1a) $y = 2x^3 + 5x^2 - 7x + 10$

$\frac{dy}{dx} = 6x^2 + 10x - 7$

b) when $x = 2$ $\frac{dy}{dx} = 6(2)^2 + 10(2) - 7$

$= \underline{\underline{37}}$

2a) $y = 3x + \frac{1}{x}$

$y = 3x + x^{-1}$

$\frac{dy}{dx} = 3 - x^{-2}$

b) $3 - x^{-2} = 0$

$3 = x^{-2}$

$3 = \frac{1}{x^2}$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}}$

$x = \frac{\sqrt{3}}{3}$ or $x = -\frac{\sqrt{3}}{3}$

3) $f(x) = 3x^{\frac{3}{2}} + \frac{3}{x^2} - 6x$

$f(x) = 3x^{3/2} + 3x^{-2} - 6x$

$f'(x) = \frac{9}{2}x^{\frac{1}{2}} - 6x^{-3} - 6$

4a) $y = 4\sqrt{x} + \frac{1}{2x} + 10$

$y = 4x^{\frac{1}{2}} + \frac{1}{2}x^{-1} + 10$

$\frac{dy}{dx} = 2x^{-\frac{1}{2}} - \frac{1}{2}x^{-2}$

b/ $\frac{d^2y}{dx^2} = -x^{-3/2} + \cancel{2}x^{-3}$

5a/ $y = \frac{2x^2 - 5x + 3}{x}$

$\cancel{dy}\ y = 2x - 5 + 3x^{-1}$

$\frac{dy}{dx} = 2 - 3x^{-2}$

b/ when $x = 3$

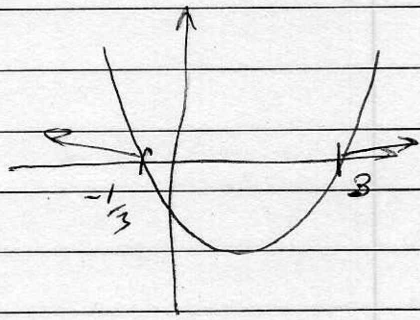
$\frac{dy}{dx} = 2 - 3(3)^{-2}$

$= \frac{5}{3}$

6a) $y = x^3 - 4x^2 - 3x + 9$

$\frac{dy}{dx} = 3x^2 - 8x - 3$

b/ Increasing when $\frac{dy}{dx} > 0$



$3x^2 - 8x - 3 > 0$

$(3x + 1)(x - 3) > 0$

$x = -\frac{1}{3} \quad x = 3$

$\underline{\underline{x < -\frac{1}{3}}} \quad \underline{\underline{x > 3}}$

7/ $y = 2x^3 + 9x^2 - 24x + 13$

$\frac{dy}{dx} = 6x^2 + 18x - 24$

Maximum (or minimum) when $\frac{dy}{dx} = 0$

$$6x^2 + 18x - 24 = 0$$
$$x^2 + 3x - 4 = 0$$
$$(x+4)(x-1) = 0$$
$$x = -4 \qquad x = 1$$

stationary points when $x = -4$ and $x = 1$

$\frac{d^2y}{dx^2} = 12x + 18$

when $x = -4$ $\quad \frac{d^2y}{dx^2} = 12(-4) + 18 = -30$ $\quad$ -ve so: MAX

$x = 1$ $\quad \frac{d^2y}{dx^2} = 12(1) + 18 = 30$ $\quad$ +ve so: MIN

Maximum when $x = -4$

$y = 2(-4)^3 + 9(-4)^2 - 24(-4) + 13$

$= 125$

$\underline{\underline{(-4, 125)}}$

8) $y = 2x^3 + 5x^2 - 7x + 10$

$\frac{dy}{dx} = 6x^2 + 10x - 7$

when $x = 1$:

$\frac{dy}{dx} = 6(1)^2 + 10(1) - 7$

$= 9$

$m = 9$

when $x = 1$:

$y = 2(1)^3 + 5(1)^2 - 7(1) + 10$

$y = 10$

$y = 9x + c$ $\quad (1, 10)$

$10 = 9(1) + c$

$c = 1$

$\underline{\underline{y = 9x + 1}}$

9/ $f(x) = 2x^3 + x^2 - 18x + 2$

$f'(x) = 6x^2 + 2x - 18$

A and B are where $f'(x) = 2$

$6x^2 + 2x - 18 = 2$

$6x^2 + 2x - 20 = 0$

$3x^2 + x - 10 = 0$

$(3x - 5)(x + 2) = 0$

$x = \frac{5}{3}$ $\quad x = -2$

$y = 2\left(\frac{5}{3}\right)^3 + \left(\frac{5}{3}\right)^2 - 18\left(\frac{5}{3}\right) + 2$ $\quad y = 2(-2)^3 + (-2)^2 - 18(-2) + 2$

