Write your name here

Surname	Other names

Pearson
Edexcel GCE

Centre Number	Candidate Number

Core Mathematics C4

Advanced

Friday 24 June 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
6666/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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PEARSON

1. Use the binomial series to find the expansion of

$$\frac{1}{(2+5x)^3}, \qquad |x| < \frac{2}{5}$$

in ascending powers of x, up to and including the term in x^3.
Give each coefficient as a fraction in its simplest form.

(6)

1) $(2+5x)^{-3}$

$2^{-3}\left(1+\frac{5}{2}x\right)^{-3}$

$\frac{1}{8}\left(1+\frac{5}{2}x\right)^{-3}$

$\frac{1}{8}\left(1+(-3)\left(\frac{5}{2}x\right)+\frac{(-3)(-4)}{2}\left(\frac{5}{2}x\right)^2+\frac{(-3)(-4)(-5)}{6}\left(\frac{5}{2}x\right)^3\right)$

$\frac{1}{8}\left(1-\frac{15}{2}x+\frac{75}{2}x^2-\frac{625}{4}x^3\right)$

$\frac{1}{8}-\frac{15}{16}x+\frac{75}{16}x^2-\frac{625}{32}x^3$

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2.

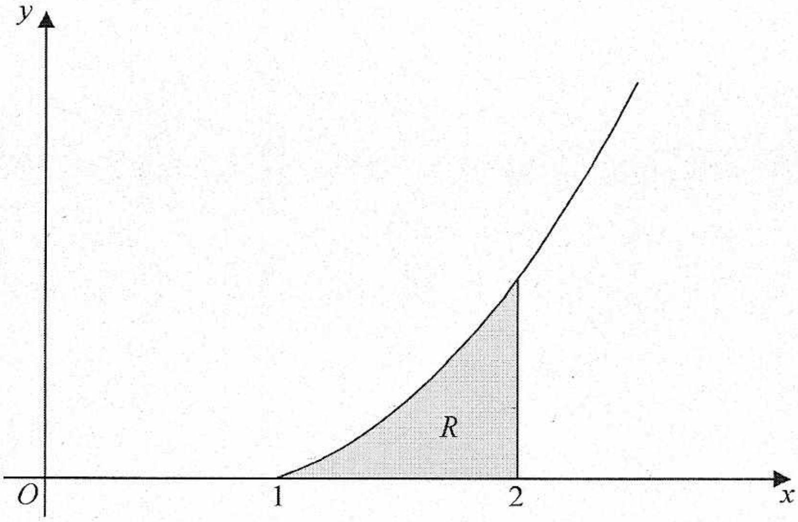

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = x^2 \ln x, \quad x \geqslant 1$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 2$

The table below shows corresponding values of x and y for $y = x^2 \ln x$

x	1	1.2	1.4	1.6	1.8	2
y	0	0.2625	0.6595	1.2032	1.9044	2.7726

(a) Complete the table above, giving the missing value of y to 4 decimal places.
(1)

(b) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of R, giving your answer to 3 decimal places.
(3)

(c) Use integration to find the exact value for the area of R.
(5)

b/

$$0.2\left(\frac{0}{2} + 0.2625 + 0.6595 + 1.2632 + 1.9044 + \frac{2.7726}{2}\right)$$

$$= 1.083 \ (3dp)$$

c/

$$\int u \frac{dv}{dx}\, dx = uv - \int v \frac{du}{dx}\, dx$$

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Question 2 continued

$u = \ln x \qquad \frac{dv}{dx} = x^2$

$\frac{du}{dx} = \frac{1}{x} \qquad v = \frac{x^3}{3}$

$\left[\frac{x^3}{3} \ln x - \int \frac{x^3}{3} \cdot \frac{1}{x} \, dx \right]_1^2$

$\left[\frac{x^3}{3} \ln x - \int \frac{x^2}{3} \, dx \right]_1^2$

$\left[\frac{x^3}{3} \ln x - \frac{x^3}{9} \right]_1^2$

$\left(\frac{8}{3} \ln 2 - \frac{8}{9} \right) - \left(\frac{1}{3} \ln 1 - \frac{1}{9} \right)$

$\frac{8}{3} \ln 2 - \frac{8}{9} - \left(-\frac{1}{9} \right)$

$\frac{8}{3} \ln 2 - \frac{7}{9}$

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3. The curve C has equation

$$2x^2y + 2x + 4y - \cos(\pi y) = 17$$

(a) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

(5)

The point P with coordinates $\left(3, \frac{1}{2}\right)$ lies on C.

