Mathematics
2022 Paper 1 (Non-Calculator)
Higher Tier

Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

Total Marks

Instructions

• Use **black** ink or ball-point pen.
• **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
• Answer **all** questions.
• Answer the questions in the spaces provided
– there may be more space than you need.
• **Calculators may not be used**.
• Diagrams are **NOT** accurately drawn, unless otherwise indicated.
• You must **show all your working out.**

Information

• The total mark for this paper is 80
• The marks for **each** question are shown in brackets
– use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.
• Keep an eye on the time.
• Try to answer every question.
• Check your answers if you have time at the end.

Higher Tier Formulae Sheet

Perimeter, area and volume

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

$$\text{Area of a trapezium} = \frac{1}{2}(a+b)\,h$$

Volume of a prism = area of cross section × length

Where r is the radius and d is the diameter:

$$\text{Circumference of a circle} = 2\pi r = \pi d$$

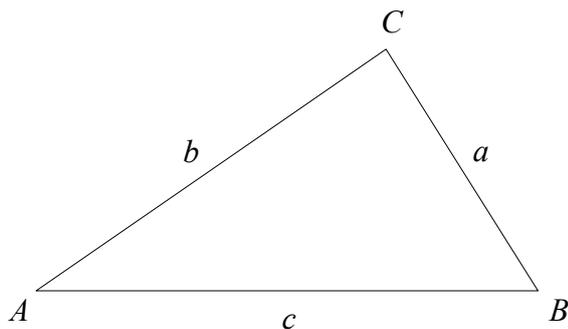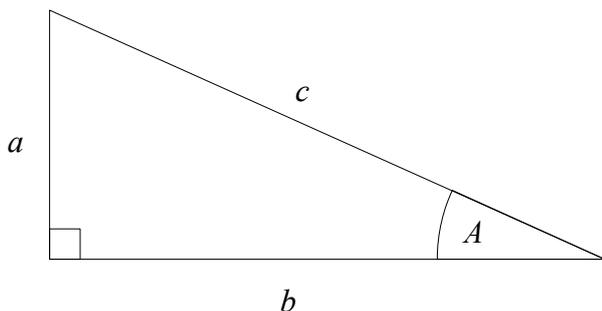$$\text{Area of a circle} = \pi r^2$$

Quadratic formula

The solution of $ax^2 + bx + c = 0$

where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagoras' Theorem and Trigonometry

In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$

In any triangle ABC where a, b and c are the length of the sides:

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\dfrac{1}{2}ab\sin C$

Compound Interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

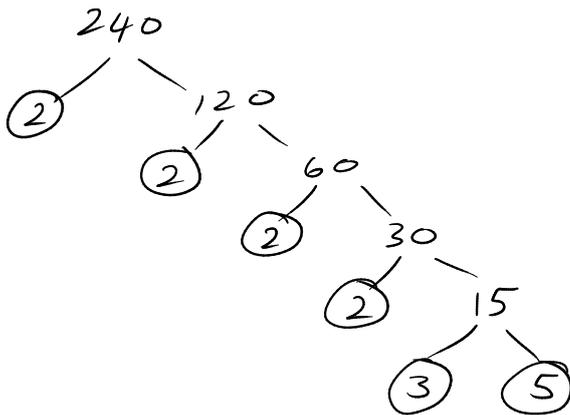$$\text{Total accrued} = P\left(1 + \frac{r}{100}\right)^n$$

Probability

Where P (A) is the probability of outcome A and P (B) is the probability of outcome B:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A \text{ given } B)\,P(B)$$

END OF EXAM AID

1 Write 240 as a product of its prime factors.

240
2   120
  2   60
    2   30
      2   15
        3   5

2×2×2×2×3×5

(Total for Question 1 is 2 marks)

2 (a) Work out $\dfrac{3}{4} - \dfrac{7}{10}$

5× over 5× on the 3/4 term, ×2 over ×2 on the 7/10 term

$$\dfrac{15}{20} - \dfrac{14}{20}$$

$\dfrac{1}{20}$

(2)

(b) Work out $2\dfrac{1}{3} \times \dfrac{3}{5}$

Give your answer as a mixed number in its simplest form.

$2\dfrac{1}{3} = \dfrac{7}{3}$

$$\dfrac{7}{3} \times \dfrac{3}{5} = \dfrac{7}{5} = 1\dfrac{2}{5}$$

$1\dfrac{2}{5}$

(2)

(Total for Question 2 is 4 marks)

3   In a bag there are only red counters, blue counters, green counters and yellow counters.

A counter is taken at random from the bag.

The table shows the probabilities that the counter will be green or will be yellow.

Colour	Red	Blue	Green	Yellow
Probability	$2x$	x	0.35	0.20

$$0.3 \qquad 0.15$$

The probability that the counter will be red is twice the probability that the counter will be blue.

There are 21 green counters in the bag.

