

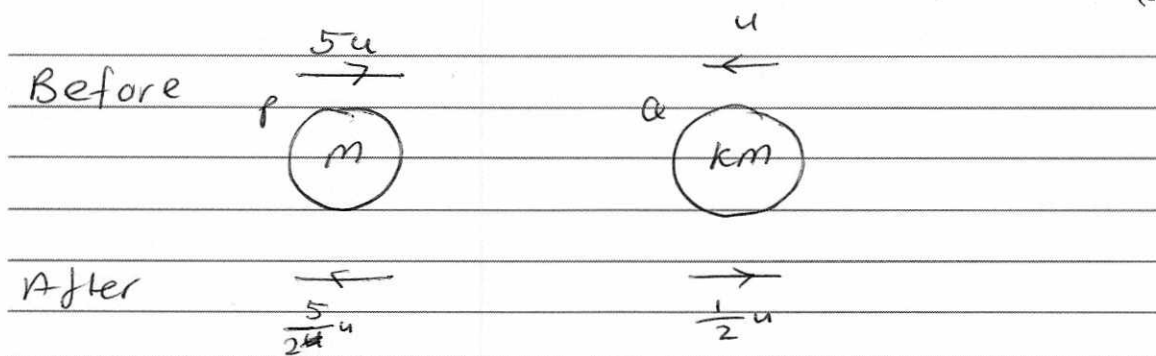


1. Particle  $P$  of mass  $m$  and particle  $Q$  of mass  $km$  are moving in opposite directions on a smooth horizontal plane when they collide directly. Immediately before the collision the speed of  $P$  is  $5u$  and the speed of  $Q$  is  $u$ . Immediately after the collision the speed of each particle is halved and the direction of motion of each particle is reversed.

Find

- (a) the value of  $k$ , (3)

- (b) the magnitude of the impulse exerted on  $P$  by  $Q$  in the collision. (3)



a/

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m(5u) + km(-u) = m\left(-\frac{5}{2}u\right) + km\left(\frac{1}{2}u\right)$$

$$5mu - kmu = -\frac{5}{2}mu + \frac{1}{2}kmu$$

$$5 - k = -\frac{5}{2} + \frac{1}{2}k$$

$$\frac{15}{2} = \frac{3}{2}k$$
~~$$k = \frac{5}{3}$$~~

$$\underline{\underline{k = 5}}$$

b/

$$I = mv - mu$$

$$= m\left(-\frac{5}{2}u\right) - m(5u)$$

$$= -\frac{5}{2}mu - 5mu$$

$$= \underline{\underline{-\frac{15}{2}mu \text{ Ns}}}$$



2. A small stone is projected vertically upwards from a point  $O$  with a speed of  $19.6 \text{ ms}^{-1}$ .

Modelling the stone as a particle moving freely under gravity,

(a) find the greatest height above  $O$  reached by the stone, (2)

(b) find the length of time for which the stone is more than  $14.7 \text{ m}$  above  $O$ . (5)

$$a) \quad s = ?$$

$$u = 19.6$$

$$v = 0$$

$$a = -9.8$$

$$t =$$

$$v^2 = u^2 + 2as$$

$$0 = (19.6)^2 + 2(-9.8)s$$

$$s = \underline{19.6 \text{ m}}$$

$$b) \quad s = 14.7$$

$$u = 19.6$$

$$v =$$

$$a = -9.8$$

$$t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$14.7 = 19.6t + \frac{1}{2}(-9.8)t^2$$

$$14.7 = 19.6t - 4.9t^2$$

$$4.9t^2 - 19.6t + 14.7 = 0$$

$$a = 4.9 \quad b = -19.6 \quad c = 14.7$$

$$t = \frac{-(-19.6) \pm \sqrt{(-19.6)^2 - 4(4.9)(14.7)}}{2(4.9)}$$

$$t = 3 \quad \text{and} \quad t = 1$$

$$3 - 1 = 2$$

2 seconds



3.

