Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	5	/	0	1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Friday 25 January 2013 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only		

Team Leader's use only		

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

$x \quad y$

Find

(a) the value of w, **(2)**

(b) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants. **(5)**

1a)

$-32 = (2w - 3)^5$

$-2 = 2w - 3$

$1 = 2w$

$\underline{w = 1/2}$

b/

$\frac{dy}{dx} = 5(2x - 3)^4 \times 2$

$= 10(2x - 3)^4$

when $x = 1/2$

$\frac{dy}{dx} = 10(2(1/2) - 3)^4$

$= 160$

$\therefore m = 160$

$y = mx + c \qquad (1/2, -32)$

$-32 = 1/2(160) + c$

$-32 = 80 + c$

$c = -112$

$\underline{\underline{y = 160x - 112}}$

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2. $$g(x) = e^{x-1} + x - 6$$

(a) Show that the equation $g(x) = 0$ can be written as

$$x = \ln(6 - x) + 1, \qquad x < 6$$

(2)

The root of $g(x) = 0$ is α.

The iterative formula

$$x_{n+1} = \ln(6 - x_n) + 1, \qquad x_0 = 2$$

is used to find an approximate value for α.

(b) Calculate the values of x_1, x_2 and x_3 to 4 decimal places.

(3)

(c) By choosing a suitable interval, show that $\alpha = 2.307$ correct to 3 decimal places.

(3)

a/ $e^{x-1} + x - 6 = 0$

$e^{x-1} = 6 - x$

$x - 1 = \ln(6 - x)$

$\underline{x = \ln(6-x) + 1}$

b/ $x_0 = 2$

$x_1 = 2.3863$

$x_2 = 2.2847$

$x_3 = 2.3125$

c/ $g(2.3065) = -2.75 \times 10^{-4}$

$g(2.3075) = 4.42 \times 10^{-3}$

change of sign $\therefore$ $\alpha = 2.307$ to 3dp.

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3.

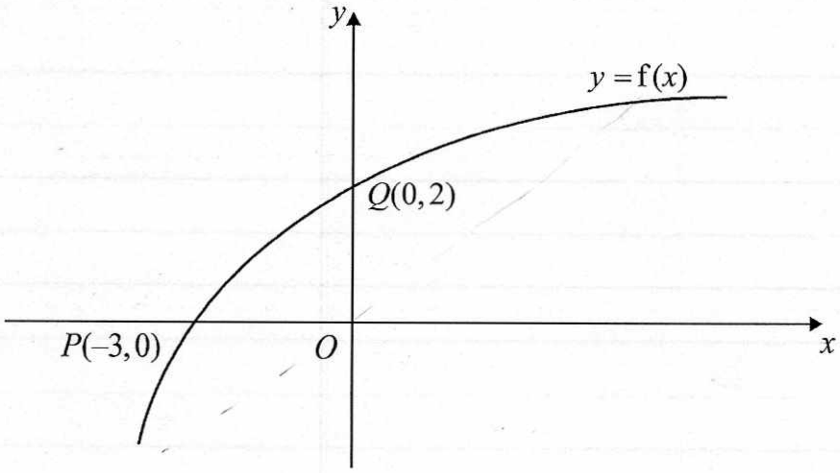


Figure 1

Figure 1 shows part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve passes through the points $Q(0,2)$ and $P(-3,0)$ as shown.

(a) Find the value of $\mathrm{ff}(-3)$.

(2)

On separate diagrams, sketch the curve with equation

(b) $y = \mathrm{f}^{-1}(x)$,

(2)

(c) $y = \mathrm{f}(|x|) - 2$,

(2)

(d) $y = 2\mathrm{f}\left(\frac{1}{2}x\right)$.

