

1. A discrete random variable X has the probability function

$$P(X=x) = \begin{cases} k(1-x)^2 & x = -1, 0, 1 \text{ and } 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $k = \frac{1}{6}$ (3)

(b) Find $E(X)$ (2)

(c) Show that $E(X^2) = \frac{4}{3}$ (2)

(d) Find $\text{Var}(1-3X)$ (3)

x	-1	0	1	2
$P(X=x)$	$4k$	k	0	k

$$4k + k + k = 1$$

$$6k = 1$$

$$k = \frac{1}{6}$$

b/

x	-1	0	1	2
$P(X=x)$	$\frac{2}{3}$	$\frac{1}{6}$	0	$\frac{1}{6}$

$$E(X) = -1\left(\frac{2}{3}\right) + 0\left(\frac{1}{6}\right) + 1(0) + 2\left(\frac{1}{6}\right) = -\frac{1}{3}$$

c/

$$E(X^2) = 1\left(\frac{2}{3}\right) + 4\left(\frac{1}{6}\right) = \frac{4}{3}$$

$$d/ \text{Var}(X) = \frac{4}{3} - \left(-\frac{1}{3}\right)^2$$

$$= \frac{11}{9}$$

$$\text{Var}(1-3X) = \frac{11}{9} \times 9$$

$$= \underline{\underline{11}}$$



2. A bank reviews its customer records at the end of each month to find out how many customers have become unemployed, u , and how many have had their house repossessed, h , during that month. The bank codes the data using variables $x = \frac{u-100}{3}$ and $y = \frac{h-20}{7}$. The results for the 12 months of 2009 are summarised below.

$$\sum x = 477 \quad S_{xx} = 5606.25 \quad \sum y = 480 \quad S_{yy} = 4244 \quad \sum xy = 23\,070$$

- (a) Calculate the value of the product moment correlation coefficient for x and y . (3)
- (b) Write down the product moment correlation coefficient for u and h . (1)

The bank claims that an increase in unemployment among its customers is associated with an increase in house repossessions.

- (c) State, with a reason, whether or not the bank's claim is supported by these data. (2)

$$a) \quad r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$$

$$\begin{aligned} S_{xy} &= \sum xy - \frac{\sum x \sum y}{n} \\ &= 23070 - \frac{(477)(480)}{12} \\ &= 3990 \end{aligned}$$

$$\begin{aligned} r &= \frac{3990}{\sqrt{(5606.25)(4244)}} \\ &= 0.818 \quad (3\text{sf}) \end{aligned}$$

$$b) \quad 0.818 \quad (3\text{sf})$$

c) There is a positive correlation so the bank's claim is supported by the data.



3. A scientist is researching whether or not birds of prey exposed to pollutants lay eggs with thinner shells. He collects a random sample of egg shells from each of 6 different nests and tests for pollutant level, p , and measures the thinning of the shell, t . The results are shown in the table below.

p	3	8	30	25	15	12
t	1	3	9	10	5	6

[You may use $\sum p^2 = 1967$ and $\sum pt = 694$]

- (a) Draw a scatter diagram on the axes on page 7 to represent these data. (2)
- (b) Explain why a linear regression model may be appropriate to describe the relationship between p and t . (1)
- (c) Calculate the value of S_{pt} and the value of S_{pp} . (4)
- (d) Find the equation of the regression line of t on p , giving your answer in the form $t = a + bp$. (4)
- (e) Plot the point (\bar{p}, \bar{t}) and draw the regression line on your scatter diagram. (2)

The scientist reviews similar studies and finds that pollutant levels above 16 are likely to result in the death of a chick soon after hatching.

- (f) Estimate the minimum thinning of the shell that is likely to result in the death of a chick. (2)