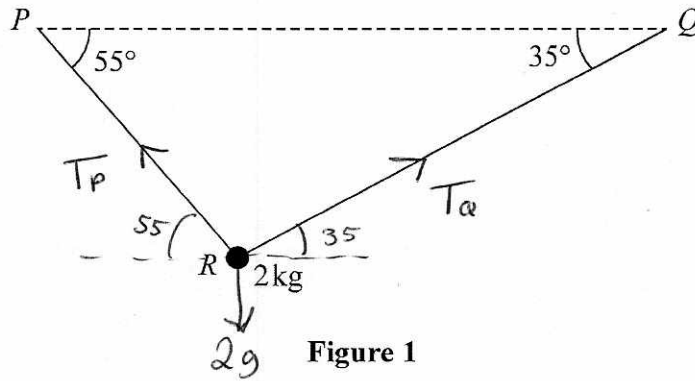


Figure 1

A particle of mass 2 kg is suspended from a horizontal ceiling by two light inextensible strings,  $PR$  and  $QR$ . The particle hangs at  $R$  in equilibrium, with the strings in a vertical plane. The string  $PR$  is inclined at  $55^\circ$  to the horizontal and the string  $QR$  is inclined at  $35^\circ$  to the horizontal, as shown in Figure 1.

Find

- (i) the tension in the string  $PR$ ,
- (ii) the tension in the string  $QR$ .

(7)

$$\leftarrow = \rightarrow$$

$$T_P \cos 55 = T_Q \cos 35$$

$$T_P = \frac{T_Q \cos 35}{\cos 55}$$

$$\uparrow = \downarrow$$

$$T_P \sin 55 + T_Q \sin 35 = 2g$$

$$\frac{T_Q \cos 35 \sin 55 + T_Q \sin 35}{\cos 55} = 2g$$

$$T_Q (\cos 35 \tan 55 + \sin 35) = 2g$$

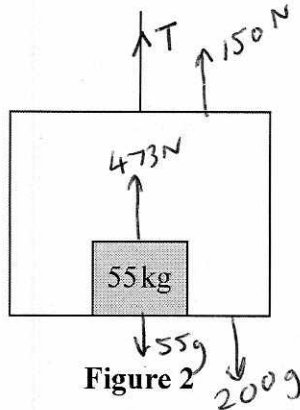
$$T_Q = \underline{\underline{11.2 \text{ N (3sf)}}}$$

$$T_P = \frac{11.2 \cos 35}{\cos 55}$$

$$= \underline{\underline{16.1 \text{ N (3sf)}}}$$



4.



A lift of mass 200 kg is being lowered into a mineshaft by a vertical cable attached to the top of the lift. A crate of mass 55 kg is on the floor inside the lift, as shown in Figure 2. The lift descends vertically with constant acceleration. There is a constant upwards resistance of magnitude 150 N on the lift. The crate experiences a constant normal reaction of magnitude 473 N from the floor of the lift.

- (a) Find the acceleration of the lift. (3)
- (b) Find the magnitude of the force exerted on the lift by the cable. (4)

a) crate:

$$55g - 473 = 55a$$

$$66 = 55a$$

$$a = \underline{\underline{1.2 \text{ m s}^{-2}}}$$

b) whole system:

$$255g - T - 150 = 255(1.2)$$

$$2499 - T - 150 = 306$$

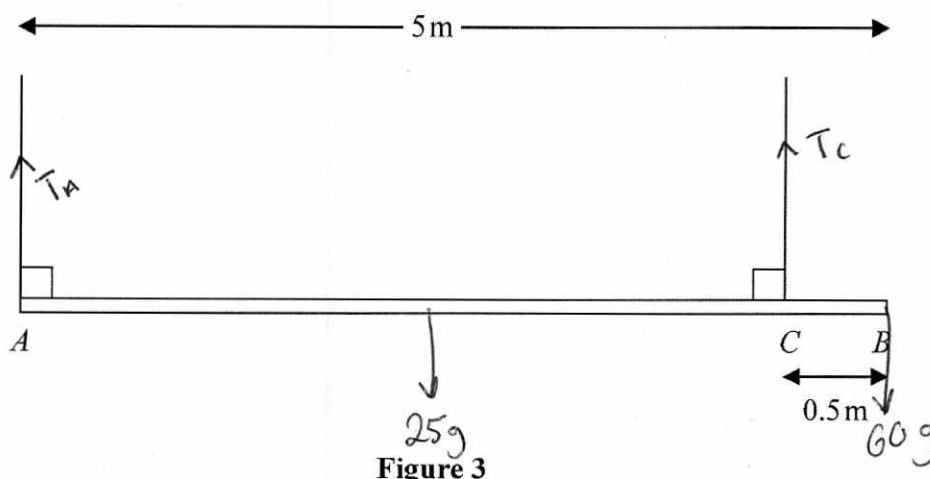
$$2499 - 150 - 306 = T$$

$$T = 2043 \text{ N}$$

$$= \underline{\underline{2040 \text{ N (3sf)}}}$$



5.



A beam  $AB$  has length  $5\text{ m}$  and mass  $25\text{ kg}$ . The beam is suspended in equilibrium in a horizontal position by two vertical ropes. One rope is attached to the beam at  $A$  and the other rope is attached to the point  $C$  on the beam where  $CB = 0.5\text{ m}$ , as shown in Figure 3. A particle  $P$  of mass  $60\text{ kg}$  is attached to the beam at  $B$  and the beam remains in equilibrium in a horizontal position. The beam is modelled as a uniform rod and the ropes are modelled as light strings.

(a) Find

- (i) the tension in the rope attached to the beam at  $A$ ,
- (ii) the tension in the rope attached to the beam at  $C$ .

(6)

Particle  $P$  is removed and replaced by a particle  $Q$  of mass  $M\text{ kg}$  at  $B$ . Given that the beam remains in equilibrium in a horizontal position,

(b) find

- (i) the greatest possible value of  $M$ ,
- (ii) the greatest possible tension in the rope attached to the beam at  $C$ .

(6)

a) Taking moments about C:

$$0.5(60g) + 4.5T_A = 2(25g)$$

$$30g + 4.5T_A = 50g$$

$$4.5T_A = 20g$$

$$T_A = \frac{40}{9}g \text{ N } (43.6\text{ N } 3\text{ sf})$$



## Question 5 continued

ii) Forces up = forces down

$$\frac{40}{9}g + T_c = 85g$$

$$T_c = \frac{725}{9}g \text{ N} \quad (789 \text{ N } 3 \text{ sf})$$

b) i)

Beam on the point of tipping about C

$$T_A = 0$$

Taking moments about C:

$$0.5 Mg = 2(25g)$$

$$0.5 Mg = 50g$$

$$M = \underline{\underline{100 \text{ kg}}}$$

ii) Forces up = forces down

$$T_c = \cancel{125} 100g + 25g$$

$$= \underline{\underline{125g}}$$



6. A particle  $P$  is moving with constant velocity. The position vector of  $P$  at time  $t$  seconds ( $t \geq 0$ ) is  $\mathbf{r}$  metres, relative to a fixed origin  $O$ , and is given by

$$\mathbf{r} = (2t - 3)\mathbf{i} + (4 - 5t)\mathbf{j}$$

(a) Find the initial position vector of  $P$ .

(1)

The particle  $P$  passes through the point with position vector  $(3.4\mathbf{i} - 12\mathbf{j})\text{m}$  at time  $T$  seconds.

(b) Find the value of  $T$ .

(3)

(c) Find the speed of  $P$ .