Work out the number of red counters in the bag.

$$0.35 + 0.2 = 0.55$$
$$1 - 0.55 = 0.45$$

$$3x = 0.45$$
$$x = 0.15$$
$$2x = 0.3 \quad (Red \ Counters)$$

21 Green counters

35% of total $= 21$
$\div 7 \qquad\qquad \div 7$
5% of total $= 3$

30% of counters are red

5% of total $= 3$
$\times 6 \qquad\qquad \times 6$
30% of total $= 18$

18

(Total for Question 3 is 4 marks)

4 Here is the graph of $y = 2x + 6 - x^2$
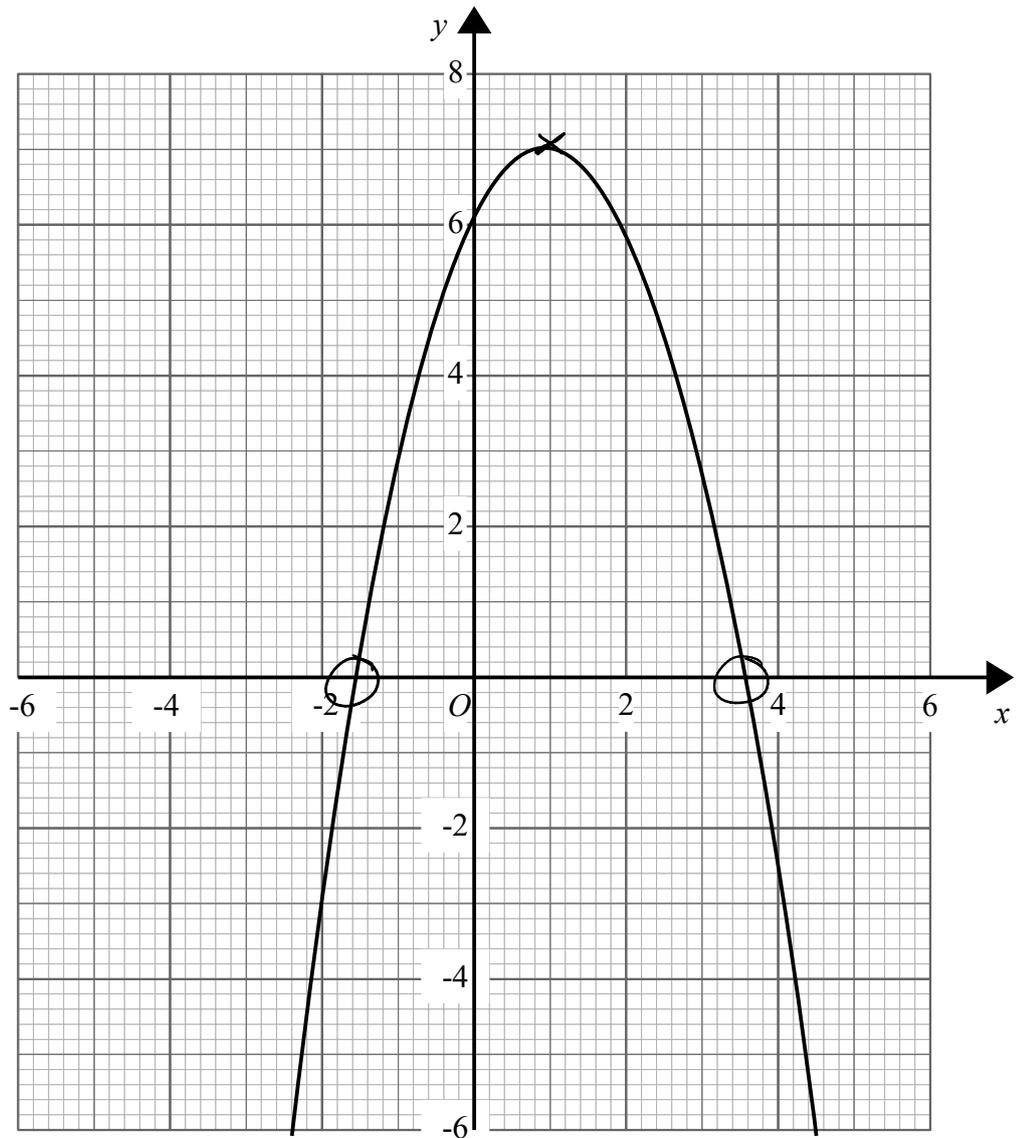
(a) Write down the turning point of the graph $y = 2x + 6 - x^2$

(................1.............. , ...7............)

(1)

(b) Use the graph to find the roots of the equation $x^2 = 2x + 6$

$$0 = 2x + 6 - x^2$$

$x = -1.6$ or $x = 3.6$

(2)

(Total for Question 4 is 3 marks)

5 $5 < 2y < 12$ where y is an integer.

(a) Write down all the possible values of *y*.

$2.5 < y < 6$

.... 3, 4 or 5 ....

(2)

(b) Solve $4 > 19 - 3x$

$3x + 4 > 19$

$3x > 15$

$x > 5$

.... $x > 5$ ....

(2)

(Total for Question 5 is 4 marks)

6   Dermot has 240 counters.
The counters are either red, or blue, or yellow or green.

15% of the counters are red.