The normal to C at P meets the x-axis at the point A.

(b) Find the x coordinate of A, giving your answer in the form $\frac{a\pi + b}{c\pi + d}$, where a, b, c and d are integers to be determined.

(4)

$$2x^2y + 2x + 4y - \cos(\pi y) = 17$$

$$\boxed{\begin{array}{ll} u = 2x^2 & v = y \\ \frac{du}{dx} = 4x & \frac{dv}{dx} = \frac{dy}{dx} \end{array}}$$

$$2x^2\frac{dy}{dx} + 4xy + 2 + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(2x^2 + 4 + \pi\sin(\pi y)\right) + 4xy + 2 = 0$$

$$\frac{dy}{dx}\left(2x^2 + 4 + \pi\sin(\pi y)\right) = -2 - 4xy$$

$$\frac{dy}{dx} = \frac{-2 - 4xy}{2x^2 + 4 + \pi\sin(\pi y)}$$

b/ gradient of tangent when $x = 3$ $y = \frac{1}{2}$

$$\frac{dy}{dx} = \frac{-2 - 4(3)(\frac{1}{2})}{2(3)^2 + 4 + \pi\sin(\frac{1}{2}\pi)}$$

$$= \frac{-8}{22 + \pi}$$

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Question 3 continued

$\therefore$ gradient of normal $= \frac{22+\pi}{8}$

$y = \left(\frac{22+\pi}{8}\right)x + c$

$\frac{1}{2} = \left(\frac{22+\pi}{8}\right)(3) + c$

$\frac{1}{2} = \cancel{22}\ \frac{66+3\pi}{8} + c$

$4 = 66 + 3\pi + 8c$

$-3\pi - 62 = 8c$

$c = \frac{-3\pi - 62}{8}$

$y = \left(\frac{22+\pi}{8}\right)x - \frac{3\pi - 62}{8}$

Crosses x when $y=0$

$0 = \left(\frac{22+\pi}{8}\right)x - \frac{3\pi - 62}{8}$

$0 = (22+\pi)x - 3\pi - 62$

$3\pi + 62 = (22+\pi)x$

$x = \frac{3\pi + 62}{22+\pi}$

$= \frac{3\pi + 62}{\pi + 22}$

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4. The rate of decay of the mass of a particular substance is modelled by the differential equation

$$\frac{dx}{dt} = -\frac{5}{2}x, \qquad t \geqslant 0$$

where x is the mass of the substance measured in grams and t is the time measured in days.

Given that $x = 60$ when $t = 0$,

(a) solve the differential equation, giving x in terms of t. You should show all steps in your working and give your answer in its simplest form.

(4)

(b) Find the time taken for the mass of the substance to decay from 60 grams to 20 grams. Give your answer to the nearest minute.

(3)

a/ $\int \frac{1}{x}\, dx = \int -\frac{5}{2}\, dt$

$\ln x = \frac{-5}{2}t + c$

$\ln(60) = \frac{-5}{2}(0) + c$

$c = \ln(60)$

$\ln x = \frac{-5}{2}t + \ln(60)$

$\ln x - \ln 60 = \frac{-5}{2}t$

$\ln \frac{x}{60} = -\frac{5}{2}t$

$\frac{x}{60} = e^{-\frac{5}{2}t}$

$x = 60e^{-\frac{5}{2}t}$

b/ $20 = 60e^{-\frac{5}{2}t}$

$\frac{1}{3} = e^{-\frac{5}{2}t}$

$\ln \frac{1}{3} = -\frac{5}{2}t$

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Question 4 continued

$t = -\frac{2}{5} \ln \frac{1}{3}$

$= 0.43944\ldots$ days

$= 633$ minutes.

