Write your name here

Surname	Other Names

Mathematics

Paper 1 (Non-Calculator)
Higher Tier

Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – there may be more space than you need.
- **Calculators may not be used**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Higher Tier Formulae Sheet

Perimeter, area and volume

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

$$\text{Area of a trapezium} = \frac{1}{2}(a + b)\, h$$

Volume of a prism = area of cross section × length

Where r is the radius and d is the diameter:

$$\text{Circumference of a circle} = 2\pi r = \pi d$$

$$\text{Area of a circle} = \pi r^2$$

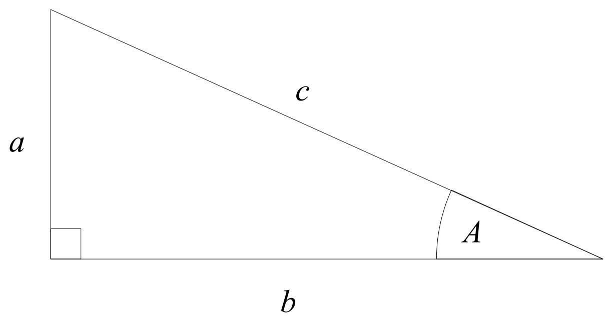
Quadratic formula

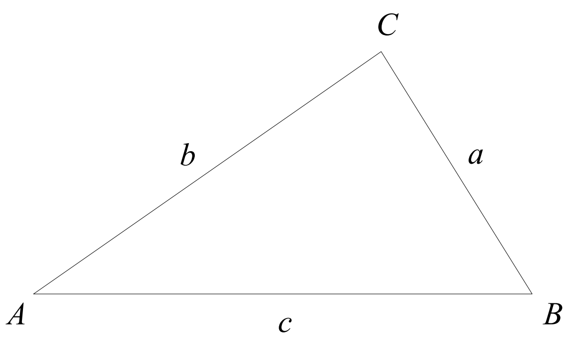
The solution of $ax^2 + bx + c = 0$

where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagoras' Theorem and Trigonometry





In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$

In any triangle ABC where a, b and c are the length of the sides:

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2} ab \sin C$

Compound Interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

$$\text{Total accrued} = P\left(1 + \frac{r}{100}\right)^n$$

Probability

Where P (A) is the probability of outcome A and P (B) is the probability of outcome B:

P (A or B) = P (A) + P (B) – P (A and B)

P (A and B) = P (A given B) P(B)

END OF EXAM AID

1 Work out 5.92 ÷ 0.16

$$\frac{5.92}{0.16} = \frac{592}{16}$$

$$\begin{array}{r} 0\,3\,7 \\ 16 \overline{)\,5^{5}9^{11}2} \end{array}$$

16
32
48
64
80
96
112
128
144
160

37

(Total for Question 1 is 3 marks)

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2 Write 324 as a product of powers of its prime factors.

324
2 162
2 81
3 27
3 9
3 3

$2 \times 2 \times 3 \times 3 \times 3 \times 3$

$2^2 \times 3^4$

(Total for Question 2 is 3 marks)

3 (a) Work out $2\frac{2}{3} + 1\frac{3}{5}$

Give your answer as a mixed number.

$$\frac{5\times 8}{5\times 3} + \frac{8\times 3}{5\times 3}$$

$$\frac{40}{15} + \frac{24}{15} = \frac{64}{15} = 4\frac{4}{15}$$

$4\frac{4}{15}$ ....

(2)

(b) Work out $\frac{2}{3} \div \frac{3}{4}$

$$\frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$$

$\frac{8}{9}$ ....

(2)

(Total for Question 3 is 4 marks)

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4 Work out the value of $\frac{5^{-3} \times 5^{7}}{5}$

$$\frac{5^4}{5^1} = 5^3 = 125$$

125 ....