$\frac{2}{5}$ of the counters are blue

The ratio of yellow counters to green counters is 3:1

Work out the number of yellow counters Dermot has.

$\frac{240}{10} = 24 \quad (10\%)$ 　　　　$\frac{240}{5} = 48 \quad \left(\frac{1}{5}\right)$

$\frac{24}{2} = 12 \quad (5\%)$ 　　　　　$48 \times 2 = 96 \quad \left(\frac{2}{5}\right)$

$24 + 12 = 36 \quad (15\%)$ 　　　　　96 Blue

36 red

　　　　　$36 + 96 = 132$ 　Red + Blue

　　　　　$240 - 132 = 108$ 　Yellow + Green

Y : G 　　　$\frac{108}{4} = \frac{54}{2} = 27$

| 27 | 27 | 27 | : | 27 |

81 : 27 　　　　　　　　　　　　　　..................... 81 ....

(Total for Question 6 is 4 marks)

7 Here is a cube.

0.05 cm

0.05 cm

0.05 cm
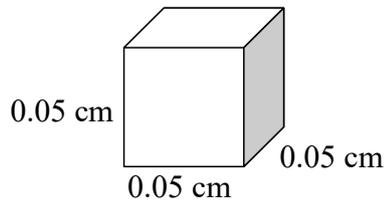
Work out the volume of this cube.
Give your answer in standard form

$$0.05 = 5 \times 10^{-2}$$

$$5 \times 10^{-2} \times 5 \times 10^{-2} \times 5 \times 10^{-2}$$
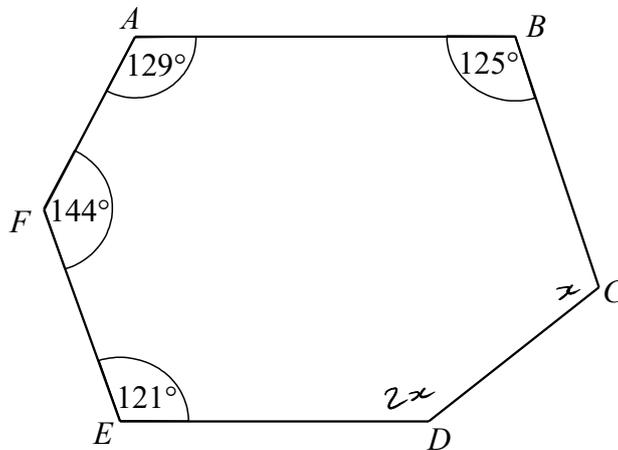
$$125 \times 10^{-6}$$

$$1.25 \times 10^{2} \times 10^{-6}$$

$$1.25 \times 10^{-4}$$

..............1.25×10^{-4}..........cm³

(Total for Question 7 is 3 marks)

8

A
129°

B
125°

F
144°

x C

121°

2x

E

D

ABCDEF is a hexagon.

Angle *CDE* = 2 × Angle *BCD*

Work out the size of angle *CDE*.

$$1\ 2\ 5$$
$$1\ 2\ 9$$
$$1\ 4\ 4$$
$$+\ \ 1\ 2\ 1$$
$$\overline{5\ 1\ 9}$$

Angles in a hexagon = $(6-2) \times 180$
$$= 4 \times 180$$
$$= 720$$

$$720 - 519 = 201$$

$$3x = 201$$

$$\frac{067}{3\overline{)20^{2}1}}$$

$$x = \frac{201}{3}$$

$$= 67$$

$$2x = 134$$

..................134..................°

(Total for Question 8 is 3 marks)

9 Liquid **A** has a density of 1.2 g/cm³

$$density = \frac{mass}{volume}$$

150 cm³ of Liquid **A** is mixed with some of Liquid **B** to make Liquid **C**.

Liquid **C** has a mass of 220 g and a density of 1.1 g/cm³

Find the density of Liquid **B**.

A//   mass = density × volume
            = 1.2 × 150
            = 180 g

C//   volume = $\frac{mass}{density}$

            = $\frac{220}{1.1}$ = $\frac{2200}{11}$

                    = 200 cm³

B//   Mass = Mass of C − Mass of A
            = 220 − 180
            = 40 g

    Volume = Volume of C − Volume of A
            = 200 − 150
            = 50 cm³

    Density = $\frac{40}{50}$ = $\frac{4}{5}$ = 0.8

............................0.8............................ g/cm³

(Total for Question 9 is 3 marks)

10 10 students have a mean height of 150 cm.
4 of the students have a mean height of 165 cm.

