1) $\int \frac{2x}{x^2+4}\,dx$ $\qquad u = x^2 + 4$

$\frac{du}{dx} = 2x$

$\int \frac{2x}{x^2+4} \frac{dx}{du}\,du$ $\qquad \frac{dx}{du} = \frac{1}{2x}$

$\int \frac{2x}{u} \cdot \frac{1}{2x}\,du$

$\int \frac{1}{u}\,du$

$\ln u + c$

$\underline{\underline{\ln(x^2+4) + c}}$

2) $\int \sin^3 x \cos x\,dx$ $\qquad u = \sin x$

$\frac{du}{dx} = \cos x$

$\int u^3 \cos x \frac{dx}{du}\,du$ $\qquad \frac{dx}{du} = \frac{1}{\cos x}$

$\int u^3 \cancel{\cos x} \frac{1}{\cos x}\,du$

$\int u^3\,du$

$\frac{1}{4}u^4 + C$

$\underline{\underline{\frac{1}{4}\sin^4 x + C}}$

3/ $\int 2x(x^2+2)^2 \, dx$

$u = x^2 + 2$

$\frac{du}{dx} = 2x$

$\frac{dx}{du} = \frac{1}{2x}$

$\int 2x(u)^2 \frac{dx}{du} \, du$

$\int 2x \, u^2 \frac{1}{2x} \, du$

$\int u^2 \, du$

$\frac{1}{3} u^3 + c$

$\underline{\frac{1}{3}(x^2+2)^3 + c}$

4/

$$\int \frac{e^{3x}}{1+e^x}\,dx \qquad u = 1 + e^x$$

$$\frac{du}{dx} = e^x$$

$$\int \frac{(e^x)^3}{u}\frac{dx}{du}\,du \qquad \frac{dx}{du} = \frac{1}{e^x}$$

$$\int \frac{(e^x)^3}{u}\cdot\frac{1}{e^x}\,du$$

$$\int \frac{(e^x)^2}{u}\,du \qquad e^x = u - 1$$

$$\int \frac{(u-1)^2}{u}\,du$$

$$\int \frac{u^2 - 2u + 1}{u}\,du$$

$$\int u - 2 + \frac{1}{u}\,du$$

$$\frac{1}{2}u^2 - 2u + \ln u + c$$

$$\frac{1}{2}(1+e^x)^2 - 2(1+e^x) + \ln(1+e^x) + c$$

$$\frac{1}{2}(1 + 2e^x + e^{2x}) - 2 - 2e^x + \ln(1+e^x) + c$$

$$\frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$$

$$-\frac{3}{2} - e^x + \frac{1}{2}e^{2x} + \ln(1+e^x) + c$$

5)

$$\int_2^3 2x^2(x^3-4)^2\,dx$$

$$u = x^3 - 4$$

$$\frac{du}{dx} = 3x^2$$

$$\int_4^{23} 2x^2(x^3-4)^2\,\frac{dx}{du}\,du$$

$$\frac{dx}{du} = \frac{1}{3x^2}$$

$$\int_4^{23} 2x^2 \cdot u^2 \cdot \frac{1}{3x^2}\,du$$

$$u = (3)^3 - 4$$
$$= 23$$
$$u = (2)^3 - 4$$
$$= 4$$

$$\int_4^{23} \frac{2}{3}u^2\,du$$

$$\left[\frac{2}{9}u^3\right]_4^{23}$$

$$\frac{2}{9}(23)^3 - \frac{2}{9}(4)^3$$

$$= \underline{\underline{2689.\dot{5}\ \text{units}^2}}$$

6)

$$\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} \, dx$$

$x = \sin u$

$\frac{dx}{du} = \cos u$

$$\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-\sin^2 u}} \, dx$$

~~$x = \sin(\frac{1}{2})$~~

$\frac{1}{2} = \sin u$

$u = \frac{1}{6}\pi$

$$\int_0^{\frac{1}{6}\pi} \frac{1}{\sqrt{1-\sin^2 u}} \frac{dx}{du} \, du$$

$0 = \sin u$

$u = 0$

$$\int_0^{\frac{1}{6}\pi} \frac{1}{\sqrt{\cos^2 u}} \cdot \cos u \, du$$

$$\int_0^{\frac{1}{6}\pi} \frac{\cos u}{\cos u} \, du$$

$$\int_0^{\frac{1}{6}\pi} 1 \, du$$

$$\cancel{\int_0^{\frac{1}{6}\pi}} \left[u \right]_0^{\frac{1}{6}\pi}$$

$$\frac{1}{6}\pi - 0$$

$$\frac{1}{6}\pi$$

7/

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos x} \, dx$$

$u = 1 + \cos x$

$\frac{du}{dx} = -\sin x$

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{u} \, dx$$

$\frac{dx}{du} = \frac{1}{-\sin x}$

$u = 1 + \cos(\frac{\pi}{2})$

$= 1$

$$\int_2^1 \frac{\sin x}{u} \frac{dx}{du} \, du$$

$u = 1 + \cos 0$

$= 2$

$$\int_2^1 \frac{\sin x}{u} \cdot \frac{1}{-\sin x} \, du$$

$$\int_2^1 -\frac{1}{u} \, du$$

$$\int_1^2 u^{-1} \, du$$

$$\Big[\ln |u| \Big]_1^2$$

$$\ln 2 - \ln 1$$

$$\underline{\underline{\ln 2}}$$