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

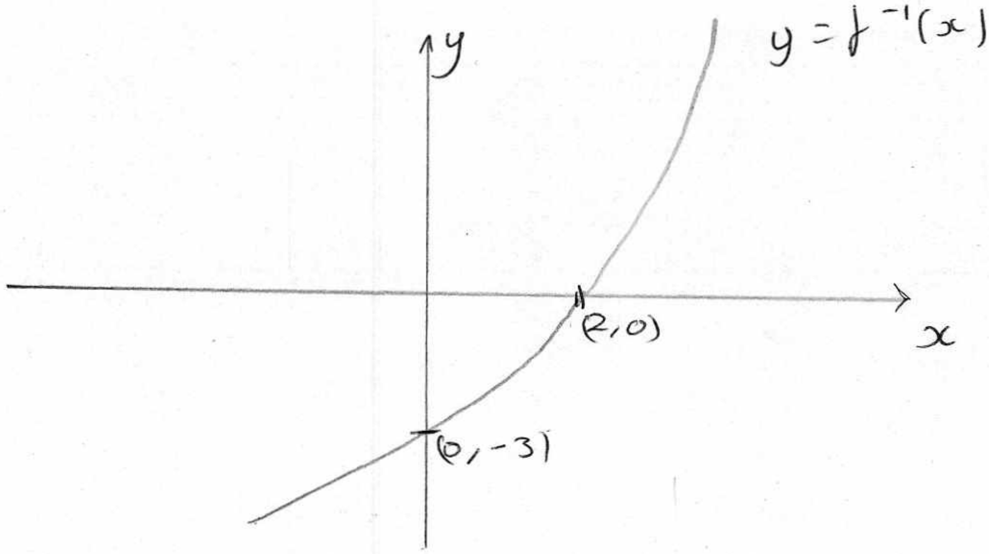
3a) $f(-3) = 0$

$f(0) = 2$

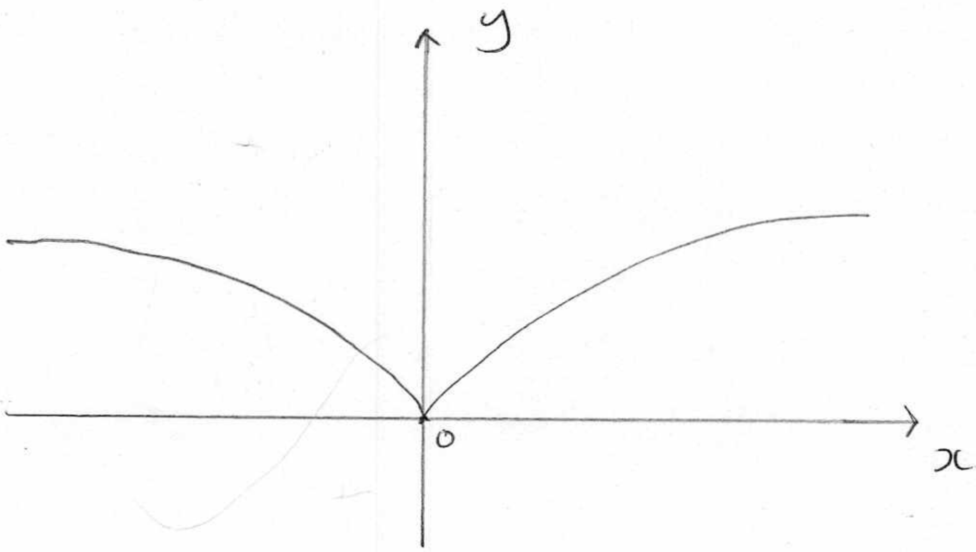
$\therefore ff(-3) = \underline{\underline{2}}$

Question 3 continued

b/



c/



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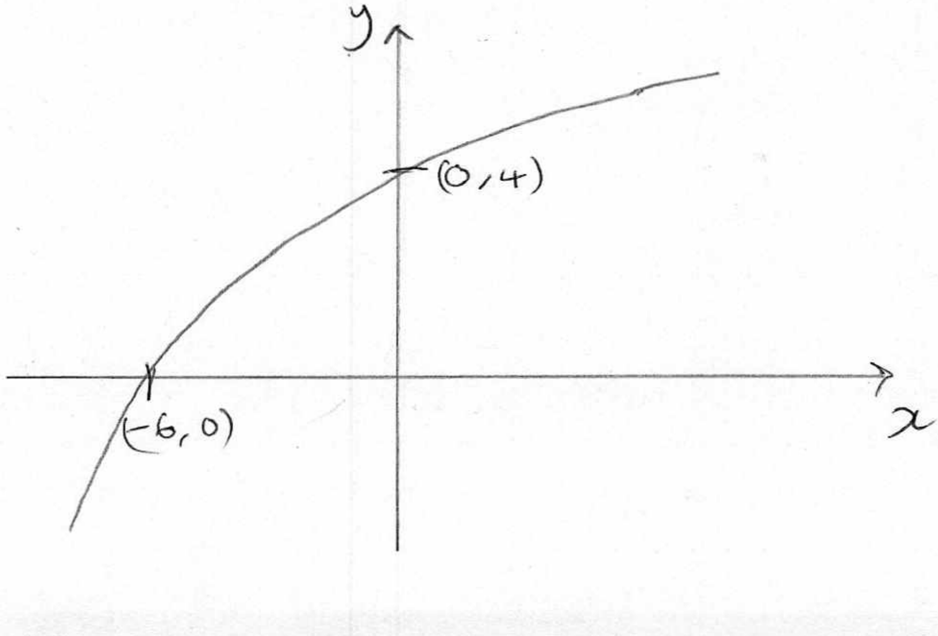

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4. (a) Express $6\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(4)

(b) $$p(\theta) = \frac{4}{12 + 6\cos\theta + 8\sin\theta}, \quad 0 \leqslant \theta \leqslant 2\pi$$

Calculate

(i) the maximum value of $p(\theta)$,

(ii) the value of θ at which the maximum occurs.