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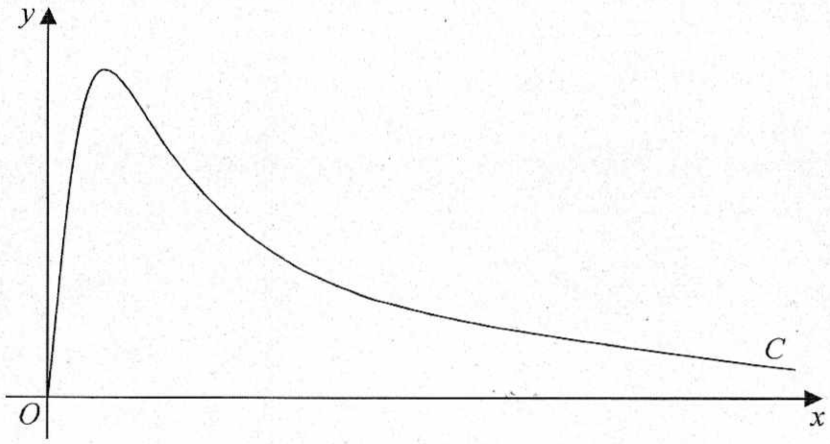

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\tan t, \qquad y = 5\sqrt{3}\sin 2t, \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(4\sqrt{3}, \frac{15}{2}\right)$.

(a) Find the exact value of $\frac{dy}{dx}$ at the point P.

Give your answer as a simplified surd.

(4)

The point Q lies on the curve C, where $\frac{dy}{dx} = 0$

(b) Find the exact coordinates of the point Q.

(2)

a/ $\frac{dx}{dt} = 4\sec^2 t \qquad \frac{dy}{dt} = 10\sqrt{3}\cos 2t$

$\frac{dy}{dx} = \frac{10\sqrt{3}\cos 2t}{4\sec^2 t}$

$= \frac{5\sqrt{3}\cos 2t}{2\sec^2 t}$

$= \frac{5}{2}\sqrt{3}\cos 2t\cos^2 t$

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Question 5 continued

when $x = 4\sqrt{3}$

$4\sqrt{3} = 4 \tan t$

$\sqrt{3} = \tan t$

$t = \pi/3$

$\frac{dy}{dx} = \frac{5}{2}\sqrt{3} \cos\left(\frac{2\pi}{3}\right)\left(\cos\left(\frac{\pi}{3}\right)\right)^2$

$= \frac{-15}{16}\sqrt{3}$

b) $\frac{10\sqrt{3} \cos 2t}{4 \sec^2 t} = 0$

$10\sqrt{3} \cos 2t = 0$

$\cos 2t = 0$

$2t = \frac{\pi}{2}, \frac{3\pi}{2}$

$t = \frac{\pi}{4}, \frac{3\pi}{4}$

between 0 and $\frac{\pi}{2}$ so $\underline{t = \frac{\pi}{4}}$

$x = 4 \tan \frac{\pi}{4}$ $\quad$ $y = 5\sqrt{3} \sin\left(\frac{\pi}{2}\right)$

$= 4$ $\quad$ $= 5\sqrt{3}$

$(4, 5\sqrt{3})$

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6. (i) Given that $y > 0$, find

$$\int \frac{3y-4}{y(3y+2)}\,dy$$

(6)

(ii) (a) Use the substitution $x = 4\sin^2\theta$ to show that

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,dx = \lambda\int_0^{\frac{\pi}{3}} \sin^2\theta\,d\theta$$

where λ is a constant to be determined.

(5)

(b) Hence use integration to find

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,dx$$

giving your answer in the form $a\pi + b$, where a and b are exact constants.