b/ there appears to be a positive correlation

$$c/ S_{pt} = \sum pt - \frac{\sum p \sum t}{n}$$

$$= 694 - \frac{(93)(34)}{6}$$

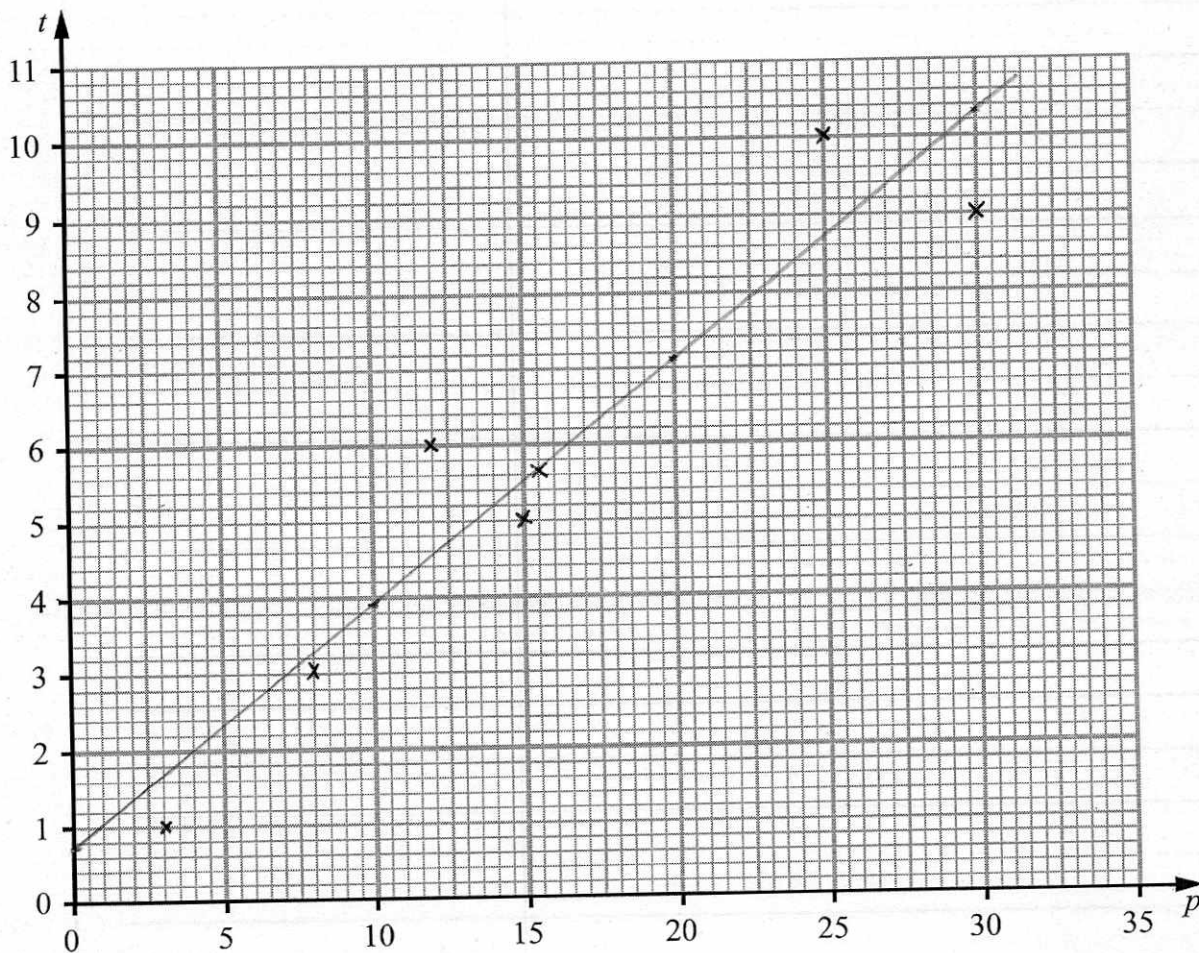
$$= \underline{167}$$

$$S_{pp} = \sum p^2 - \frac{(\sum p)^2}{n}$$

$$= 1967 - \frac{93^2}{6} = \underline{525.5}$$



Question 3 continued



$$d) \quad b = \frac{S_{pt}}{S_{pp}} = \frac{167}{525.5}$$

$$= 0.318 \quad 3st$$

$$a = \bar{t} - b\bar{p} \quad \bar{p} = 15.5 \quad \bar{t} = \frac{17}{3}$$

$$= 0.741 \quad 3st$$

$$t = 0.741 + 0.318p$$

$$P) \quad p=16 \quad t = 0.741 + 0.318(16)$$

$$= 5.8 \quad 1dp$$



4.

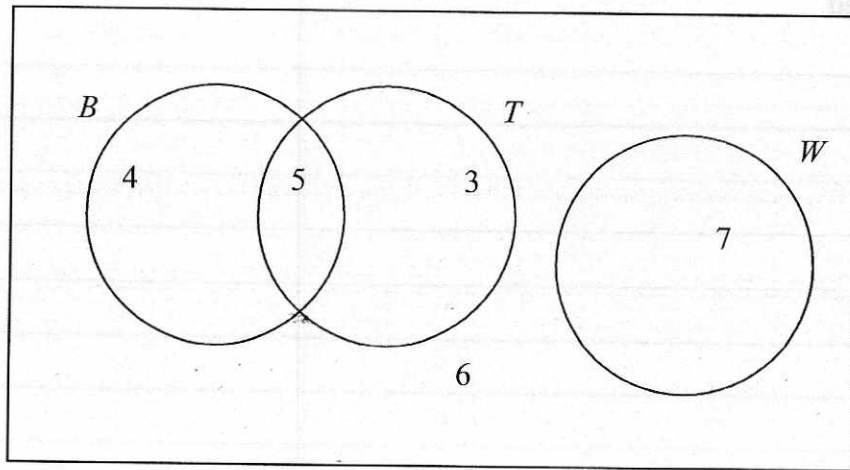


Figure 1

Figure 1 shows how 25 people travelled to work.

