

1. At time $t = 0$ a ball is projected vertically upwards from a point O and rises to a maximum height of 40 m above O . The ball is modelled as a particle moving freely under gravity.

(a) Show that the speed of projection is 28 m s^{-1} . (3)

(b) Find the times, in seconds, when the ball is 33.6 m above O . (5)

$$s = 40$$

$$u = ?$$

$$v = 0$$

$$a = -9.8$$

$$t = .$$

$$v^2 = u^2 + 2as$$

$$0 = u^2 + 2(-9.8)(40)$$

$$784 = u^2$$

$$u = \underline{\underline{28 \text{ m s}^{-1}}}$$

b/

$$s = 33.6$$

$$u = 28$$

$$v =$$

$$a = -9.8$$

$$t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$33.6 = 28t + \frac{1}{2}(-9.8)t^2$$

$$33.6 = 28t - 4.9t^2$$

$$0 = 4.9t^2 - 28t + 33.6$$

$$t = \frac{-(-28) \pm \sqrt{(-28)^2 - 4(4.9)(33.6)}}{2(4.9)}$$

$$= \underline{\underline{4}} \quad \text{and} \quad \underline{\underline{\frac{12}{7} \text{ seconds}}}$$

$$\underline{\underline{1.71 \text{ (3sf)}}}$$



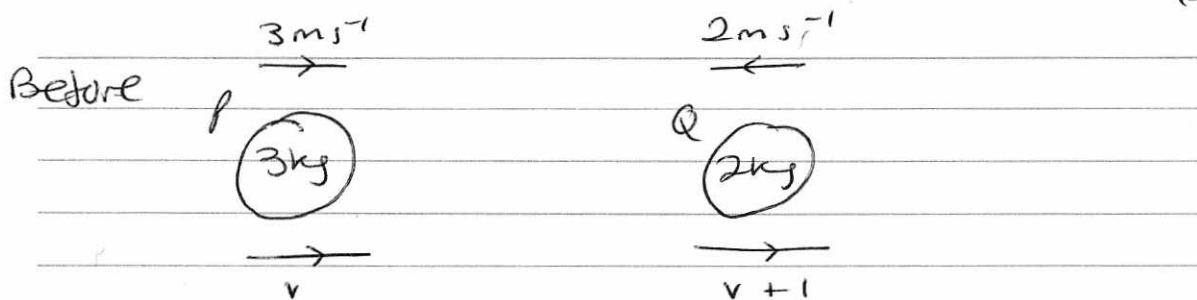
2. Particle P has mass 3 kg and particle Q has mass 2 kg. The particles are moving in opposite directions on a smooth horizontal plane when they collide directly. Immediately before the collision, P has speed 3 m s^{-1} and Q has speed 2 m s^{-1} . Immediately after the collision, both particles move in the same direction and the difference in their speeds is 1 m s^{-1} .

(a) Find the speed of each particle after the collision.

(5)

(b) Find the magnitude of the impulse exerted on P by Q .

(3)



$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$3(3) + 2(-2) = 3(v) + 2(v+1)$$

$$9 - 4 = 3v + 2v + 2$$

$$5 = 5v + 2$$

$$3 = 5v$$

$$v = \frac{0.6 \text{ m s}^{-1}}{[P]} \quad \text{and} \quad \frac{1.6 \text{ m s}^{-1}}{[Q]}$$

b/

$$\begin{aligned}
 I &= mv - mu \\
 &= 3(0.6) - 3(3) \\
 &= 1.8 - 9 \\
 &= 7.2 \text{ N s}
 \end{aligned}$$



3.

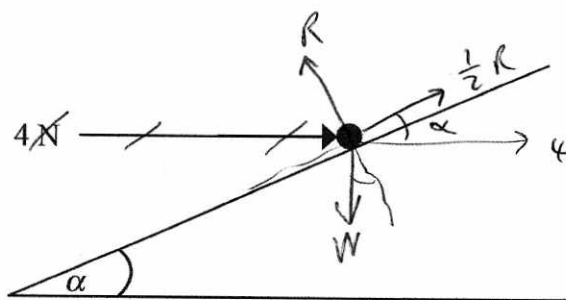


Figure 1

A particle of weight W newtons is held in equilibrium on a rough inclined plane by a horizontal force of magnitude 4 N. The force acts in a vertical plane containing a line of greatest slope of the inclined plane. The plane is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{3}{4}$, as shown in Figure 1.