(Total for Question 4 is 2 marks)

5 Tracey writes down three numbers a, b and c.

$$a : b = 3 : 5 \quad \times 4$$
$$a : c = 4 : 7 \quad \times 3$$

(a) Find $a : b : c$

$a : b$ $\quad$ $a : c$

$12 : 20$ $\quad$ $12 : 21$

$12 : 20 : 21$

(2)

Jamie writes down three numbers d, e and f.

$$d = 2e$$
$$f = 3d$$

(b) Find $e : d : f$

Let $e = 1$

$d = 2(1) = 2$

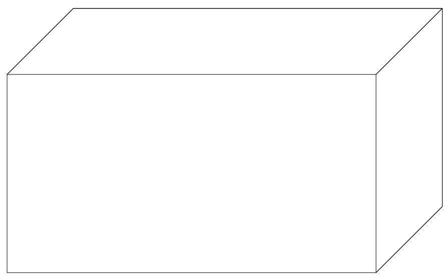
$f = 3(2) = 6$

$1 : 2 : 6$

(2)

(Total for Question 5 is 4 marks)

6 The diagram shows a cuboid.



3 m

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

The cuboid has height 3 m
The volume of the cuboid is 21 m^3
The pressure on the floor due to the cuboid is 25 newtons/m^2 (pressure)

Work out the force exerted by the cuboid on the floor.

$$\text{Area of base} = \frac{21}{3} = 7\,m^2 \qquad (\text{area})$$

$$25 = \frac{\text{force}}{7}$$

$$\text{force} = 25 \times 7$$

175 newtons

(Total for Question 6 is 3 marks)

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7 In a bag there are counters.
The counters are all either red or blue or yellow.

The number of red counters : The number of blue counters : The number of yellow counters = 4 : 5 : 8

The number of yellow counters is 24 more than the numbers of blue counters.

Work out the total number of counters in the bag.

Red	8	8	8	8					32
Blue	8	8	8	8	8				40
Yellow	8	8	8	8	8	8	8	8	64

24

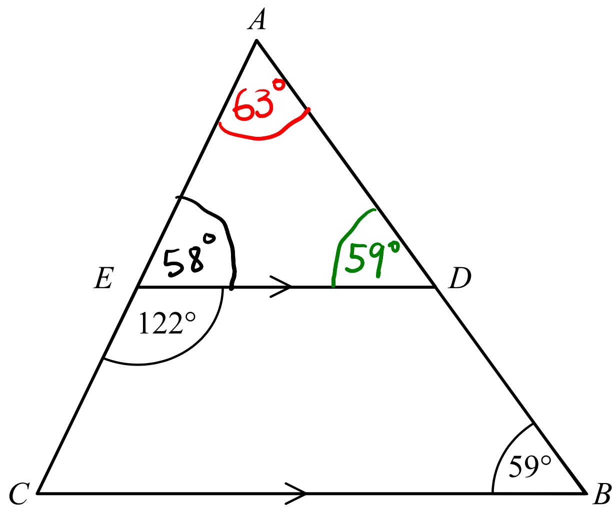
$24 \div 3 = 8$

$$\begin{array}{r} 32 \\ 40 \\ 64 \\ \hline 136 \end{array}$$

136

(Total for Question 7 is 3 marks)

8 ABC is a triangle.



AEC and ADB are straight lines.
ED is parallel to CB.
Angle $CED = 122°$
Angle $ABC = 59°$

Work out the size of angle CAB.
You must give a reason for each stage of your working.

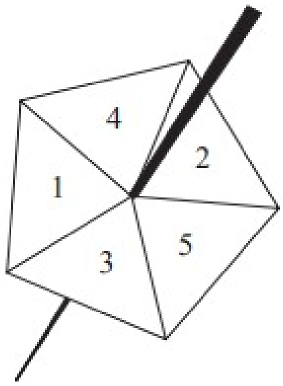
$AED = 58°$ Angles on a straight line add to 180°

$ADE = 59°$ Corresponding angles are equal

$CAB = \underline{\underline{63°}}$ Angles in a triangle add to 180°

(Total for Question 8 is 5 marks)

9 Roy spins a biased 5-sided spinner 48 times.



Here are his results.

Score	1	2	3	4	5
Frequency	9	10	6	7	16

Roy is now going to spin the spinner another two times.