Their travel to work is represented by the events

B bicycle

T train

W walk

(a) Write down 2 of these events that are mutually exclusive. Give a reason for your answer. (2)

(b) Determine whether or not B and T are independent events. (3)

One person is chosen at random.

Find the probability that this person

(c) walks to work, (1)

(d) travels to work by bicycle and train. (1)

(e) Given that this person travels to work by bicycle, find the probability that they will also take the train. (2)

a/ B and W

b/ If independent $P(B) \times P(T) = P(B \cap T)$



Question 4 continued

$$P(B) = 9/25$$

$$P(T) = 8/25$$

$$P(B) \times P(T) = \frac{72}{625}$$

$P(B) \times P(T) \neq \frac{5}{25} \therefore B$ and T are not independent.

c/ $7/25$

d/ $5/25$ [$1/5$]

e/ $5/9$

$$\left[\frac{P(B \cap T)}{P(B)} \right]$$

5.

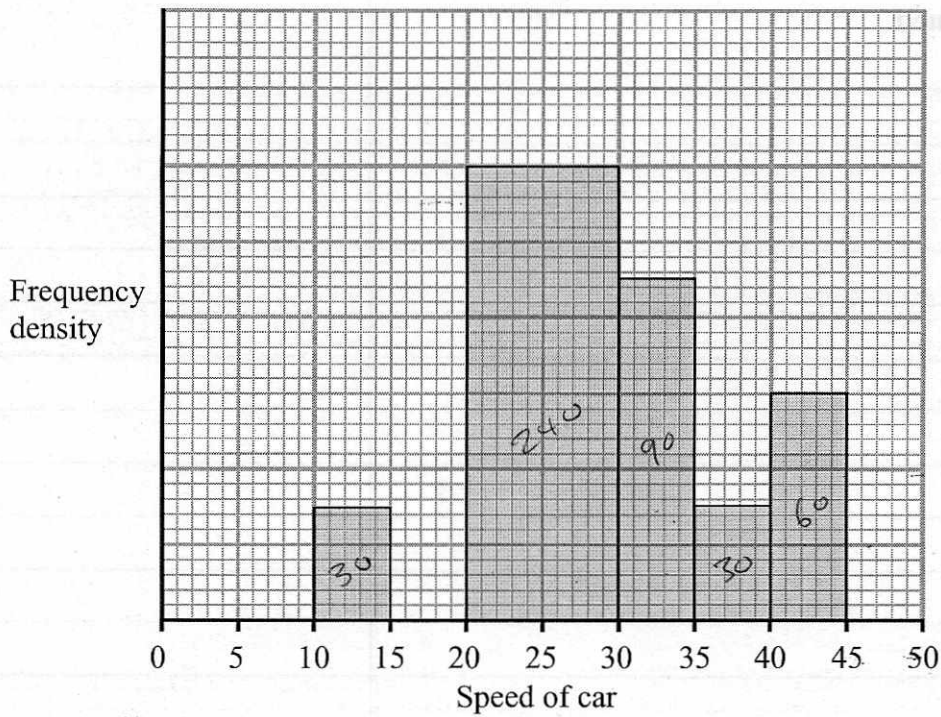


Figure 2

A policeman records the speed of the traffic on a busy road with a 30 mph speed limit. He records the speeds of a sample of 450 cars. The histogram in Figure 2 represents the results.

- (a) Calculate the number of cars that were exceeding the speed limit by at least 5 mph in the sample. (4)
- (b) Estimate the value of the mean speed of the cars in the sample. (3)
- (c) Estimate, to 1 decimal place, the value of the median speed of the cars in the sample. (2)
- (d) Comment on the shape of the distribution. Give a reason for your answer. (2)
- (e) State, with a reason, whether the estimate of the mean or the median is a better representation of the average speed of the traffic on the road. (2)

$$\frac{450}{22.5} = 20 \text{ cars per box}$$

~~$$90 + 30 + 60 = 180$$~~

$$30 + 60 = 90$$



Question 5 continued

speed	frequency	fx
10-15	30	375
20-30	240	6000
30-35	90	2925
35-40	30	1125
40-45	60	2550
		<hr/> 12975

$$\frac{12975}{450} = 28.8 \text{ (3sf) mph}$$

c/ 225th car [Interpolation]

$$20 + \frac{195}{240} \times 10 = 28.1 \text{ (3sf) mph}$$

d/ mean > median ∴ positive skew

e/ there is a skew ∴ the median is a better representation

[median is not affected by extreme values]