(4)

a) $R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$R\cos\alpha = 6$

$R\sin\alpha = 8$

$\tan\alpha = 8/6$

$\alpha = \tan^{-1}(8/6)$

$= 0.927^c$ (3dp)

$R = \sqrt{8^2 + 6^2}$

$= 10$

$\underline{10\cos(\theta - 0.927)}$

b)

i) Max value of $p(\theta)$ is where $6\cos\theta + 8\sin\theta = -10$

$\frac{4}{12-10} = \underline{\underline{2}}$

ii) $\cos(\theta - 0.927) = -1$

$\theta - 0.927 = \pi$

$\theta = \pi + 0.927$

Leave blank

5. (i) Differentiate with respect to x

(a) $y = x^3 \ln 2x$

(b) $y = (x + \sin 2x)^3$

(6)

Given that $x = \cot y$,

(ii) show that $\frac{dy}{dx} = \frac{-1}{1+x^2}$

(5)

5i a) $u = x^3 \quad v = \ln 2x$

$\frac{du}{dx} = 3x^2 \quad \frac{dv}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = x^3\left(\frac{1}{x}\right) + 3x^2 \ln 2x$

$= x^2 + 3x^2 \ln 2x$

b/ $y = (x + \sin 2x)^3$

$\frac{dy}{dx} = 3(x + \sin 2x)^2 (1 + 2\cos 2x)$

ii/ $x = \cot y$

$\frac{dx}{dy} = -\operatorname{cosec}^2 y$

$\frac{dy}{dx} = \frac{-1}{\cancel{8}\operatorname{cosec}^2 y}$

$\boxed{1 + \cot^2 y = \operatorname{cosec}^2 y}$

$= \frac{-1}{1 + \cot^2 y}$

$= \frac{-1}{1 + x^2}$

Leave blank

6. (i) Without using a calculator, find the exact value of

$$(\sin 22.5° + \cos 22.5°)^2$$

You must show each stage of your working.

(5)

(ii) (a) Show that $\cos 2\theta + \sin\theta = 1$ may be written in the form

$$k\sin^2\theta - \sin\theta = 0, \text{ stating the value of } k.$$

(2)

(b) Hence solve, for $0 \leqslant \theta < 360°$, the equation

$$\cos 2\theta + \sin\theta = 1$$

(4)

6i/ ~~$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$~~

$\sin^2 22.5 + 2\sin 22.5\cos 22.5 + \cos^2 22.5$

$\cos 2A = 2\cos^2 A - 1$ $\quad\cos 45 = 2\cos^2 22.5 - 1$
$\cos 2A = 1 - 2\sin^2 A$ $\quad\cos 45 = 1 - 2\sin^2 22.5$
$\sin 2A = 2\sin A\cos A.$ $\quad\sin 45 = 2\sin 22.5\cos 22.5$

$\frac{1-\cos 45}{2} + \cancel{2}\sin 45 + \frac{\cos 45 + 1}{2}$

$\frac{1 - \sqrt{2}/2}{2} + \cancel{2}.\frac{\sqrt{2}}{2} + \frac{\sqrt{2}/2 + 1}{2}$

$\frac{1}{2} - \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{4} + \frac{1}{2}$

~~$\frac{1+\sqrt{2}}{2}$~~

$1 + \frac{1}{2}\sqrt{2}$ or $1 + \frac{\sqrt{2}}{2}$

Leave blank

Question 6 continued

ii a/ $\cos 2\theta = \cancel{2} \; 1 - 2\sin^2\theta$

$1 - 2\sin^2\theta + \sin\theta = 1$

$0 = 2\sin^2\theta - \sin\theta$

$k = 2$

b/ $\sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0 \qquad \sin\theta = \frac{1}{2}$

$\theta = 0°, 180° \qquad \theta = 30°, 150°$

Turn over

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7. $$\mathrm{h}(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \qquad x \geqslant 0$$