The coefficient of friction between the particle and the plane is $\frac{1}{2}$.

Given that the particle is on the point of sliding down the plane,

- (i) show that the magnitude of the normal reaction between the particle and the plane is 20 N,
- (ii) find the value of W .

(9)

$$\tan \alpha = \frac{3}{4} \quad \cos \alpha = \frac{4}{5} \quad \sin \alpha = \frac{3}{5}$$

Resolving \uparrow

$$R = 4 \sin \alpha + W \cos \alpha$$

$$R = 4 \left(\frac{3}{5} \right) + W \left(\frac{4}{5} \right)$$

$$R = \frac{12}{5} + \frac{4}{5} W \quad (1)$$

Resolving \nearrow

$$\frac{1}{2} R + 4 \cos \alpha = W \sin \alpha$$

$$\frac{1}{2} R + 4 \left(\frac{4}{5} \right) = W \left(\frac{3}{5} \right)$$

$$\frac{1}{2} R + \frac{16}{5} = \frac{3}{5} W \quad (2)$$

$$(1) \times 10$$

$$10R = 24 + 8W$$

$$(2) \times 10$$

$$10R = 12W - 64$$

$$24 + 8W = 12W - 64$$

$$88 = 4W$$

$$W = 22 \text{ N}$$



Question 3 continued

$$R = \frac{12}{5} + \frac{4}{5}(22)$$
$$= \underline{\underline{20}} \quad N$$

(Total 9 marks)

Q3



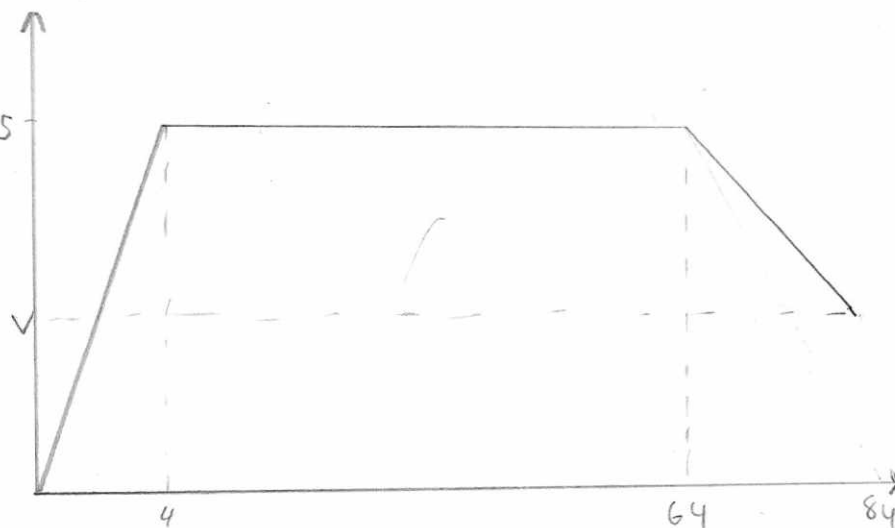
4. A girl runs a 400 m race in a time of 84 s. In a model of this race, it is assumed that, starting from rest, she moves with constant acceleration for 4 s, reaching a speed of 5 m s^{-1} . She maintains this speed for 60 s and then moves with constant deceleration for 20 s, crossing the finishing line with a speed of $V \text{ m s}^{-1}$.