Work out an estimate for the probability that he gets a score of 5 both times

$\frac{16}{48} = \frac{8}{24} = \frac{4}{12} = \frac{1}{3}$

$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$ .... $\frac{1}{9}$

(Total for Question 9 is 2 marks)

10 Solve the simultaneous equations

$$2x - y = 4 \quad \times 5$$
$$5x + 2y = 7 \quad \times 2$$

$$10x - 5y = 20$$
$$10x + 4y = 14$$

$$-9y = 6$$
$$y = \frac{6}{-9} = -\frac{2}{3}$$

$$2x - -\frac{2}{3} = 4$$
$$2x + \frac{2}{3} = 4$$
$$6x + 2 = 12$$
$$6x = 10$$
$$x = \frac{10}{6} = \frac{5}{3}$$

$x =$ $\frac{5}{3}$

$y =$ $-\frac{2}{3}$

(Total for Question 10 is 4 marks)

11 Work out the value of $8^{\frac{4}{3}} + \left(\frac{1}{3}\right)^{-3}$

$$8^{\frac{4}{3}} = 2^4 = 16$$

$$\left(\frac{1}{3}\right)^{-3} = 3^3 = 27$$

$$16 + 27 = \underline{\underline{43}}$$

43

(Total for Question 11 is 3 marks)

12 There are p counters in a bag.
60 of the counters are white.

Jill takes at random 50 counters from the bag.
8 of these 50 counters are white.

Work out an estimate for the value of p.

$$\times\frac{15}{2}$$

$$\frac{60}{p} = \frac{8}{50}$$

$$\times\frac{15}{2}$$

$$\frac{60}{8} = \frac{30}{4} = \frac{15}{2}$$

$$50 \times \frac{15}{2}$$

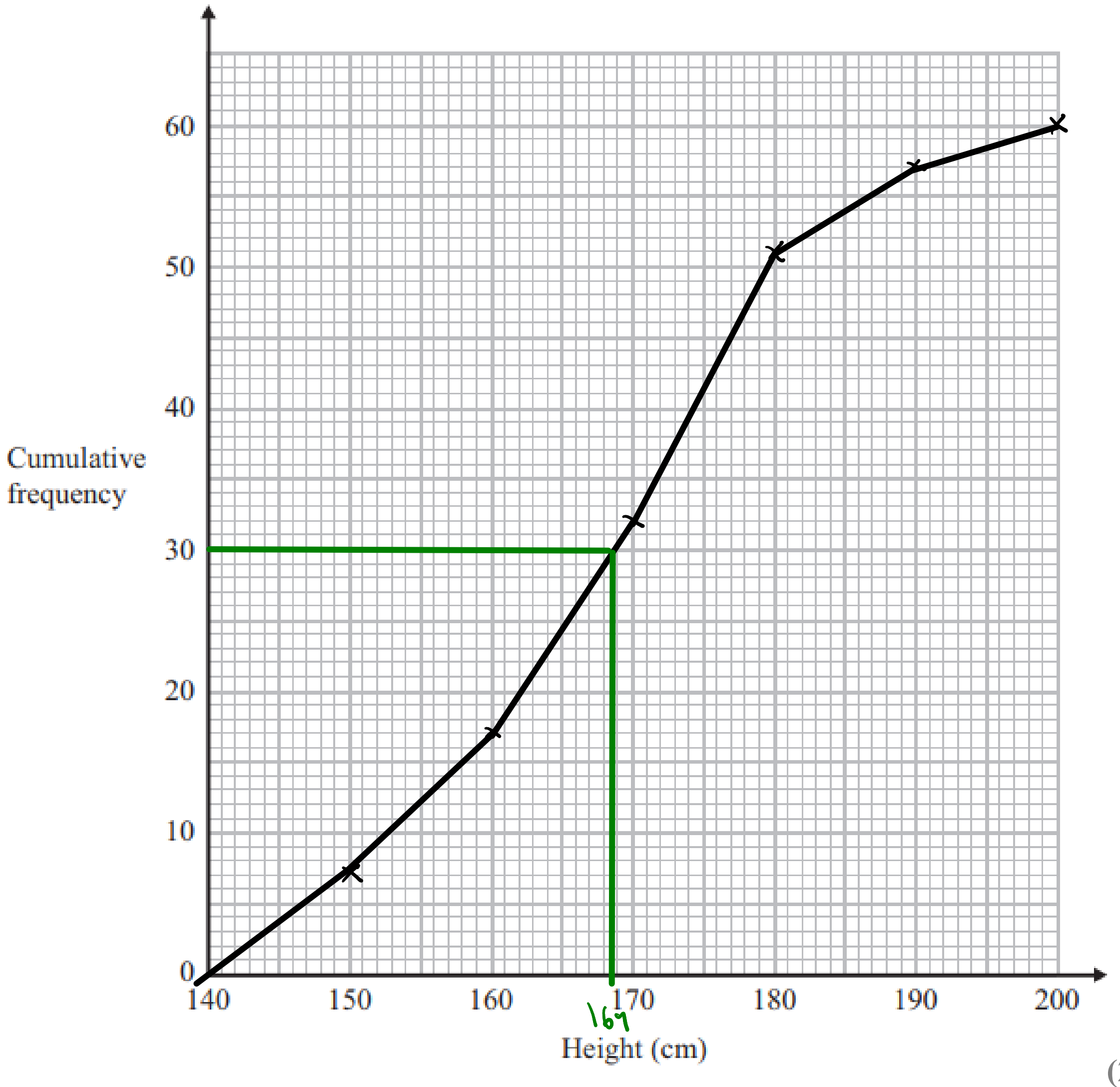
$$25 \times 15 = \underline{\underline{375}}$$