(a) Show that $\mathrm{h}(x) = \dfrac{2x}{x^2+5}$ **(4)**

(b) Hence, or otherwise, find $\mathrm{h}'(x)$ in its simplest form. **(3)**

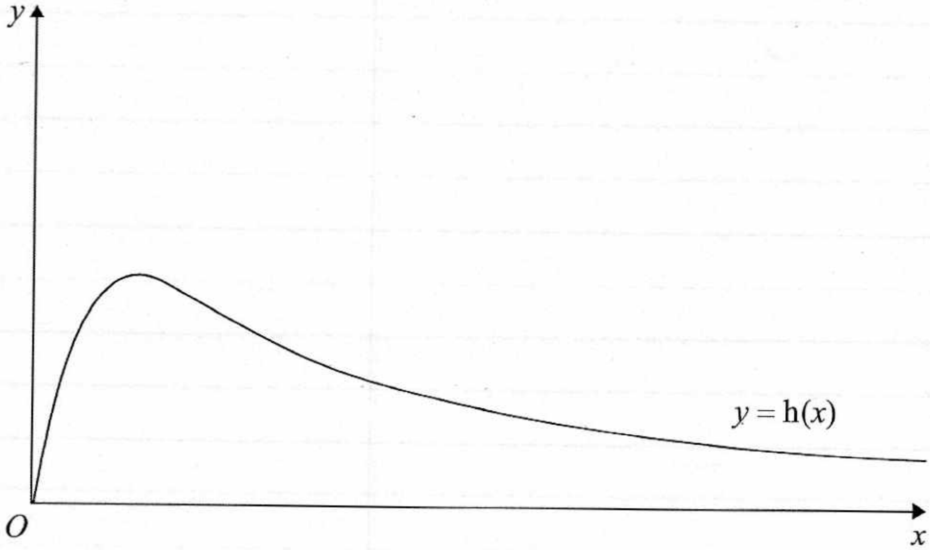

Figure 2

Figure 2 shows a graph of the curve with equation $y = \mathrm{h}(x)$.

(c) Calculate the range of $\mathrm{h}(x)$. **(5)**

$$h(x) = \frac{2(x^2+5)}{(x^2+5)(x+2)} + \frac{4(x+2)}{(x^2+5)(x+2)} - \frac{18}{(x^2+5)(x+2)}$$

$$= \frac{2(x^2+5) + 4(x+2) - 18}{(x^2+5)(x+2)}$$

$$= \frac{2x^2 + 10 + 4x + 8 - 18}{(x^2+5)(x+2)}$$

$$= \frac{2x^2 + 4x}{(x^2+5)(x+2)}$$

$$= \frac{2x\cancel{(x+2)}}{(x^2+5)\cancel{(x+2)}} = \frac{2x}{x^2+5}$$

Question 7 continued

b/

$$h(x) = \frac{2x}{x^2+5}$$

$u = 2x \qquad v = x^2 + 5$

$\frac{du}{dx} = 2 \qquad \frac{dv}{dx} = 2x$

$$h'(x) = \frac{(x^2+5)(2) - (2x)(2x)}{(x^2+5)^2}$$

$$= \frac{2x^2 + 10 - 4x^2}{(x^2+5)^2}$$

$$= \frac{10 - 2x^2}{(x^2+5)^2}$$

$$= \frac{-2(x^2-5)}{(x^2+5)^2}$$

c/ Max is where $\frac{dy}{dx} = 0$

$$\frac{10-2x^2}{(x^2+5)^2} = 0$$

$10 - 2x^2 = 0$

$5 = x^2$

$x = \sqrt{5}$

$y = \frac{2\sqrt{5}}{(\sqrt{5})^2+5}$

$= \frac{\sqrt{5}}{5}$

$0 \leq h(x) \leq \frac{\sqrt{5}}{5}$

Leave blank

8. The value of Bob's car can be calculated from the formula

$$V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$$

where V is the value of the car in pounds (£) and t is the age in years.

(a) Find the value of the car when $t = 0$ **(1)**

(b) Calculate the exact value of t when $V = 9500$ **(4)**

(c) Find the rate at which the value of the car is decreasing at the instant when $t = 8$. Give your answer in pounds per year to the nearest pound. **(4)**

8a/ $V_0 = 17000 + 2000 + 500$

$= £19500$

b/ $9500 = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$

$9000 = 17000e^{-0.25t} + 2000e^{-0.5t}$

$0 = 2000e^{-0.5t} + 17000e^{-0.25t} - 9000$

$0 = 2e^{-0.5t} + 17e^{-0.25t} - 9$

$0 = (2e^{-0.25t} - 1)(e^{-0.25t} + 9)$

$e^{-0.25t} = \frac{1}{2}$ $\quad e^{-0.25t} = -9$

$-0.25t = \ln(1/2)$ – NO SOLUTION –

~~$t = 2.77$~~

$t = -4\ln\left(\frac{1}{2}\right)$

$= 4\ln 2$

Leave blank

Question 8 continued

c/ $\frac{dV}{dt} = -4250e^{-0.25t} - 1000e^{-0.5t}$

when $t = 8$

$\frac{dV}{dt} = -4250e^{-2} - 1000e^{-4}$

$= £-593.49$ £/year

$= \underline{£593}$ ~~£37~~. £/ year