

1. Find the first 3 terms, in ascending powers of x , in the binomial expansion of

$$(2-5x)^6$$

Give each term in its simplest form.

(4)

$$1 \quad 6 \quad 15$$

$$1(2)^6 + 6(2)^5(-5x) + 15(2)^4(-5x)^2$$

$$64 - 960x + 6000x^2$$

Q1

(Total 4 marks)



2. $f(x) = ax^3 + bx^2 - 4x - 3$, where a and b are constants.

Given that $(x - 1)$ is a factor of $f(x)$,

(a) show that

$$a + b = 7$$

(2)

Given also that, when $f(x)$ is divided by $(x + 2)$, the remainder is 9,

(b) find the value of a and the value of b , showing each step in your working.

(4)

$$a) \quad f(1) = 0$$

$$a(1)^3 + b(1)^2 - 4(1) - 3 = 0$$

$$a + b - 4 - 3 = 0$$

$$a + b = 7 \quad (1)$$

$$b) \quad f(-2) = 9$$

$$a(-2)^3 + b(-2)^2 - 4(-2) - 3 = 9$$

$$-8a + 4b + 8 - 3 = 9$$

$$-8a + 4b = 4$$

$$-2a + b = 1 \quad (2)$$

$$(1) - (2)$$

$$3a = 6$$

$$a = 2$$

$$b = 5$$



3. A company predicts a yearly profit of £120 000 in the year 2013. The company predicts that the yearly profit will rise each year by 5%. The predicted yearly profit forms a geometric sequence with common ratio 1.05

- (a) Show that the predicted profit in the year 2016 is £138 915 (1)
- (b) Find the first year in which the yearly predicted profit exceeds £200 000 (5)
- (c) Find the total predicted profit for the years 2013 to 2023 inclusive, giving your answer to the nearest pound. (3)

$$a) \quad u_n = ar^{n-1}$$

$$2016 = \text{term } 4$$

$$u_4 = 120000 (1.05)^3 \\ = \underline{\underline{£138915}}$$

$$b) \quad ar^{n-1} > 200000$$

$$120000 (1.05)^{n-1} > 200000$$

$$(1.05)^{n-1} > 5/3$$

$$(n-1) \log 1.05 > \log(5/3)$$

$$n-1 > \frac{\log(5/3)}{\log(1.05)}$$

$$n-1 > 10.4698\dots$$

$$n > 11.4698$$

The 12th term exceeds £200 000

2024

$$c) \quad S_{11} = \frac{a(1-r^n)}{1-r}$$

$$= \frac{120000(1-1.05^{11})}{1-1.05}$$

$$= 1704814.46$$

$$= \underline{\underline{£1704814}}$$

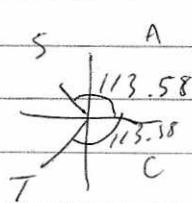


4. Solve, for $0 \leq x < 180^\circ$,

$$\cos(3x - 10^\circ) = -0.4$$

giving your answers to 1 decimal place. You should show each step in your working.

(7)



$$\cos(3x - 10) = -0.4$$

$$3x - 10 = \cos^{-1}(-0.4)$$

$$3x - 10 = 113.5781785^\circ,$$

$$246.4218215^\circ,$$

$$473.5781785^\circ$$

$$x = 41.2^\circ, 85.5^\circ, 161.2^\circ$$



5. The circle C has equation

$$x^2 + y^2 - 20x - 24y + 195 = 0$$

The centre of C is at the point M .

(a) Find

- (i) the coordinates of the point M ,
- (ii) the radius of the circle C .

(5)

N is the point with coordinates $(25, 32)$.

(b) Find the length of the line MN .

(2)

The tangent to C at a point P on the circle passes through point N .

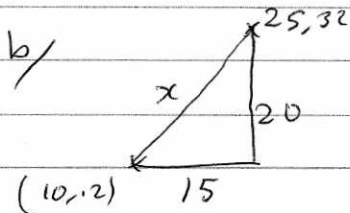
(c) Find the length of the line NP .