(a) Sketch, in the space below, a speed-time graph for the motion of the girl during the whole race. (2)

(b) Find the distance run by the girl in the first 64 s of the race. (3)

(c) Find the value of V . (5)

(d) Find the deceleration of the girl in the final 20 s of her race. (2)



b/
$$\frac{60 + 64}{2} \times 5 = \underline{\underline{310 \text{ m}}}$$

c/
$$\frac{5 + V}{2} \times 20 = 90$$

$$\underline{\underline{V = 4}}$$

d/
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1}{20} \text{ m s}^{-2}$$

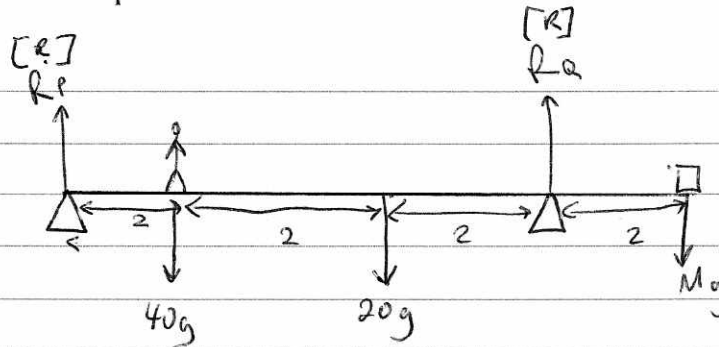


5. A plank PQR , of length 8 m and mass 20 kg, is in equilibrium in a horizontal position on two supports at P and Q , where $PQ = 6$ m.

A child of mass 40 kg stands on the plank at a distance of 2 m from P and a block of mass M kg is placed on the plank at the end R . The plank remains horizontal and in equilibrium. The force exerted on the plank by the support at P is equal to the force exerted on the plank by the support at Q .

By modelling the plank as a uniform rod, and the child and the block as particles,

- (a) (i) find the magnitude of the force exerted on the plank by the support at P ,
 (ii) find the value of M . (10)
- (b) State how, in your calculations, you have used the fact that the child and the block can be modelled as particles. (1)



Forces up = Forces down

$$2R = 60g + Mg \quad (1)$$

Taking moments about R:

$$2R + 8R = 4(20g) + 6(40g)$$

$$10R = 320g$$

$$2R = 64g \quad (2)$$

$$60g + mg = 64g$$

$$\underline{m = 4}$$

$$2R = 60g + 4g$$

$$\underline{R = 32g}$$

b) Forces act at one point



6.

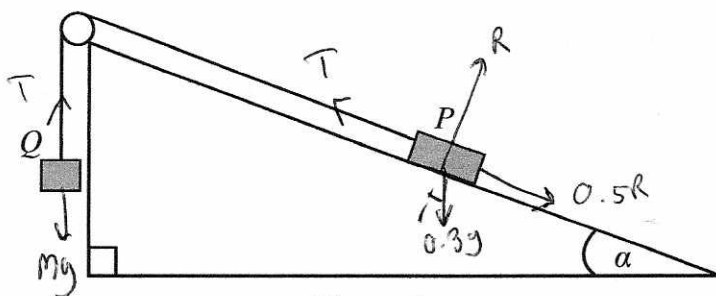


Figure 2

Two particles P and Q have masses 0.3 kg and $m \text{ kg}$ respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a fixed rough plane. The plane is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between P and the plane is $\frac{1}{2}$.

The string lies in a vertical plane through a line of greatest slope of the inclined plane. The particle P is held at rest on the inclined plane and the particle Q hangs freely below the pulley with the string taut, as shown in Figure 2.

The system is released from rest and Q accelerates vertically downwards at 1.4 m s^{-2} . Find

(a) the magnitude of the normal reaction of the inclined plane on P , (2)

(b) the value of m . (8)

When the particles have been moving for 0.5 s , the string breaks. Assuming that P does not reach the pulley,