(4)

$$\frac{3y-4}{y(3y+2)} = \frac{A}{y} + \frac{B}{3y+2}$$

$$3y-4 = A(3y+2) + By$$

let $y = 0$

$$-4 = 2A$$
$$A = -2$$

Let $y = \frac{-2}{3}$

$$-6 = \frac{-2}{3}B$$

$$B = 9$$

$$\int \frac{-2}{y} + \frac{9}{3y+2}\,dy$$

$$-2\ln y + 3\ln(3y+2) + c$$

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Question 6 continued

ii/ $\int_0^3 \sqrt{\frac{x}{4-x}}\, dx$

$x = 4\sin^2\theta$
$= 4(\sin\theta)^2$

$\frac{dx}{d\theta} = 8\sin\theta\cos\theta$

when $x=3$
$3 = 4\sin^2\theta$
$\frac{3}{4} = \sin^2\theta$
$\frac{\sqrt{3}}{2} = \sin\theta$
$\theta = \frac{\pi}{3}$

when $x=0$
$0 = 4\sin^2\theta$
$0 = \sin^2\theta$
$0 = \sin\theta$
$\theta = 0$

$\int_0^{\frac{\pi}{3}} \sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}\ \frac{dx}{d\theta}\ d\theta$

$\int_0^{\frac{\pi}{3}} \sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}\ 8\sin\theta\cos\theta\ d\theta$

$\int_0^{\frac{\pi}{3}} \sqrt{\frac{\cancel{4}\sin^2\theta}{\cancel{4}\cos^2\theta}} \cdot 8\sin\theta\cos\theta\ d\theta$

$\int_0^{\frac{\pi}{3}} \frac{\sin\theta}{\cancel{\cos\theta}} \cdot 8\sin\theta\ \cancel{\cos\theta}\ d\theta$

$8\int_0^{\frac{\pi}{3}} \sin^2\theta\ d\theta.$

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Question 6 continued

b/ $8\int_0^{\frac{\pi}{3}} \sin^2\theta \quad d\theta$

$$\boxed{\begin{aligned} \cos 2\theta &= \cos^2\theta - \sin^2\theta \\ \cos 2\theta &= 1 - 2\sin^2\theta \\ \sin^2\theta &= \tfrac{1}{2} - \tfrac{1}{2}\cos 2\theta \end{aligned}}$$

$$8\int_0^{\frac{\pi}{3}} \frac{1}{2} - \frac{1}{2}\cos 2\theta \quad d\theta$$

$$4\int_0^{\frac{\pi}{3}} 1 - \cos 2\theta \quad d\theta$$

$$4\left[\theta - \frac{1}{2}\sin 2\theta + c\right]_0^{\frac{\pi}{3}}$$

$$4\left[\left(\frac{\pi}{3} - \frac{1}{2}\sin\frac{2\pi}{3}\right) - \left(\cancel{\tfrac{1}{2}}\,0\right)\right]$$

$$4\left(\frac{\pi}{3} - \frac{1}{2}\cdot\frac{\sqrt{3}}{2}\right)$$

$$4\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$$

$$\underline{\underline{\frac{4}{3}\pi - \sqrt{3}}}$$

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Question 6 continued

Q6

(Total 15 marks)

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7. (a) Find

$$\int (2x - 1)^{\frac{3}{2}} \, dx$$

giving your answer in its simplest form.

(2)