(Total for Question 12 is 2 marks)

13 The cumulative frequency table shows the height, in cm, of some tomato plants.

Height	**Cumulative Frequency**
$140 < h \leqslant 150$	7
$140 < h \leqslant 160$	17
$140 < h \leqslant 170$	32
$140 < h \leqslant 180$	51
$140 < h \leqslant 190$	57
$140 < h \leqslant 200$	60

(a) On the grid, plot a cumulative frequency graph for this information.



(2)

(b) Use the graph to find an estimate for the median height of the plants.

169 cm

(1)

(Total for Question 13 is 3 marks)

14 x is inversely proportional to y

Complete the table of values.

x	80	16	5	4
y	2	10	32	40

$x = \frac{k}{y}$

$80 = \frac{k}{2}$

$k = 160$

$x = \frac{160}{y}$

$xy = 160$

(Total for Question 14 is 3 marks)

15 The straight line **L** has equation $2y + 3x - 9 = 0$

Find an equation of the straight line perpendicular to **L** that passes through $(3, -7)$

$2y = -3x + 9$

$y = -\frac{3}{2}x + \frac{9}{2}$

$m = -\frac{3}{2}$ $\therefore$ perp. $m = \frac{2}{3}$

$y = \frac{2}{3}x + c$ $(3, -7)$ x y

$-7 = \frac{2}{3}(3) + c$

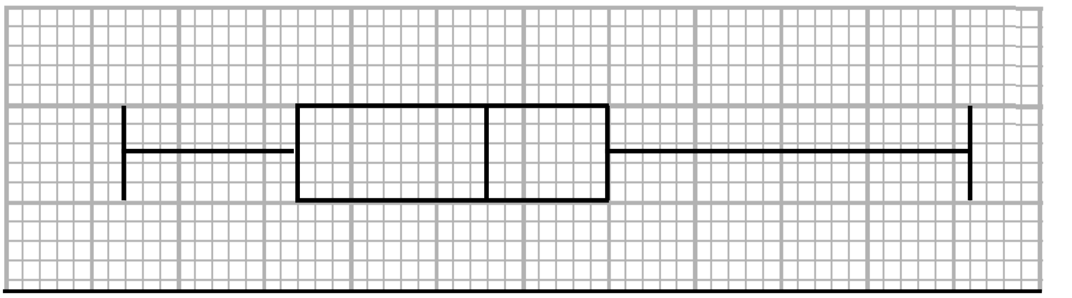
$-7 = 2 + c$

$c = -9$

$y = \frac{2}{3}x - 9$

(Total for Question 15 is 3 marks)