Work out the mean height of the other 6 students.

$$mean = \frac{total}{n}$$

$$150 = \frac{total}{10}$$

$$1500 = total \text{ (all 10 students)}$$

$$165 = \frac{total (4)}{4}$$

$$660 = total \text{ (4 students)}$$

$$1500 - 660 = 840 \quad \text{(other 6 students)}$$

$$Mean = \frac{840}{6} = \frac{420}{3}$$

$$= 140$$

_____**140** cm_____

(Total for Question 10 is 3 marks)

11 The points A, B, C and D lie in order on a straight line.

$AB{:}BD = 2{:}5$ and $AC{:}CD = 4{:}7$

Find $AB{:}BC{:}CD$

7 parts   11 parts
×11   ×7   (Make the total number
                        of parts the same)

AB : BD   AC : CD
22 : 55   28 : 49

A   B   C   D
← 22 → ← 55 →
← 29 → ← 49 →
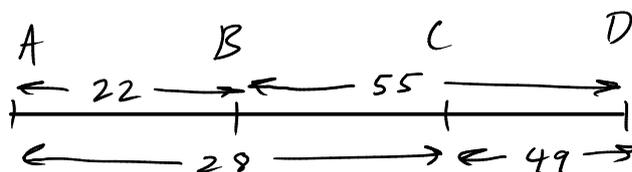
AB : BC : CD
22 : 6 : 49

_____**22 : 6 : 49**_____
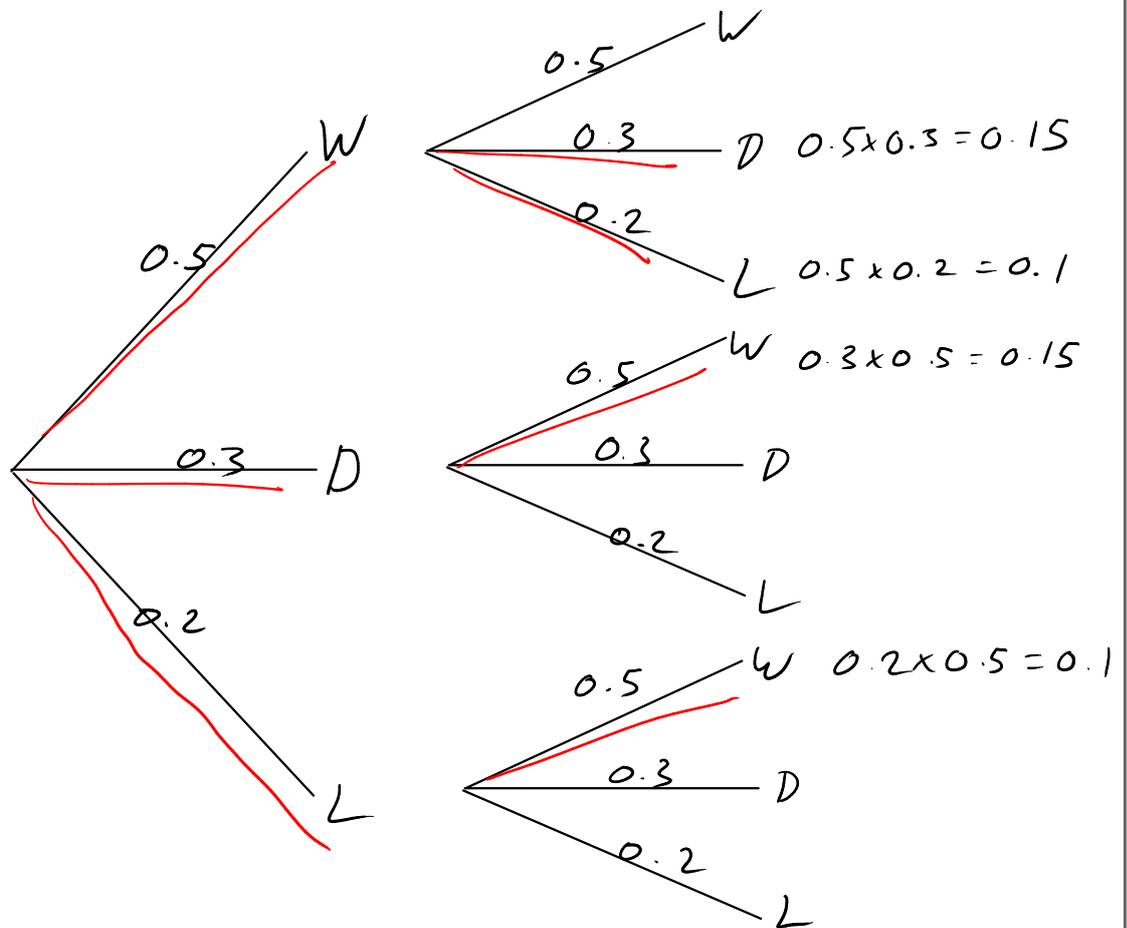
(Total for Question 11 is 3 marks)

12 Jon plays a game where he can win, draw or lose.

The probability Jon wins any game 0.5.
The probability Jon draws any game is 0.3

Jon plays two games.

Work out the probability Jon wins exactly one game.