$= -\frac{431}{27}$ $\quad = 26$

$\left(\frac{5}{3}, -\frac{431}{27}\right)$ and $(-2, 26)$

10/ $y = \dfrac{(4x-1)(x+2)}{2x}$

$$y = \frac{4x^2 + 8x - x - 2}{2x}$$

$$y = \frac{4x^2 + 7x - 2}{2x}$$

$$y = 2x + \frac{7}{2} - x^{-1}$$

$$\frac{dy}{dx} = 2 + x^{-2}$$

when $x = -2$

$$\frac{dy}{dx} = 2 + (-2)^{-2}$$

$$= \frac{9}{4}$$

$\therefore$ gradient of normal $= -4/9$

$y = -\frac{4}{9}x + c$ when $x = -2$

$y = 2(-2) + \frac{7}{2} - (-2)^{-1}$

$= 0$

$$0 = -\frac{4}{9}(-2) + c$$

$$0 = \frac{8}{9} + c$$

$$c = -\frac{8}{9}$$

$$y = -\frac{4}{9}x - \frac{8}{9}$$

$$9y = -4x - 8$$

$$\underline{4x + 9y + 8 = 0}$$

11) $C = \frac{4500}{v} + v + 10$

$C = 4500v^{-1} + v + 10$

$\frac{dC}{dv} = -4500v^{-2} + 1$

Minimum where $\frac{dC}{dv} = 0$

$-\frac{4500}{v^2} + 1 = 0$

$1 = \frac{4500}{v^2}$

$v^2 = 4500$

$v = 67.1$ mph $\quad (30\sqrt{5})$

b) $\frac{d^2C}{dv^2} = 9000v^{-3}$

$= 9000(30\sqrt{5})^{-3}$

$=$ 0.0298142…

positive $\therefore$ it is a minimum.

c) $C = \frac{4500}{30\sqrt{5}} + 30\sqrt{5} + 10$

$= £144.16$

12/ surface area $= 2\pi r^2 + 2\pi rh$

$$500 = 2\pi r^2 + 2\pi rh$$

volume $= \pi r^2 h$

$$2\pi rh = 500 - 2\pi r^2$$

$$h = \frac{500 - 2\pi r^2}{2\pi r}$$

$$\text{volume} = \pi r^2 \left(\frac{500 - 2\pi r^2}{2\pi r} \right)$$

$$V = \frac{\cancel{\pi} r^{\cancel{2}}(500 - 2\pi r^2)}{2\cancel{\pi}\cancel{r}}$$

$$V = \frac{500r - 2\pi r^3}{2}$$

$$V = \underline{\underline{250r - \pi r^3}}$$

b/ $$\frac{dv}{dr} = 250 - 3\pi r^2$$

Max where $\frac{dv}{dr} = 0$ $\quad 250 - 3\pi r^2 = 0$

$$250 = 3\pi r^2$$

$$\frac{250}{3\pi} = r^2$$

$$r = \sqrt{\frac{250}{3\pi}}$$

$$= 5.15 \text{ cm}$$

$$\max V = 250(5.15) - \pi(5.15)^3$$

$$= \underline{\underline{858 \text{ cm}^3}}$$

c/ $$\frac{d^2v}{dr^2} = -6\pi r$$

$$= -6\pi(5.15)$$

$$= -97.1$$

negative $\therefore$ it is a maximum.

13) $y = 4x^3 + 15x^2 - 18x + 5$

$\frac{dy}{dx} = 12x^2 + 30x - 18$

stationary points are where $\frac{dy}{dx} = 0$

$$12x^2 + 30x - 18 = 0$$
$$4x^2 + 10x - 6 = 0$$
$$2x^2 + 5x - 3 = 0$$
$$(2x - 1)(x + 3) = 0$$
$$x = \frac{1}{2} \qquad x = -3$$

$$\frac{d^2y}{dx^2} = 24x + 30$$

when $x = \frac{1}{2}$ $\frac{d^2y}{dx^2} = 42$ positive $\therefore$ Minimum

when $x = -3$ $\frac{d^2y}{dx^2} = -42$ negative $\therefore$ Maximum

when $x = \frac{1}{2}$ $y = 4(\frac{1}{2})^3 + 15(\frac{1}{2})^2 - 18(\frac{1}{2}) + 5$
$= \frac{1}{4}$

when $x = -3$ $y = 4(-3)^3 + 15(-3)^2 - 18(-3) + 5$
$= 86$

Minimum at $(\frac{1}{2}, \frac{1}{4})$

Maximum at $(-3, 86)$