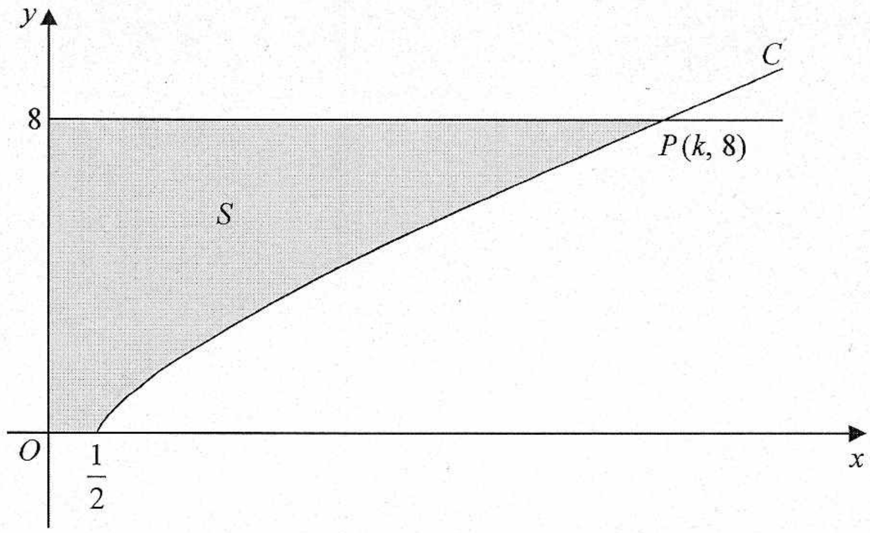

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = (2x - 1)^{\frac{3}{4}}, \qquad x \geqslant \frac{1}{2}$$

The curve C cuts the line $y = 8$ at the point P with coordinates $(k, 8)$, where k is a constant.

(b) Find the value of k.

(2)

The finite region S, shown shaded in Figure 3, is bounded by the curve C, the x-axis, the y-axis and the line $y = 8$. This region is rotated through 2π radians about the x-axis to form a solid of revolution.

(c) Find the exact value of the volume of the solid generated.

(4)

a) $\dfrac{\frac{2}{5}(2x-1)^{\frac{5}{2}}}{2}$

$\frac{1}{5}(2x-1)^{5/2} + C$

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Question 7 continued

b/ $y = (2x-1)^{3/4}$ $\quad (\underset{x}{k}, \underset{y}{8})$

$8 = (2k-1)^{3/4}$

$4096 = (2k-1)^3$

$16 = 2k - 1$

$17 = 2k$

$\underline{k = 17/2}$

c/ Vol of S = Vol. of Cylinder − Area Under Curve.

vol. of cyclinder $= \pi r^2 h.$

$= \pi (8)^2 (\frac{17}{2})$

$= \underline{544\pi}$

Area under curve $= \pi \int_{\frac{1}{2}}^{\frac{17}{2}} y^2 \, dx$

$= \pi \int_{\frac{1}{2}}^{17/2} (2x-1)^{3/2} \, dx$

$= \pi \left[\frac{1}{5} (2x-1)^{5/2} \right]_{\frac{1}{2}}^{17/2}$

$= \pi \left[\left(\frac{1}{5} (2(\frac{17}{2}) - 1 \right)^{5/2} - \left(\frac{1}{5} (2(\frac{1}{2}) - 1 \right)^{5/2} \right]$

$= \frac{1024}{5} \pi$

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Question 7 continued

$$\text{vol. of } S = 544\pi - \frac{1024}{5}\pi$$

$$= \frac{1696}{5}\pi$$

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8. With respect to a fixed origin O, the line l_1 is given by the equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

where μ is a scalar parameter.

The point A lies on l_1 where $\mu = 1$

(a) Find the coordinates of A.

(1)

The point P has position vector $\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$.

The line l_2 passes through the point P and is parallel to the line l_1

(b) Write down a vector equation for the line l_2

(2)

(c) Find the exact value of the distance AP.
Give your answer in the form $k\sqrt{2}$, where k is a constant to be determined.

(2)

The acute angle between AP and l_2 is θ.

(d) Find the value of $\cos\theta$

(3)

A point E lies on the line l_2
Given that $AP = PE$,

(e) find the area of triangle APE,

(2)

(f) find the coordinates of the two possible positions of E.