(4)

6a)  $-3\mathbf{i} + 4\mathbf{j}$   $t=0$

b/ i)  $2t - 3 = 3.4$   
 $2t = 6.4$   
 $t = 3.2$

c/  $\mathbf{r} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} + t \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

$$\sqrt{2^2 + 5^2} = \sqrt{29} \text{ ms}^{-1}$$

$$= 5.39 \text{ ms}^{-1} \text{ (3sf)}$$

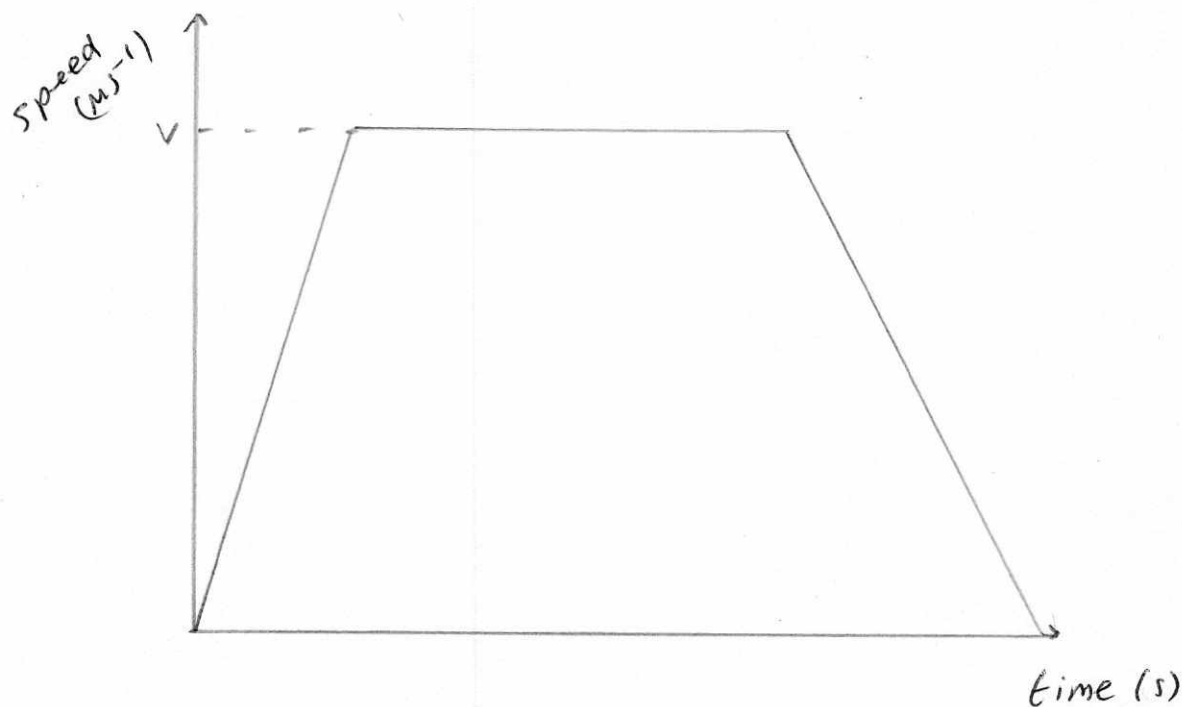




7. A train travels along a straight horizontal track between two stations,  $A$  and  $B$ . The train starts from rest at  $A$  and moves with constant acceleration  $0.5 \text{ m s}^{-2}$  until it reaches a speed of  $V \text{ m s}^{-1}$ , ( $V < 50$ ). The train then travels at this constant speed before it moves with constant deceleration  $0.25 \text{ m s}^{-2}$  until it comes to rest at  $B$ .

(a) Sketch in the space below a speed-time graph for the motion of the train between the two stations  $A$  and  $B$ .

(2)



The total time for the journey from  $A$  to  $B$  is 5 minutes.

(b) Find, in terms of  $V$ , the length of time, in seconds, for which the train is

- (i) accelerating,
- (ii) decelerating,
- (iii) moving with constant speed.

(5)

Given that the distance between the two stations  $A$  and  $B$  is 6.3 km,

(c) find the value of  $V$ .