16 The box plot shows the number of visitors to a park on each of 180 days.



Work out an estimate for the number of days there were fewer than 350 visitors to the park.

upper quartile

$\frac{3}{4}$ of 180

$\frac{180}{4} = 45$ $45 \times 3 = 135$

135

(Total for Question 16 is 2 marks)

17 Prove that the difference between the squares of two consecutive odd numbers is a multiple of 8.

odd number = $2n+1$ next odd number = $2n+3$

$(2n+3)^2 - (2n+1)^2$

$(2n+3)(2n+3) - (2n+1)(2n+1)$

$(4n^2 + 6n + 6n + 9) - (4n^2 + 2n + 2n + 1)$

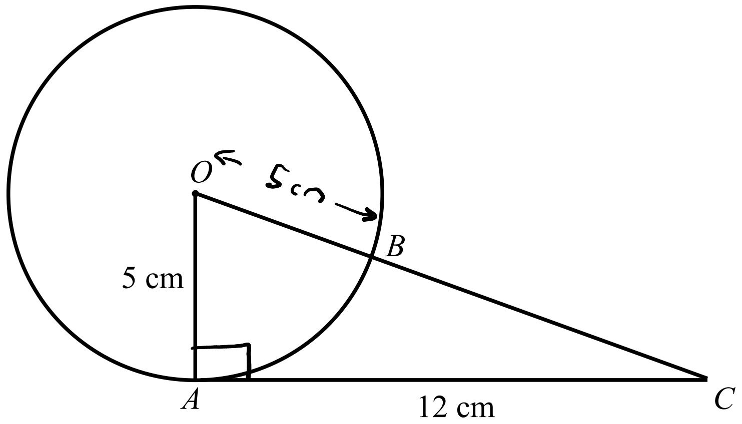
$4n^2 + 12n + 9 - 4n^2 - 4n - 1$

$8n + 8$

$8(n+1)$ ∴ always a multiple of 8

(Total for Question 17 is 4 marks)

18



A and *B* are points on the circumference of a circle, centre *O*.
AC is a tangent to the circle.
OBC is a straight line.

OA = 5 cm
AC = 12 cm

Find the length of *BC*.
You must show all your working.

$OA^2 + AC^2 = OC^2$
$5^2 + 12^2 = OC^2$
$169 = OC^2$
$OC = 13$ cm

OB is a radius ∴ 5cm

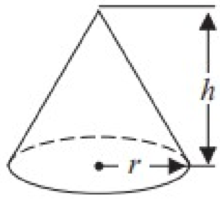
$BC = 13 - 5$
$= 8$ cm

8 cm

(Total for Question 18 is 4 marks)

19 A cone has height 12 cm and volume 72π cm^3.

Volume of a cone $= \frac{1}{3}\pi r^2 h$



Find the diameter of the cone.
Give your answer in the form $a\sqrt{b}$ where a is an integer and b is a prime number.

$$72\cancel{\pi} = \frac{1}{3}\cancel{\pi} r^2 (12)$$

$$72 = 4r^2$$

$$\frac{72}{4} = \frac{36}{2} = 18$$

$$r^2 = 18$$

$$r = \sqrt{18}$$

$$= \sqrt{9}\sqrt{2}$$

$$= 3\sqrt{2}$$

$$\text{Diameter} = 2 \times 3\sqrt{2}$$

$$= 6\sqrt{2}$$

$6\sqrt{2}$ cm

(Total for Question 19 is 4 marks)

20 A, B and C are three points such that

$\overrightarrow{AB} = 6\mathbf{a} + 9\mathbf{b}$

$\overrightarrow{AC} = 10\mathbf{a} + 15\mathbf{b}$

(a) Prove that A, B and C lie on a straight line.

$\overrightarrow{AB} = 3(2a + 3b)$

$\overrightarrow{AC} = 5(2a + 3b)$

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are both multiples of $2a+3b$ and both pass through A. $\therefore$ on a straight line.