W
0.5 → W
0.3 → D   0.5 × 0.5 = 0.15
0.2 → L   0.5 × 0.2 = 0.1

0.5

0.3 → D
0.5 → W   0.3 × 0.5 = 0.15
0.3 → D
0.2 → L

0.2 → L
0.5 → W   0.2 × 0.5 = 0.1
0.3 → D
0.2 → L
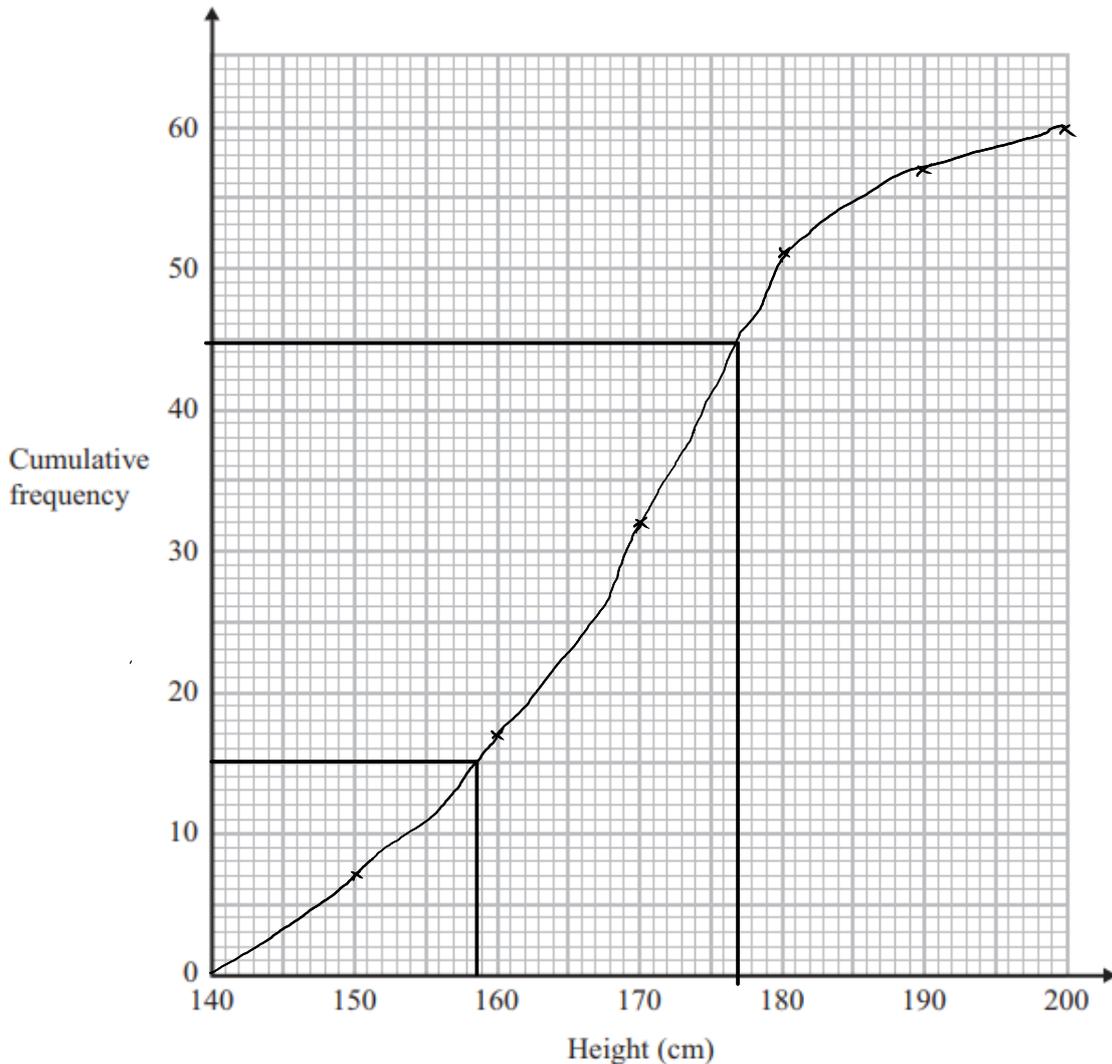
0.15 + 0.1 + 0.15 + 0.1

= 0.5

0.5

(Total for Question 12 is 3 marks)

13 The cumulative frequency table shows the height, in cm, of some tomato plants.

Height	Cumulative Frequency
$140 < h \leqslant 150$	7
$140 < h \leqslant 160$	17
$140 < h \leqslant 170$	32
$140 < h \leqslant 180$	51
$140 < h \leqslant 190$	57
$140 < h \leqslant 200$	60

(a) On the grid, plot a cumulative frequency graph for this information.

(2)

(b) Find an estimate for the inter quartile range.

UQ   45
LQ   15

$177 - 159$

..............18..........cm

(17 to 19)   (1)

(Total for Question 13 is 3 marks)

UQ   176 or 177   LQ   156 or 159

14 Find the value of $\left(\dfrac{64}{125}\right)^{-\frac{2}{3}}$

flip $\downarrow$

$\frac{2}{3}$ ← square

← cube root

$\left(\dfrac{4}{5}\right)^{-2}$

$\left(\dfrac{16}{25}\right)^{-1}$

.......$\dfrac{25}{16}$.......

