

GCE Examinations  
Advanced Subsidiary

# Core Mathematics C4

Paper L

## MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



*Written by Shaun Armstrong*

© *Solomon Press*

*These sheets may be copied for use solely by the purchaser's institute.*

## C4 Paper L – Marking Guide

1. (a)  $\frac{dn}{dt} = 0 \Rightarrow e^{0.5t} = 5$  M1  
 $t = 2 \ln 5 = 3.219 \text{ mins} = 3 \text{ mins } 13 \text{ secs}$  M1 A1
- (b)  $\int dn = \int (e^{0.5t} - 5) dt$   
 $n = 2e^{0.5t} - 5t + c$  M1 A1  
 $t = 0, n = 20 \Rightarrow 20 = 2 + c, \quad c = 18$  M1  
 $n = 2e^{0.5t} - 5t + 18$  A1
- (c) as  $t$  increases,  $n$  rapidly becomes very large  $\therefore$  not realistic B1 **(8)**
- 
2.  $6x + y + x \frac{dy}{dx} - 4y \frac{dy}{dx} = 0$  M1 A2
- $(1, 4) \Rightarrow 6 + 4 + \frac{dy}{dx} - 16 \frac{dy}{dx} = 0, \quad \frac{dy}{dx} = \frac{2}{3}$  M1 A1
- grad of normal =  $-\frac{3}{2}$  M1
- $\therefore y - 4 = -\frac{3}{2}(x - 1)$  M1
- $2y - 8 = -3x + 3$   
 $3x + 2y - 11 = 0$  A1 **(8)**
- 
3. (a)  $u = 2 - x^2 \Rightarrow \frac{du}{dx} = -2x$  M1
- $I = \int \frac{1}{u} \times (-\frac{1}{2}) du = -\frac{1}{2} \int \frac{1}{u} du$  A1  
 $= -\frac{1}{2} \ln |u| + c = -\frac{1}{2} \ln |2 - x^2| + c$  M1 A1
- (b)  $= \int_0^{\frac{\pi}{4}} (\frac{1}{2} \sin 4x + \frac{1}{2} \sin 2x) dx$  M1 A1
- $= [-\frac{1}{8} \cos 4x - \frac{1}{4} \cos 2x]_0^{\frac{\pi}{4}}$  M1 A1
- $= (\frac{1}{8} - 0) - (-\frac{1}{8} - \frac{1}{4}) = \frac{1}{2}$  M1 A1 **(10)**
- 
4. (a)  $\begin{array}{ccc} x & 1 & 2 & 3 \\ y & 0 & 1.665 & 3.144 \end{array}$  B1  
 $\text{area} \approx \frac{1}{2} \times 1 \times [0 + 3.144 + 2(1.665)] = 3.24 \text{ (3sf)}$  B1 M1 A1
- (b)  $\text{volume} = \pi \int_1^3 x^2 \ln x \, dx$  M1
- $u = \ln x, \quad u' = \frac{1}{x}, \quad v' = x^2, \quad v = \frac{1}{3} x^3$
- $I = \frac{1}{3} x^3 \ln x - \int \frac{1}{3} x^2 dx$  M1 A2  
 $= \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 + c$  A1
- $\text{volume} = \pi [\frac{1}{3} x^3 \ln x - \frac{1}{9} x^3]_1^3$   
 $= \pi \{(9 \ln 3 - 3) - (0 - \frac{1}{9})\}$  M1  
 $= \pi(9 \ln 3 - \frac{26}{9})$  A1 **(11)**
-

