1a)

$$\tan x = \frac{\sin x}{\cos x}$$

$u = \sin x \qquad v = \cos x$

$\frac{du}{dx} = \cos x \qquad \frac{dv}{dx} = -\sin x$

$$\frac{d(\tan x)}{dx} = \frac{\cos x \cos x - (\sin x)(-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \underline{\underline{\sec^2 x}}$$

b)

$$\sec x = \frac{1}{\cos x}$$

$u = 1 \qquad v = \cos x$

$\frac{du}{dx} = 0 \qquad \frac{dv}{dx} = -\sin x$

$$\frac{d(\sec x)}{dx} = \frac{0 - (-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$= \tan x \cdot \sec x$$

$$= \underline{\underline{\sec x \tan x}}$$

c/ $\cot x = \dfrac{\cos x}{\sin x}$

$u = \cos x \qquad v = \sin x$

$\dfrac{du}{dx} = -\sin x \qquad \dfrac{dv}{dx} = \cos x$

$$\frac{d(\cot x)}{dx} = \frac{-\sin x \sin x - \cos x \cos x}{(\sin x)^2}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x}$$

$$= \underline{\underline{-\operatorname{cosec}^2 x}}$$

d/ $\operatorname{cosec} x = \dfrac{1}{\sin x}$

$u = 1 \qquad v = \sin x$

$\dfrac{du}{dx} = 0 \qquad \dfrac{dv}{dx} = \cos x$

$$\frac{d(\operatorname{cosec} x)}{dx} = \frac{0 - \cos x}{\sin^2 x}$$

$$= \frac{-\cos x}{\sin^2 x}$$

$$= -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}$$

$$= \underline{\underline{-\operatorname{cosec} x \cot x}}$$

2a/

$u = x^2 \qquad v = \cos 2x$

$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = -2 \sin 2x$

$\frac{d(x^2 \cos 2x)}{dx} = 2x \cos 2x - 2x^2 \sin 2x$

b/ $\frac{d(3 \sin(2x+1))}{dx} = 6 \cos(2x+1)$

3a/ $y = e^{3x}(\cos 2x + \sin x)$

$u = e^{3x} \qquad v = \cos 2x + \sin x$

$\frac{du}{dx} = 3e^{3x} \qquad \frac{dv}{dx} = -2 \sin 2x + \cos x$

$\frac{dy}{dx} = 3e^{3x}(\cos 2x + \sin x) + e^{3x}(-2 \sin 2x + \cos x)$

b/ $y = \ln \sin x$

$\frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x$

$= \frac{\cos x}{\sin x}$

4a/ $x = 2 \tan y$

$$\frac{dx}{dy} = 2 \sec^2 y$$

b/ $$\frac{dy}{dx} = \frac{1}{2\sec^2 y}$$ $[1 + \tan^2 y = \sec^2 y]$

$$= \frac{1}{2(1+\tan^2 y)}$$

$$= \frac{1}{2 + 2\tan^2 y}$$

$\left[\begin{array}{l} \frac{x}{2} = \tan y \\ \frac{x^2}{4} = \tan^2 y \end{array}\right]$

$$= \frac{1}{2 + 2\left(\frac{x^2}{4}\right)}$$

$$= \frac{1}{2 + \frac{\cancel{2}x^2}{\cancel{4}2}}$$

$$= \frac{2}{4 + x^2}$$

5/ $y = \operatorname{cosec} x + \cos 2x$

$$\frac{dy}{dx} = -\operatorname{cosec} x \cot x - 2\sin 2x$$

b/ $$\frac{dy}{dx} = -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} - 2\sin 2x$$

when $x = \frac{\pi}{4}$ $\quad \frac{dy}{dx} = -2 - \sqrt{2}$

$$y = \frac{1}{\sin x} + \cos 2x$$

$$= \sqrt{2}$$

$y = (-2 - \sqrt{2})x + c \qquad \left(\frac{\pi}{4}, \sqrt{2}\right)$

$\sqrt{2} = (-2 - \sqrt{2}) \cdot \frac{\pi}{4} + c$

$\sqrt{2} = \frac{-2\pi}{4} - \frac{\pi\sqrt{2}}{4} + c$

$c = \sqrt{2} + \frac{\pi}{2} + \frac{\pi\sqrt{2}}{4}$

$y = (-2 - \sqrt{2})x + \sqrt{2} + \frac{\pi}{2} + \frac{\pi\sqrt{2}}{4}$

6/

$$y = \sec 2x$$

$$\frac{dy}{dx} = 2\sec 2x \tan 2x$$

when $x = \frac{\pi}{6}$ $\quad \frac{dy}{dx} = 4\sqrt{3}$

$$y = 2$$

Normal gradient : $-\frac{1}{4\sqrt{3}} = -\frac{\sqrt{3}}{12}$

$$y = -\frac{\sqrt{3}}{12}x + c \qquad \left(\frac{\pi}{6}, 2\right)$$

$$2 = -\frac{\sqrt{3}}{12} \cdot \frac{\pi}{6} + c$$

$$2 = -\frac{\pi\sqrt{3}}{72} + c$$

$$c = 2 + \frac{\pi\sqrt{3}}{72}$$

$$y = -\frac{\sqrt{3}}{12}x + 2 + \frac{\pi\sqrt{3}}{72}$$