(Total for Question 14 is 2 marks)

---

15 Write $0.1\dot{6}$ as a fraction in its simplest form.

$$0.1\dot{6} = x$$
$$1.\dot{6} = 10x$$
$$16.\dot{6} = 100x$$
$$15 = 90x$$
$$x = \dfrac{15}{90}$$
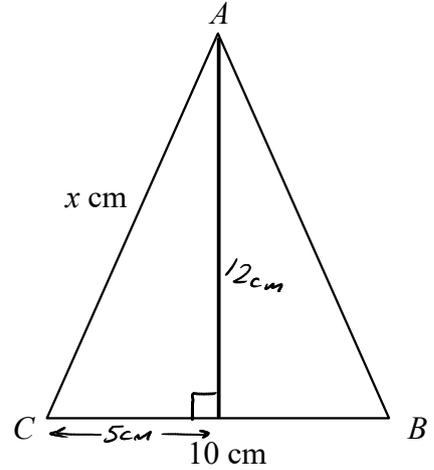$$= \dfrac{1}{6}$$

.......$\dfrac{1}{6}$.......

(Total for Question 15 is 2 marks)

16 *ABC* is an isosceles triangle.
Angle *ACB* = Angle *ABC*

The area of the triangle is 60 cm²
Calculate the perimeter of the triangle.

Area = $\frac{1}{2}$ base × height

$60 = \frac{1}{2}(10)h$

$60 = 5h$

$h = 12$ cm

$5^2 + 12^2 = x^2$

$25 + 144 = x^2$

$169 = x^2$

$x = \sqrt{169}$

$= 13$ cm

perimeter $= 10 + 13 + 13$

$= 36$

...............36...............cm

(Total for Question 16 is 3 marks)

---

17 Here are the equations of 5 straight lines.

$P: \ y = 2x + 5$
$Q: \ y = -2x + 5$
$R: \ y = x + 5$
$S: \ y = -\frac{1}{2}x + 6$
$T: \ y = \frac{1}{2}x + 1$

(a) Write down the letter of the line that is parallel to $y = x + 6$

R

(1)

(b) Write down the letter of the line that is perpendicular to $y = 2x - 1$
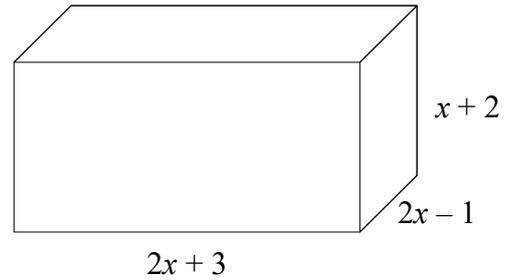
perp m $= -\frac{1}{2}$

S

(1)

(Total for Question 17 is 2 marks)

18 The diagram shows a cuboid.

All the measurements are in centimetres.
The total surface area of the cuboid is 172 cm^2

Find the value of x.

$x + 2$

$2x - 1$

$2x + 3$

$2(2x+3)(x+2) + 2(2x+3)(2x-1) + 2(2x-1)(x+2) = 172$

$2(2x^2 + 4x + 3x + 6) + 2(4x^2 - 2x + 6x - 3) + 2(2x^2 + 4x - x - 2) = 172$

$2(2x^2 + 7x + 6) + 2(4x^2 + 4x - 3) + 2(2x^2 + 3x - 2) = 172$

$4x^2 + 14x + 12 + 8x^2 + 8x - 6 + 4x^2 + 6x - 4 = 172$

$16x^2 + 28x + 2 = 172$

$16x^2 + 28x - 170 = 0$

$8x^2 + 14x - 85 = 0$

$(4x + 17)(2x - 5)$

$x = -\frac{17}{4} \qquad x = \frac{5}{2}$

x cannot be -ve

$\left[\text{OR } 2.5 \right]$

2.5

(Total for Question 18 is 3 marks)

19 Write $(3 - \sqrt{8})^2$ in the form $a + b\sqrt{2}$, where a and b are integers.

$$(3 - \sqrt{8})(3 - \sqrt{8})$$

$$9 - 3\sqrt{8} - 3\sqrt{8} + 8$$

$$17 - 6\sqrt{8}$$

$$17 - 6(2\sqrt{2})$$

$$17 - 12\sqrt{2}$$

$$\sqrt{8} = \sqrt{4}\sqrt{2}$$
$$= 2\sqrt{2}$$

$17 - 12\sqrt{2}$

(Total for Question 19 is 2 marks)

20 Given that

$$2x + 1 : x + 2 = x + 8 : 3x - 4$$

Find the possible values of x.