5. (a)  $\frac{5-8x}{(1+2x)(1-x)^2} \equiv \frac{A}{1+2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$   
 $5-8x \equiv A(1-x)^2 + B(1+2x)(1-x) + C(1+2x)(1-x)$  M1  
 $x = -\frac{1}{2} \Rightarrow 9 = \frac{9}{4}A \Rightarrow A = 4$  A1  
 $x = 1 \Rightarrow -3 = 3C \Rightarrow C = -1$  A1  
coeffs  $x^2 \Rightarrow 0 = A - 2B \Rightarrow B = 2$  M1 A1  
 $f(x) = \frac{4}{1+2x} + \frac{2}{1-x} - \frac{1}{(1-x)^2}$
- (b)  $f(x) = 4(1+2x)^{-1} + 2(1-x)^{-1} - (1-x)^{-2}$   
 $(1+2x)^{-1} = 1 + (-1)(2x) + \frac{(-1)(-2)}{2}(2x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(2x)^3 + \dots$  M1  
 $= 1 - 2x + 4x^2 - 8x^3 + \dots$  A1  
 $(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots$  B1  
 $(1-x)^{-2} = 1 + (-2)(-x) + \frac{(-2)(-3)}{2}(-x)^2 + \frac{(-2)(-3)(-4)}{3 \times 2}(-x)^3 + \dots$   
 $= 1 + 2x + 3x^2 + 4x^3 + \dots$  A1  
 $f(x) = 4(1 - 2x + 4x^2 - 8x^3) + 2(1 + x + x^2 + x^3) - (1 + 2x + 3x^2 + 4x^3)$  M1  
 $= 5 - 8x + 15x^2 - 34x^3 + \dots$  A1
- (c)  $|x| < \frac{1}{2}$  A1 (12)

6. (a)  $\frac{dx}{dt} = 1 + \cos t, \quad \frac{dy}{dt} = \cos t$  M1  
 $\frac{dy}{dx} = \frac{\cos t}{1 + \cos t}$  M1 A1
- (b)  $\frac{\cos t}{1 + \cos t} = 0, \cos t = 0, t = \frac{\pi}{2}$  M1 A1  
 $\therefore (\frac{\pi}{2} + 1, 1)$  A1
- (c)  $= \int_0^{\pi} \sin t \times (1 + \cos t) dt = \int_0^{\pi} (\sin t + \frac{1}{2} \sin 2t) dt$  M1 A1  
 $= [-\cos t - \frac{1}{4} \cos 2t]_0^{\pi}$  M1 A1  
 $= (1 - \frac{1}{4}) - (-1 - \frac{1}{4}) = 2$  M1 A1 (12)

7. (a)  $\vec{AB} = (8\mathbf{j} - 6\mathbf{k}) - (3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}) = (-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$  M1  
 $\therefore \mathbf{r} = (3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}) + \lambda(-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$  A1
- (b)  $3 - 3\lambda = -2 + 7\mu$  (1)  
 $6 + 2\lambda = 10 - 4\mu$  (2)  
 $-8 + 2\lambda = 6 + 6\mu$  (3) B1  
(3) - (2):  $-14 = -4 + 10\mu, \mu = -1, \lambda = 4$  M1 A1  
check (1)  $3 - 12 = -2 - 7, \text{ true } \therefore \text{ intersect}$  B1
- (c)  $\mathbf{r} = (-2\mathbf{i} + 10\mathbf{j} + 6\mathbf{k}) - (7\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}) \therefore (-9, 14, 0)$  M1 A1
- (d)  $\vec{OC} = [(-2 + 7\mu)\mathbf{i} + (10 - 4\mu)\mathbf{j} + (6 + 6\mu)\mathbf{k}]$   
 $\vec{AC} = \vec{OC} - \vec{OA} = [(-5 + 7\mu)\mathbf{i} + (4 - 4\mu)\mathbf{j} + (14 + 6\mu)\mathbf{k}]$  M1 A1  
 $\therefore [(-5 + 7\mu)\mathbf{i} + (4 - 4\mu)\mathbf{j} + (14 + 6\mu)\mathbf{k}] \cdot (-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) = 0$  M1  
 $15 - 21\mu + 8 - 8\mu + 28 + 12\mu = 0$  A1  
 $\mu = 3 \therefore \vec{OC} = (19\mathbf{i} - 2\mathbf{j} + 24\mathbf{k})$  M1 A1 (14)

Total (75)

## Performance Record – C4 Paper L

Question no.	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	Total
Topic(s)	differential equation	differentiation	integration	trapezium rule, integration	partial fractions, binomial series	parametric equations	vectors	
Marks	8	8	10	11	12	12	14	75
Student								