(6)



## Question 7 continued

$$\text{bi/ } \text{acceleration} = \frac{\text{velocity}}{\text{time}} \quad \begin{array}{l} \text{(change in)} \\ \text{(change in)} \end{array}$$

$$0.5 = \frac{v}{t}$$

$$\underline{t = 2v}$$

$$\text{ii/ } -0.25 = \frac{-v}{t}$$

$$\underline{t = 4v}$$

$$\text{iii/ } \underline{300 - 6v} \quad \boxed{300 \text{ seconds total}}$$

$$\text{c/ } \text{Total distance} = 6300\text{m} \quad \boxed{\text{Area}}$$

$$\left( \frac{300 + 300 - 6v}{2} \right) (v) = 6300$$

$$(300 - 3v)(v) = 6300$$

$$300v - 3v^2 = 6300$$

$$100v - v^2 = 2100$$

$$v^2 - 100v + 2100 = 0$$

$$(v - 30)(v - 70) = 0$$

$$v = 30 \quad v = 70$$

$v$  is less than 50

$$\therefore \underline{v = 30 \text{ ms}^{-1}}$$



8.

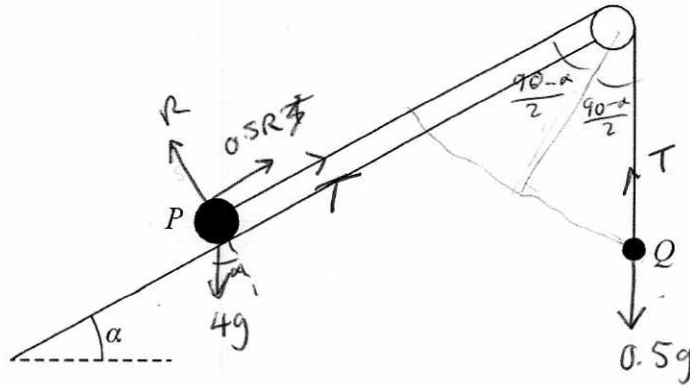


Figure 4

$\mu = 0.5$

Two particles  $P$  and  $Q$  have mass 4 kg and 0.5 kg respectively. The particles are attached to the ends of a light inextensible string. Particle  $P$  is held at rest on a fixed rough plane, which is inclined to the horizontal at an angle  $\alpha$  where  $\tan \alpha = \frac{4}{3}$ . The coefficient of friction between  $P$  and the plane is 0.5. The string lies along the plane and passes over a small smooth light pulley which is fixed at the top of the plane. Particle  $Q$  hangs freely at rest vertically below the pulley. The string lies in the vertical plane which contains the pulley and a line of greatest slope of the inclined plane, as shown in Figure 4. Particle  $P$  is released from rest with the string taut and slides down the plane.

Given that  $Q$  has not hit the pulley, find

- (a) the tension in the string during the motion, (11)
- (b) the magnitude of the resultant force exerted by the string on the pulley. (4)

$$\tan \alpha = \frac{4}{3} \quad \cos \alpha = \frac{3}{5} \quad \sin \alpha = \frac{4}{5}$$

P//  $R = 4g \cos \alpha$   
 $= \frac{12}{5}g$

$$4g \sin \alpha - 0.5 \left( \frac{12}{5}g \right) - T = 4a$$

$$\frac{16}{5}g - \frac{6}{5}g - T = 4a$$

$$2g - T = 4a \quad (1)$$

Q//  $T - 0.5g$   
 ~~$0.5g - T = 0.5a$~~   
 ~~$4g - 8T = 4a$~~  (2)  
 $8T - 4g = 4a$



## Question 8 continued

$$2g - T = 8T - 4g$$

$$6g = 9T$$

$$T = \frac{2}{3}g \text{ N} \quad (6.53 \text{ N } 3\text{sf})$$

b/

$$2 T \cos\left(\frac{90-\alpha}{2}\right)$$

$$2 \left(\frac{2}{3}g\right) \cos(18.4\dots)$$

$$12.4 \text{ N } (3\text{sf})$$