(2)

Three points D, E and F lie on a straight line such that

$\overrightarrow{DE} = 4\mathbf{a} - 5\mathbf{b}$

$\overrightarrow{EF} = -12\mathbf{a} + 15\mathbf{b}$

Find the ratio

length of DF : length of DE

$$\overrightarrow{DF} = \overrightarrow{DE} + \overrightarrow{EF}$$
$$= \begin{pmatrix} 4 \\ -5 \end{pmatrix} + \begin{pmatrix} -12 \\ 15 \end{pmatrix} = \begin{pmatrix} -8 \\ 10 \end{pmatrix}$$
$$= -8a + 10b$$

$\overrightarrow{DE} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$ $\quad$ $\overrightarrow{DF} = \begin{pmatrix} -8 \\ 10 \end{pmatrix}$

DF is twice as long

2 : 1

(3)

(Total for Question 20 is 5 marks)

21 The functions f and g are such that

$$f(x) = 3x^2 + 1 \quad \text{for} \quad x > 0 \qquad \text{and} \qquad g(x) = 2x - 3$$

(a) Find $f^{-1}(x)$

$\rightarrow$ square $\rightarrow$ $\times 3$ $\rightarrow$ $+1$

$$f^{-1}(x) = \sqrt{\frac{x-1}{3}}$$

$f^{-1}(x) = \sqrt{\frac{x-1}{3}}$

(2)

(b) Solve gf(x) = 95

$$2(3x^2 + 1) - 3 = 95$$

$$6x^2 + 2 - 3 = 95$$

$$6x^2 - 1 = 95$$

$$6x^2 = 96$$

$\frac{96}{6} = \frac{48}{3} = 16$

$$x^2 = 16$$

$$x = 4$$

$x = 4$

(3)

(Total for Question 21 is 5 marks)

22 Write $\dfrac{\sqrt{8}}{3-\sqrt{2}}$ in the form $\dfrac{a\sqrt{2}+b}{c}$ where a, b and c are integers.

$\sqrt{8} = 2\sqrt{2}$

$$\frac{2\sqrt{2}\,(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})}$$

$$\frac{6\sqrt{2}+4}{9+3\sqrt{2}-3\sqrt{2}-2}$$

$$\frac{6\sqrt{2}+4}{7}$$

$$\frac{6\sqrt{2}+4}{7}$$

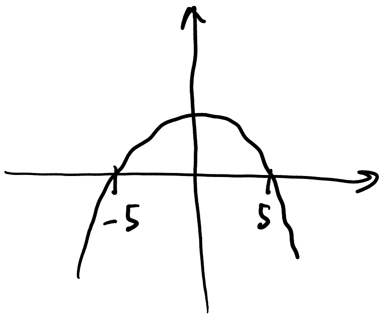
(Total for Question 22 is 4 marks)

23 Find the set of values of x for which

$$25 - x^2 > 0 \qquad \textbf{and} \qquad 3x^2 - 17x - 6 < 0$$

You must show all your working.

$(5 + x)(5 - x) > 0$



$\underline{-5 < x < 5}$

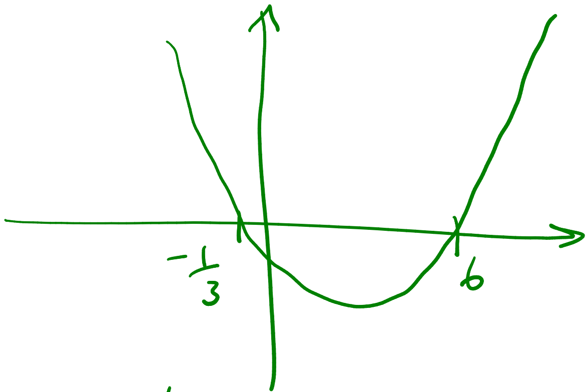
$3x^2 - 17x - 6 < 0$

3×6
$= 18$
$1 \quad 18$

$3x^2 + x - 18x - 6$

$(x - 6)(3x + 1)$

$x = 6 \qquad x = -\frac{1}{3}$



$\underline{-\frac{1}{3} < x < 6}$

Both satisfied when

$\underline{\underline{-\frac{1}{3} < x < 5}}$

$-\frac{1}{3} < x < 5$

(Total for Question 23 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS