

1. The first three terms of a geometric series are

18, 12 and p

respectively, where p is a constant.

Find

- (a) the value of the common ratio of the series, (1)
- (b) the value of p , (1)
- (c) the sum of the first 15 terms of the series, giving your answer to 3 decimal places. (2)

$$a) \quad a = 18$$

$$12 = 18r$$

$$r = \frac{12}{18} = \underline{\underline{\frac{2}{3}}}$$

$$b) \quad 12 \times \frac{2}{3} = \underline{\underline{8}}$$

$$c) \quad S_{15} = \frac{a(1-r^n)}{1-r}$$
$$= \frac{18(1-(\frac{2}{3})^{15})}{1-(\frac{2}{3})}$$

$$= \underline{\underline{53.877}}$$

2. (a) Use the binomial theorem to find all the terms of the expansion of

$$(2 + 3x)^4$$

Give each term in its simplest form.

(4)

- (b) Write down the expansion of

$$(2 - 3x)^4$$

in ascending powers of x , giving each term in its simplest form.

(1)

1 4 6 4 1

$$a) (2)^4 + 4(2)^3(3x) + 6(2)^2(3x)^2 + 4(2)(3x)^3 + (3x)^4$$

$$16 + 96x + 216x^2 + 216x^3 + 81x^4$$

$$b/ 16 - 96x + 216x^2 - 216x^3 + 81x^4$$

3.

$$f(x) = 2x^3 - 5x^2 + ax + 18$$

where a is a constant.

Given that $(x - 3)$ is a factor of $f(x)$,

(a) show that $a = -9$ (2)

(b) factorise $f(x)$ completely. (4)

Given that

$$g(y) = 2(3^{3y}) - 5(3^{2y}) - 9(3^y) + 18$$

(c) find the values of y that satisfy $g(y) = 0$, giving your answers to 2 decimal places where appropriate. (3)

a) $(x-3)$ is a factor $\therefore f(3) = 0$

$$2(3)^3 - 5(3)^2 + a(3) + 18 = 0$$

$$54 - 45 + 3a + 18 = 0$$

$$3a + 27 = 0$$

$$3a = -27$$

$$a = -9$$

b)

$$\begin{array}{r} 2x^2 + x - 6 \\ x-3 \overline{) 2x^3 - 5x^2 - 9x + 18} \\ \underline{2x^3 - 6x^2} \\ x^2 - 9x \\ \underline{x^2 - 3x} \\ -6x + 18 \\ \underline{-6x + 18} \\ 0 \end{array}$$

$$(x-3)(2x^2 + x - 6)$$

$$(x-3)(2x-3)(x+2)$$

c) $x=3$ $x=3/2$ $x=-2$

$$3^y = 3$$

$$3^y = 3/2$$

$$3^y = -2$$

(no solutions)



Question 3 continued

$$y = \underline{\underline{1}} \quad y = \log_3^{3/2}$$
$$= \underline{\underline{0.37}}$$



4.

$$y = \frac{5}{(x^2 + 1)}$$

(a) Complete the table below, giving the missing value of y to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
y	5	4	2.5	1.538	1	0.690	0.5

(1)

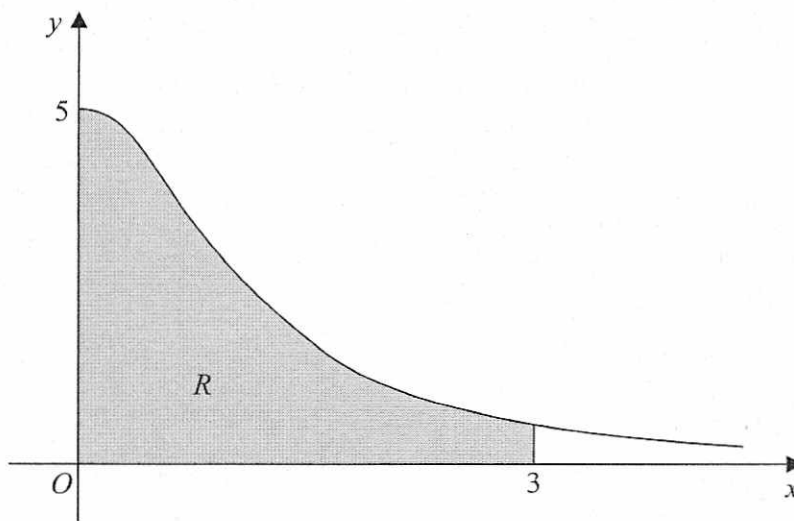


Figure 1

Figure 1 shows the region R which is bounded by the curve with equation $y = \frac{5}{(x^2 + 1)}$, the x -axis and the lines $x = 0$ and $x = 3$

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for the area of R .