(5)

a/ $r_A = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + 1 \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix}$ $\Rightarrow$ so $(3, 5, 0)$

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Question 8 continued

b/ $r = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$

c/ $(3, 5, 0)$ and $(1, 5, 2)$

$AP^2 = 2^2 + 0^2 + 2^2$

$AP^2 = 8$

$AP = \sqrt{8} = \underline{\underline{2\sqrt{2}}}$

d/

$a.b = |a||b| \cos\theta.$

$\overrightarrow{AP} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$

$a.b = (-2)(-5) + (0)(4) + (2)(3)$
$= 16$

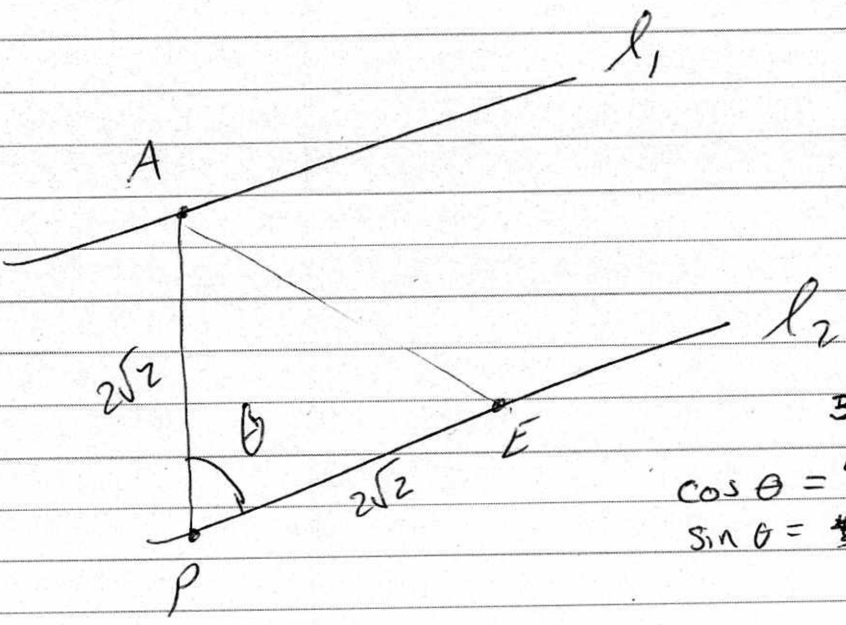
$|a| = 2\sqrt{2}$

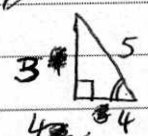
$|b| = \sqrt{5^2 + 4^2 + 3^2}$
$= \sqrt{50}$
$= 5\sqrt{2}$

$\cos\theta = \dfrac{16}{(2\sqrt{2})(5\sqrt{2})}$

$= \dfrac{16}{20} = \dfrac{4}{5}$

Question 8 continued





$\cos\theta = 4/5$

$\sin\theta = 3/5$

$\text{Area} = \frac{1}{2}ab\sin C$

$= \frac{1}{2}(2\sqrt{2})(2\sqrt{2})\sin\theta$

$= \frac{1}{2}(2\sqrt{2})(2\sqrt{2})\left(\frac{3}{5}\right)$

$= \underline{\underline{2.4 \text{ units}^2}}$

f/ direction of l_2 $\begin{pmatrix}-5\\4\\3\end{pmatrix}$

Distance of $\begin{pmatrix}-5\\4\\3\end{pmatrix} = \sqrt{50} = 5\sqrt{2}$

Distance $\overrightarrow{PE} = 2\sqrt{2} = \frac{2}{5}\begin{pmatrix}-5\\4\\3\end{pmatrix}$

$= \begin{pmatrix}-2\\8/5\\6/5\end{pmatrix}$

$P_E = \begin{pmatrix}1\\5\\2\end{pmatrix} + \begin{pmatrix}-2\\8/5\\6/5\end{pmatrix}$ OR $\begin{pmatrix}1\\5\\2\end{pmatrix} - \begin{pmatrix}-2\\8/5\\6/5\end{pmatrix}$

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Question 8 continued

$$\begin{pmatrix} -1 \\ 33/5 \\ 16/5 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 3 \\ 17/5 \\ 4/5 \end{pmatrix}$$

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