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tangent. → 1 solution.

$$y = (mx - 2) \qquad x^2 + 6x + y^2 - 8y + 5 = 0$$

$$x^2 + 6x + (mx-2)^2 - 8(mx-2) + 5 = 0$$

$$x^2 + 6x + (mx-2)(mx-2) - 8mx + 16 + 5 = 0$$

$$x^2 + 6x + m^2x^2 - 2mx - 2mx + 4 - 8mx + 21 = 0$$

$$(m^2 + 1)x^2 + (6 - 12m)x + 25 = 0$$

$$a = m^2 + 1 \qquad b = 6 - 12m \qquad c = 25$$

$$(6 - 12m)^2 - 4(m^2 + 1)(25) = 0$$

$$(6 - 12m)(6 - 12m) - 100(m^2 + 1) = 0$$

$$36 - 72m - 72m + 144m^2 - 100m^2 - 100 = 0$$

$$44m^2 - 144m - 64 = 0$$

$$11m^2 - 36m - 16 = 0$$

$$m = \frac{-(-36) \pm \sqrt{(-36)^2 - 4(11)(-16)}}{2(11)}$$

$$m = \frac{18 + 10\sqrt{5}}{11} \qquad m = \frac{18 - 10\sqrt{5}}{11}$$

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$$y = mx + 2 \qquad (x-5)^2 + (y+1)^2 = 15$$

$$(x-5)^2 + (mx + 2 + 1)^2 = 15$$

$$(x-5)(x-5) + (mx+3)(nx+3) = 15$$

$$x^2 - 5x - 5x + 25 + m^2 x^2 + 3mx + 3mx + 9 = 15$$

$$x^2 + m^2 x^2 - 10x + 6mx + 19 = 0$$

$$(1 + m^2)x^2 + (6m - 10)x + 19 = 0$$

$$1 \text{ solution} \therefore b^2 - 4ac = 0$$

$$(6m - 10)^2 - 4(19)(1 + m^2) = 0$$

$$(6m - 10)(6m - 10) - 76(1 + m^2) = 0$$

$$36m^2 - 60m - 60m + 100 - 76 - 76m^2 = 0$$

$$-40m^2 - 120m + 24 = 0$$

$$5m^2 + 15m - 3 = 0$$

$$m = \frac{-(15) \pm \sqrt{(15)^2 - 4(5)(-3)}}{2(5)}$$

$$m = \frac{-15 + \sqrt{285}}{10} \qquad m = \frac{-15 - \sqrt{285}}{10}$$