(c) find the further time that elapses until P comes to instantaneous rest. (6)

$$\tan \alpha = \frac{3}{4} \quad \cos \alpha = \frac{4}{5} \quad \sin \alpha = \frac{3}{5}$$

a/ Q: ~~At~~

$$R = 0.3g \cos \alpha$$

$$= \frac{6}{25}g$$

b/ P: $T - 0.5 \left(\frac{6}{25}g \right) - 0.3g \sin \alpha = 0.3(1.4)$

$$T - \frac{3}{25}g - \frac{96}{50}g = \frac{21}{50}$$

$$T = \frac{84}{25}$$



Question 6 continued

$$Q: \quad mg - T = m(1.4)$$

$$mg - \frac{84}{25} = 1.4m$$

$$mg - 1.4m = \frac{84}{25}$$

$$m(g - 1.4) = \frac{84}{25}$$

$$m = \underline{\underline{0.4 \text{ kg}}}$$

b) Before break :

$$s =$$

$$u = 0$$

$$v =$$

$$a = 1.4$$

$$t = 0.5$$

$$v = u + at$$

$$= 0 + 1.4(0.5)$$

$$= \underline{\underline{0.7}}$$

$$F = ma$$

$$0 - 0.3g \sin \alpha - 0.5 \left(\frac{6}{25}g \right) = 0.3a$$

$$- 2.94 = 0.3a$$

$$a = -9.8$$

After break.

$$s =$$

$$u = 0.7$$

$$v = 0$$

$$a = -9.8$$

$$t = ?$$

$$v = u + at$$

$$0 = 0.7 + (-9.8)t$$

$$t = \frac{1}{14} \text{ seconds}$$

$$= \underline{\underline{0.0714 \text{ seconds (3sf)}}$$

(Total 5 marks)

Q6



7. [In this question \mathbf{i} and \mathbf{j} are unit vectors due east and due north respectively. Position vectors are given relative to a fixed origin O .]

Two ships P and Q are moving with constant velocities. Ship P moves with velocity $(2\mathbf{i} - 3\mathbf{j}) \text{ km h}^{-1}$ and ship Q moves with velocity $(3\mathbf{i} + 4\mathbf{j}) \text{ km h}^{-1}$.

(a) Find, to the nearest degree, the bearing on which Q is moving. (2)

At 2 pm, ship P is at the point with position vector $(\mathbf{i} + \mathbf{j}) \text{ km}$ and ship Q is at the point with position vector $(-2\mathbf{j}) \text{ km}$.

At time t hours after 2 pm, the position vector of P is $\mathbf{p} \text{ km}$ and the position vector of Q is $\mathbf{q} \text{ km}$.

(b) Write down expressions, in terms of t , for

(i) \mathbf{p} ,

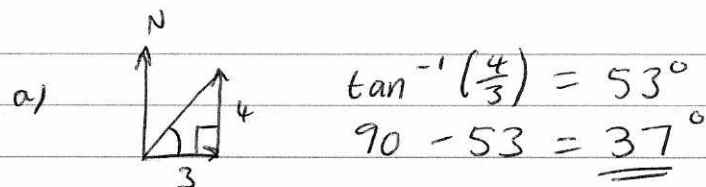
(ii) \mathbf{q} ,

(iii) \overrightarrow{PQ} . (5)

(c) Find the time when

(i) Q is due north of P ,

(ii) Q is north-west of P . (4)



b/ i)

$$\mathbf{p} = \mathbf{i} + \mathbf{j} + t(2\mathbf{i} - 3\mathbf{j})$$

$$= (\mathbf{i} + 2t)\mathbf{i} + (\mathbf{j} - 3t)\mathbf{j}$$

$$\mathbf{q} = -2\mathbf{j} + t(3\mathbf{i} + 4\mathbf{j})$$

$$= 3t\mathbf{i} + (-2 + 4t)\mathbf{j}$$

[a-p] $\overrightarrow{PQ} = (-1 + t)\mathbf{i} + (-3 + 7t)\mathbf{j}$

c/ i) $\mathbf{i} = 0$ $-1 + t = 0$
 $t = 1$ 3pm

ii) $\mathbf{i} = -\mathbf{j}$ $-1 + t = -(-3 + 7t)$
 $-1 + t = 3 - 7t$
 $8t = 4$ $t = \frac{1}{2}$ 2.30pm

