

1a/

$$f(x) = \ln x + e^x$$

$$f'(x) = \frac{1}{x} + e^x$$

b/

$$f''(x) = -x^{-2} + e^x$$

2a/

$$y = 2x^3 + e^{4x}$$

$$\frac{dy}{dx} = 6x^2 + 4e^{4x}$$

b/

$$y = (x^2 + 5)^3$$

$$\frac{dy}{dx} = 3(x^2 + 5)^2 \cdot 2x$$

$$= 6x(x^2 + 5)^2$$

3a/

$$y = x + 5(x^2 + 4)^{-1}$$

$$\frac{dy}{dx} = 1 - 5(x^2 + 4)^{-2} \cdot 2x$$

$$= 1 - 10x(x^2 + 4)^{-2}$$

b/

$$y = 8 + e^{x^2}$$

$$\frac{dy}{dx} = 2x e^{x^2}$$

4

$$y = 2 + \ln(3 - 2x)$$

$$\frac{dy}{dx} = \frac{1}{3 - 2x} \cdot -2$$

$$= \frac{-2}{3 - 2x}$$

$$\text{when } x = 1 \quad y = 2 + \ln(3 - 2)$$
$$= 2$$

$$\frac{dy}{dx} = \frac{-2}{3 - 2} = -2$$

$$y = -2x + c \quad (1, 2)$$

$$2 = -2 + c$$

$$c = 4$$

$$y = \underline{\underline{-2x + 4}}$$

5

$$y = 3(2x + 1)^{-1}$$

$$\frac{dy}{dx} = -3(2x + 1)^{-2} \cdot 2$$

$$= -6(2x + 1)^{-2}$$

when $x = 1$

$$\frac{dy}{dx} = -6(3)^{-2}$$

$$= -\frac{2}{3}$$

$$y = 3(3)^{-1}$$

$$= 1$$

perpendicular gradient = $\frac{3}{2}$ $(1, 1)$

$$y = \frac{3}{2}x + c$$

$$1 = \frac{3}{2} + c$$

$$c = -\frac{1}{2}$$

$$y = \frac{3}{2}x - \frac{1}{2}$$

b/

$$\frac{3}{2x+1} = \frac{3}{2}x - \frac{1}{2}$$

$$\frac{6}{2x+1} = 3x - 1$$

$$6 = (3x - 1)(2x + 1)$$

$$6 = 6x^2 + 3x - 2x - 1$$

$$0 = 6x^2 + x - 7$$

$$0 = (6x + 7)(x - 1)$$

$$x = -\frac{7}{6} \quad x = 1$$

$$y = \frac{3}{2}\left(-\frac{7}{6}\right) - \frac{1}{2}$$

$$= \frac{-21}{12} - \frac{1}{2}$$

$$= -\frac{9}{4}$$

$$\left(-\frac{7}{6}, -\frac{9}{4}\right)$$

6

$$y = 2(2x+1)^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = -(2x+1)^{-\frac{3}{2}} \cdot 2$$

$$= -2(2x+1)^{-\frac{3}{2}}$$

$$\text{when } x=4 \quad \frac{dy}{dx} = -2(9)^{-\frac{3}{2}}$$

$$= -\frac{2}{27}$$

$$y = 2(9)^{-\frac{1}{2}}$$

$$= \frac{2}{3}$$

$$y = -\frac{2}{27}x + c$$

$$\frac{2}{3} = -\frac{2}{27} \cdot 4 + c$$

$$\frac{2}{3} = -\frac{8}{27} + c$$

$$c = \frac{26}{27}$$

$$y = \underline{\underline{-\frac{2}{27}x + \frac{26}{27}}}$$

$$b/ \quad B : (0, \frac{26}{27})$$

$$\text{Crosses } x \text{ when } y=0 \quad 0 = \frac{-2}{27}x + \frac{26}{27}$$

$$2x = 26$$

$$x = 13$$

$$(13, 0)$$

$$\text{Area} = \frac{1}{2} (13) \left(\frac{26}{27} \right)$$

$$= \frac{169}{27} \text{ units}^2$$