$$\frac{2x + 1}{x + 2} = \frac{x + 8}{3x - 4}$$

$$(2x + 1)(3x - 4) = (x + 8)(x + 2)$$

$$6x^2 - 8x + 3x - 4 = x^2 + 2x + 8x + 16$$

$$6x^2 - 5x - 4 = x^2 + 10x + 16$$
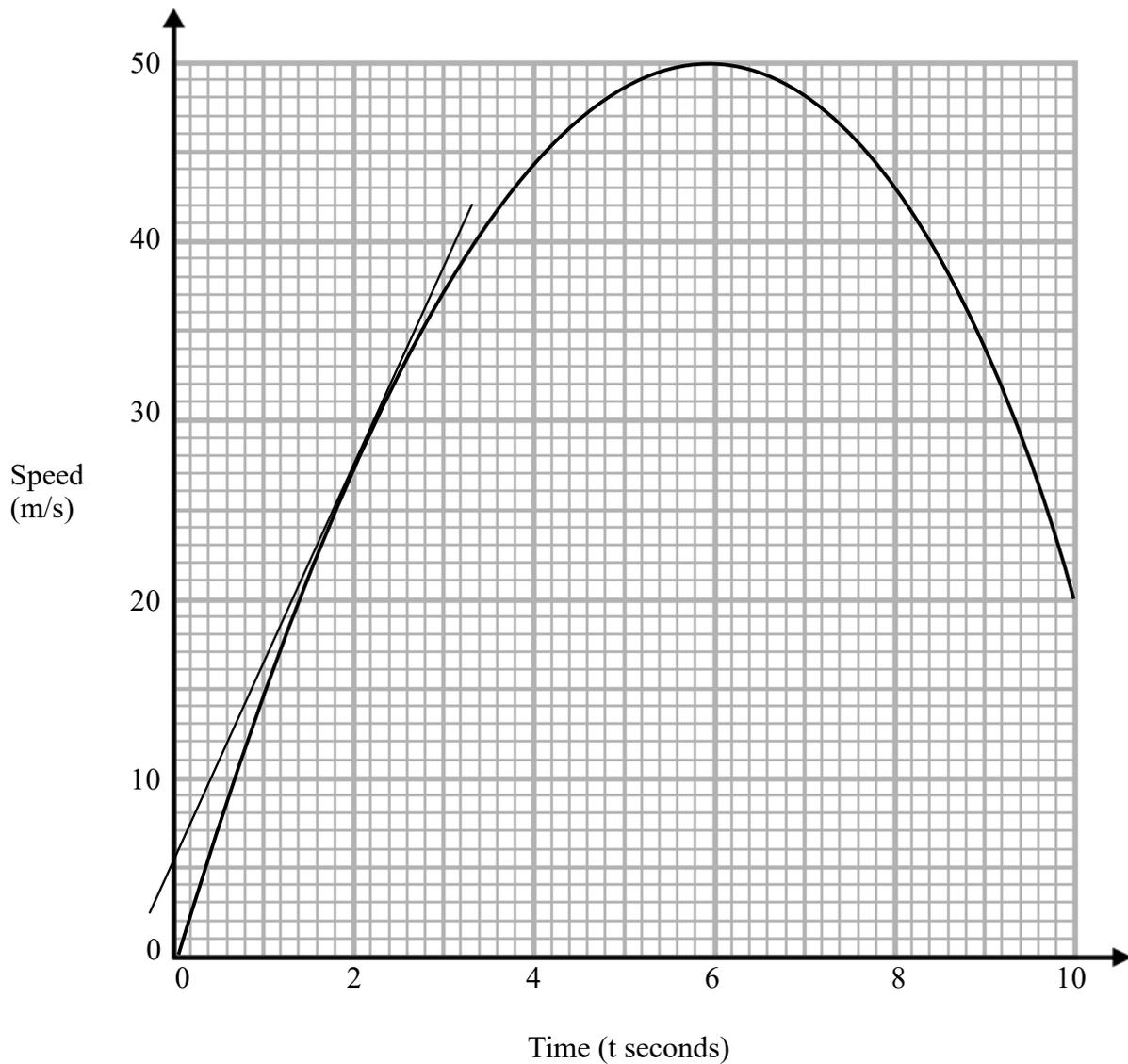
$$5x^2 - 15x - 20 = 0$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$x = 4 \qquad x = -1$$

$x = 4$ or $x = -1$

(Total for Question 20 is 4 marks)

21 Here is a speed-time graph.

Work out an estimate for the acceleration when t = 2.

$(0, 6)$ $(3, 39)$ $\dfrac{39 - 6}{3 - 0} = \dfrac{33}{3}$

…..........11............ ms^{-2}

(Total for Question 21 is 2 marks)

22 The table shows pairs of values for x and y

x	2	3
y	32	72

(i) Tick the correct statement below.

If $y \propto x$
$y = kx$
$32 = k(2)$
$k = 16$
$y = 16x$
$72 \neq 16(3)$

If $y \propto x^2$
$y = kx^2$
$32 = k(2)^2$
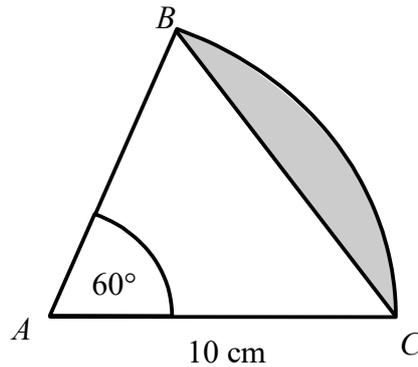$32 = 4k$
$k = 8$
$y = 8x^2$
$72 = 8(3)^2$
$72 = 8(9)$ ✓

$y \propto x$ ....