(4)

(c) Use your answer to part (b) to find an approximate value for

$$\int_0^3 \left(4 + \frac{5}{(x^2 + 1)} \right) dx$$

giving your answer to 2 decimal places.

(2)

$$b) \quad 0.5 \left(\frac{5}{2} + 4 + 2.5 + 1.538 + 1 + 0.69 + \frac{0.5}{2} \right)$$

$$= 6.239 \text{ units}^2$$

Question 4 continued

c/ translated vertically upwards 4
spaces

$$6.239 + 4 \times 3$$

$$6.239 + 12$$

$$\underline{18.239 \text{ units}^1}$$

$$\underline{18.24 \text{ (2dp) units}^2}$$

Q4

(Total 7 marks)

5.

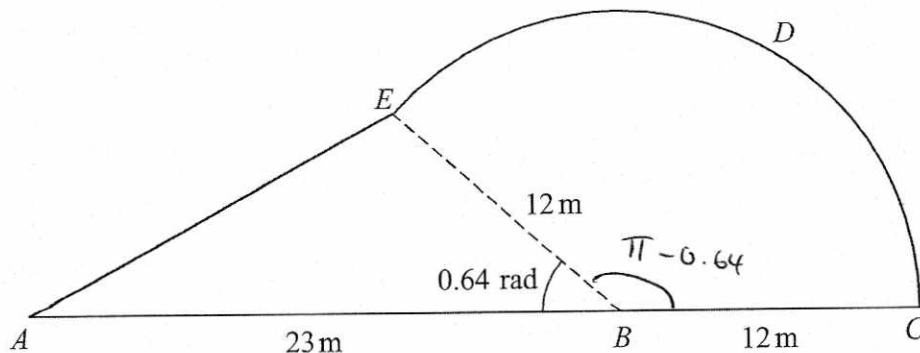


Figure 2

Figure 2 shows a plan view of a garden.

The plan of the garden $ABCDEA$ consists of a triangle ABE joined to a sector $BCDE$ of a circle with radius 12m and centre B .

The points A , B and C lie on a straight line with $AB = 23\text{m}$ and $BC = 12\text{m}$.

Given that the size of angle ABE is exactly 0.64 radians, find

(a) the area of the garden, giving your answer in m^2 , to 1 decimal place, (4)

(b) the perimeter of the garden, giving your answer in metres, to 1 decimal place. (5)

$$\begin{aligned} \text{a) Area of triangle} &= \frac{1}{2}ab \sin C \\ &= \frac{1}{2}(23)(12) \sin(0.64) \\ &= 82.41297691 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of sector} &= \frac{\theta}{2\pi} \times \pi r^2 \\ &= \frac{\pi - 0.64}{2} \times 12^2 \\ &= 180.1146711 \text{ m}^2 \end{aligned}$$

$$\text{Total Area} = \underline{262.5 \text{ m}^2}$$

$$\begin{aligned} \text{b/ AE} &= a^2 = b^2 + c^2 - 2bc \cos A \\ &= (23)^2 + (12)^2 - 2(23)(12) \cos(0.64) \\ &= 230.24... \\ &a = 15.17376491 \end{aligned}$$

Question 5 continued

$$\begin{aligned}\text{Arc length} &= \frac{\theta}{2\pi} \times 2\pi r \\ &= \theta r \\ &= (\pi - 0.64) 12 \\ &= 30.0191184\end{aligned}$$

$$\begin{aligned}\text{Total Perimeter} &= 23 + 12 + AE + CE \\ &= \underline{80.2 \text{ m ldp}}\end{aligned}$$

6.

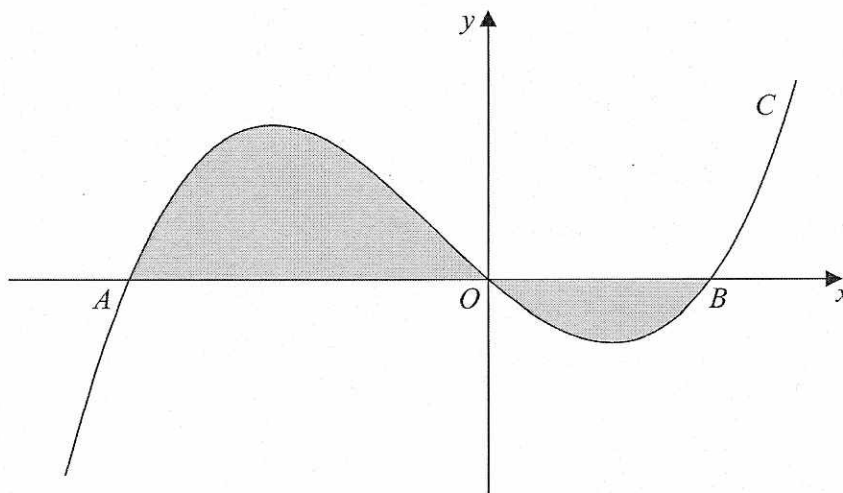


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = x(x + 4)(x - 2)$$

The curve C crosses the x -axis at the origin O and at the points A and B .