$y \propto x^2$ ....✓..........

$y \propto x^3$ ....

(ii) Write a formula for y in terms of x

$y = 8x^2$

(Total for Question 22 is 4 marks)

23 *BAC* is a sector of a circle, centre *A*.
AC is 10 cm.

Find the area of the shaded segment.
Give your answer in terms of π.

$$\text{Area of sector} = \frac{60}{360} \pi r^2$$

$$= \frac{1}{6} \pi (10)^2$$

$$= \frac{1}{6} \times 100 \pi$$

$$= \frac{100}{6} \pi = \frac{50}{3} \pi$$

$$\text{Area of triangle} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} (10)(10) \sin 60 \qquad \left[\sin 60 = \frac{\sqrt{3}}{2} \right]$$

$$= 50 \times \frac{\sqrt{3}}{2}$$

$$= 25\sqrt{3}$$

$$\text{Shaded segment} = \frac{50}{3} \pi - 25\sqrt{3}$$
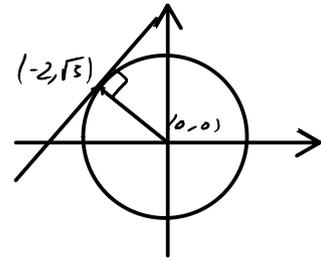
.......$\frac{50}{3}\pi - 25\sqrt{3}$....... cm²

(Total for Question 23 is 4 marks)

24 A circle has the equation $x^2 + y^2 = 7$

P is the point $(-2, \sqrt{3})$ on the circle $x^2 + y^2 = 7$

Work out the equation of the tangent to the circle at P.

$$\text{Gradient} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Gradient of radius} = \frac{-\sqrt{3}}{2}$$

$$\text{Perp gradient} = \frac{2}{\sqrt{3}} \quad (\text{gradient of tangent})$$

$$y = \frac{2}{\sqrt{3}} x + c \qquad \begin{matrix} (-2, & \sqrt{3}) \\ x & y \end{matrix}$$

$$\sqrt{3} = \frac{2}{\sqrt{3}} (-2) + c$$

$$\sqrt{3} = -\frac{4}{\sqrt{3}} + c$$

$$c = \sqrt{3} + \frac{4}{\sqrt{3}}$$

$$\underline{\underline{y = \frac{2}{\sqrt{3}} x + \sqrt{3} + \frac{4}{\sqrt{3}}}}$$
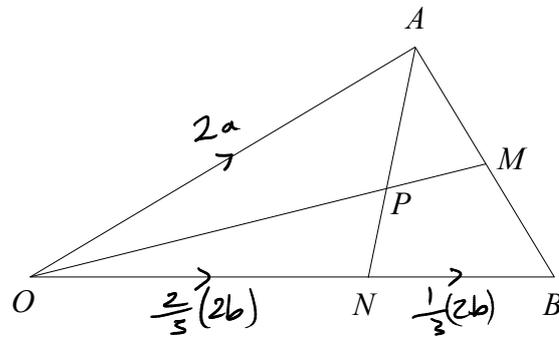
OR SIMPLIFIED:

$$y = \frac{2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} x + \frac{3\sqrt{3}}{3} + \frac{4\sqrt{3}}{3}$$

$$\underline{y = \frac{2\sqrt{3}}{3} x + \frac{7\sqrt{3}}{3}}$$

$$y = \frac{2\sqrt{3}}{3} x + \frac{7\sqrt{3}}{3}$$

(Total for Question 24 is 4 marks)

25

$\overrightarrow{OA} = 2a$

$\overrightarrow{OB} = 2b$

$\frac{2}{3}$ and $\frac{1}{3}$

N is the point on OB such that ON:NB = 2:1
M is the midpoint of AB
P is the point on OM such that APN is a straight line.

Find OP:PM

$\overrightarrow{AB} = -2a + 2b$

$\overrightarrow{AM} = -a + b$

$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$
$= 2a + (-a + b)$
$= a + b$

$\overrightarrow{OP} = x(a+b)$

$\overrightarrow{AN} = \overrightarrow{AO} + \overrightarrow{ON}$
$= -2a + \frac{4}{3}b$

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$
$= 2a + y(-2a + \frac{4}{3}b)$

$x(a+b) = 2a + y(-2a + \frac{4}{3}b)$

$xa + xb = 2a - 2ya + \frac{4}{3}yb$

$xa + xb = (2 - 2y)a + \frac{4}{3}yb$

a// $\quad x = 2 - 2y$

b// $\quad x = \frac{4}{3}y$

$\frac{3}{4}x = y$

$x = 2 - 2(\frac{3}{4}x)$

$x = 2 - \frac{3}{2}x$

$\frac{5}{2}x = 2 \qquad x = \frac{4}{5}$

P is $\frac{4}{5}$ of the way along OM

OP : PM
4 : 1

(Total for Question 25 is 5 marks)