(a) Write down the x -coordinates of the points A and B .

(1)

The finite region, shown shaded in Figure 3, is bounded by the curve C and the x -axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.

(7)

a) $A: (-4, 0)$ $B: (2, 0)$

b/ $y = x(x^2 - 2x + 4x - 8)$
 $= x(x^2 + 2x - 8)$
 $= x^3 + 2x^2 - 8x$

$$\int_{-4}^2 (x^3 + 2x^2 - 8x) dx + \int_{-4}^0 (x^3 + 2x^2 - 8x) dx$$

$$\left[\frac{x^4}{4} + \frac{2x^3}{3} - 4x^2 + C \right]_{-4}^2 + \left[\frac{x^4}{4} + \frac{2x^3}{3} - 4x^2 + C \right]_{-4}^0$$

$$\left[\left(\frac{(2)^4}{4} + \frac{2(2)^3}{3} - 4(2)^2 \right) - (0) \right] + \left[(0) - \left(\frac{(-4)^4}{4} + \frac{2(-4)^3}{3} - 4(-4)^2 \right) \right]$$

$$\frac{20}{3} + \frac{128}{3} = \frac{148}{3} \text{ units}^2$$

7. (i) Find the exact value of x for which

$$\log_2(2x) = \log_2(5x + 4) - 3 \quad (4)$$

- (ii) Given that

$$\log_a y + 3\log_a 2 = 5$$

express y in terms of a .
Give your answer in its simplest form.

(3)

$$i/ \log_2 2x - \log_2(5x + 4) = -3$$

$$\log_2\left(\frac{2x}{5x+4}\right) = -3$$

$$2^{-3} = \frac{2x}{5x+4}$$

$$\frac{1}{8} = \frac{2x}{5x+4}$$

$$\frac{5x+4}{8} = 2x$$

$$5x+4 = 16x$$

$$4 = 11x$$

$$x = \frac{4}{11}$$

$$ii/ \log_a y + \log_a 2^3 = 5$$

$$\log_a y + \log_a 8 = 5$$

$$\log_a 8y = 5$$

$$a^5 = 8y$$

$$y = \frac{1}{8}a^5$$

8. (i) Solve, for $-180^\circ \leq x < 180^\circ$,

$$\tan(x - 40^\circ) = 1.5$$

giving your answers to 1 decimal place.

(3)

- (ii) (a) Show that the equation

$$\sin \theta \tan \theta = 3 \cos \theta + 2$$

can be written in the form

$$4 \cos^2 \theta + 2 \cos \theta - 1 = 0$$

(3)

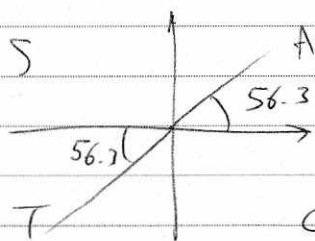
- (b) Hence solve, for $0 \leq \theta < 360^\circ$,

$$\sin \theta \tan \theta = 3 \cos \theta + 2$$

showing each stage of your working.

(5)

$$\begin{aligned} \text{i/} \quad \tan(x - 40) &= 1.5 \\ x - 40 &= \tan^{-1}(1.5) \\ x - 40 &= 56.3, -123.7 \end{aligned}$$



$$x = 96.3^\circ, -83.7^\circ$$

$$\begin{aligned} \text{ii/} \quad \sin \theta \tan \theta &= 3 \cos \theta + 2 \\ \sin \theta \frac{\sin \theta}{\cos \theta} &= 3 \cos \theta + 2 \\ \sin^2 \theta &= 3 \cos^2 \theta + 2 \cos \theta \\ 1 - \cos^2 \theta &= 3 \cos^2 \theta + 2 \cos \theta \\ &= 4 \cos^2 \theta + 2 \cos \theta - 1 \end{aligned}$$

Question 8 continued

$$b) \quad 4 \cos^2 \theta + 2 \cos \theta - 1 = 0$$

$$\left(\quad a=4 \quad b=2 \quad c=-1 \right)$$

$$\cos \theta = \frac{- (2) \pm \sqrt{(2)^2 - 4(4)(-1)}}{2(4)}$$

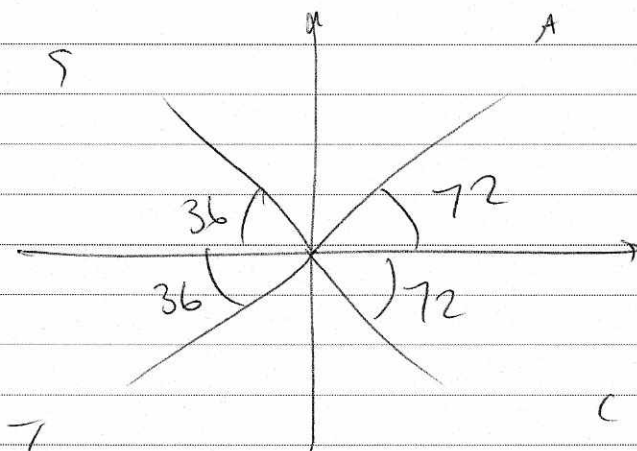
$$= \frac{-1 + \sqrt{5}}{4}, \quad \frac{-1 - \sqrt{5}}{4}$$

$$\cos \theta = \frac{-1 + \sqrt{5}}{4}$$

$$= 72$$

$$\cos \theta = \frac{-1 - \sqrt{5}}{4}$$

$$= 144$$



$$\theta = 72^\circ, 144^\circ, 216^\circ, 288^\circ$$

9. The curve with equation

$$y = x^2 - 32\sqrt{x} + 20, \quad x > 0$$

has a stationary point P .

Use calculus

(a) to find the coordinates of P ,

(6)

(b) to determine the nature of the stationary point P .

(3)

$$y = x^2 - 32x^{1/2} + 20$$

$$\frac{dy}{dx} = 2x - 16x^{-1/2}$$

Stationary point is where $\frac{dy}{dx} = 0$

$$2x - 16x^{-1/2} = 0$$

$$2x - \frac{16}{x^{1/2}} = 0$$

$$2x = \frac{16}{x^{1/2}}$$

$$2x^{3/2} = 16$$

$$x^{3/2} = 8$$

$$x^{1/2} = 2$$

$$\underline{\underline{x = 4}}$$

When $x = 4$

$$y = (4)^2 - 32(4)^{1/2} + 20$$

$$= 16 - 64 + 20$$

$$= -28$$

$$(4, -28)$$

Question 9 continued

$$b) \frac{d^2y}{dx^2} = 2 + 8x^{-3/2}$$

$$\text{when } x = 4$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= 2 + 8(4)^{-3/2} \\ &= 3 \end{aligned}$$

$\frac{d^2y}{dx^2}$ is positive \therefore it is a minimum point.

10.

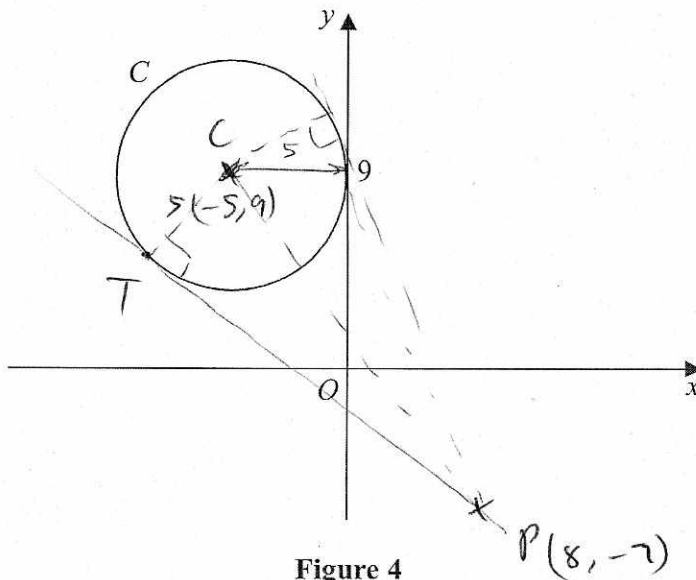


Figure 4

The circle C has radius 5 and touches the y -axis at the point $(0, 9)$, as shown in Figure 4.

- (a) Write down an equation for the circle C , that is shown in Figure 4. (3)

A line through the point $P(8, -7)$ is a tangent to the circle C at the point T .

- (b) Find the length of PT . (3)

a) $(x + 5)^2 + (y - 9)^2 = 25$

b) Length of CP :

$$x^2 = 13^2 + 16^2$$

$$x^2 = 425$$

$$x = \sqrt{425}$$

Length of PT :

$$x^2 + 5^2 = (\sqrt{425})^2$$

$$x^2 + 25 = 425$$

$$x^2 = 400$$

$$x = 20$$