(2)

a) i) $x^2 - 20x + y^2 - 24y + 195 = 0$
 $(x-10)^2 - 100 + (y-12)^2 - 144 + 195 = 0$
 $(x-10)^2 + (y-12)^2 = 49$

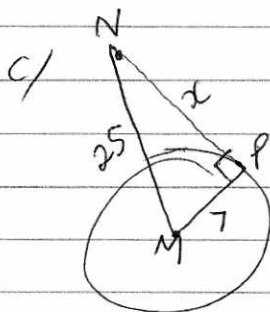
centre $(10, 12)$

ii) radius $\sqrt{49} = \underline{7}$



$$x^2 = 20^2 + 15^2$$

$$\underline{x = 25}$$



$$x^2 = 25^2 - 7^2$$

$$\underline{x = 24}$$



6. Given that

$$2\log_2(x+15) - \log_2 x = 6$$

(a) Show that

$$x^2 - 34x + 225 = 0$$

(5)

(b) Hence, or otherwise, solve the equation

$$2\log_2(x+15) - \log_2 x = 6$$

(2)

$$a) \quad 2 \log_2 (x+15) - \log_2 x = 6$$

$$\log_2 (x+15)^2 - \log_2 x = 6$$

$$\log_2 \left(\frac{(x+15)^2}{x} \right) = 6$$

$$\frac{(x+15)^2}{x} = 2^6$$

$$\frac{(x+15)^2}{x} = 64$$

$$(x+15)^2 = 64x$$

$$x^2 + 30x + 225 = 64x$$

$$x^2 - 34x + 225 = 0$$

$$b) \quad (x - 25)(x - 9) = 0$$

$$\underline{\underline{x = 25}}$$

$$\underline{\underline{x = 9}}$$



7.

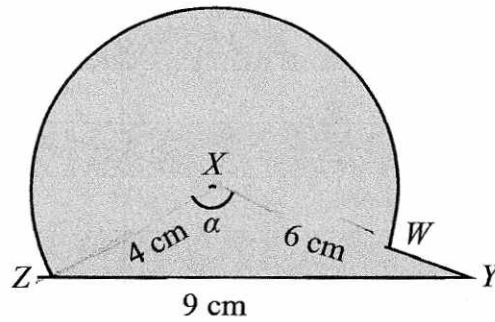


Figure 1

The triangle XYZ in Figure 1 has $XY = 6$ cm, $YZ = 9$ cm, $ZX = 4$ cm and angle $ZXY = \alpha$. The point W lies on the line XY .

The circular arc ZW , in Figure 1 is a major arc of the circle with centre X and radius 4 cm.

(a) Show that, to 3 significant figures, $\alpha = 2.22$ radians. (2)

(b) Find the area, in cm^2 , of the major sector $XZWX$. (3)

The region enclosed by the major arc ZW of the circle and the lines WY and YZ is shown shaded in Figure 1.

Calculate

(c) the area of this shaded region, (3)

(d) the perimeter $ZWYZ$ of this shaded region. (4)

$$\begin{aligned}
 \text{a) } \cos \alpha &= \frac{b^2 + c^2 - a^2}{2bc} \\
 &= \frac{(4)^2 + (6)^2 - (9)^2}{2(4)(6)} \\
 &= \frac{-29}{48} \\
 \alpha &= 2.219516005^\circ \\
 &= 2.22^\circ \text{ (3sf)}
 \end{aligned}$$

$$\begin{aligned}
 \text{b/ Area of sector} &= \frac{\theta}{2} r^2 \\
 &= \frac{2\pi - 2.22}{2} (4)^2 \\
 &= 32.50548246 \text{ cm}^2
 \end{aligned}$$



Question 7 continued

$$\begin{aligned}c/ \text{ Area of triangle} &= \frac{1}{2} a b \sin C \\ &= \frac{1}{2} (4)(6) \sin (2.22) \\ &= 9.558785667 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Total Area} &= \text{sector} + \text{triangle} \\ &= 42.0642812 \text{ cm}^2 \\ &= 42.1 \text{ cm}^2 \text{ (3sf)}\end{aligned}$$

$$\begin{aligned}d/ \text{ Arc length} &= \theta r \\ &= (2\pi - 2.22) 4 \\ &= 16.25274123 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{perimeter} &= \text{arc length} + 4 + 2 \\ &= 27.25274123 \\ &= 27.3 \text{ cm (3sf)}\end{aligned}$$



8. The curve C has equation $y = 6 - 3x - \frac{4}{x^3}$, $x \neq 0$

(a) Use calculus to show that the curve has a turning point P when $x = \sqrt{2}$ (4)

(b) Find the x -coordinate of the other turning point Q on the curve. (1)

(c) Find $\frac{d^2y}{dx^2}$. (1)

(d) Hence or otherwise, state with justification, the nature of each of these turning points P and Q . (3)

$$a/ \quad y = 6 - 3x - 4x^{-3}$$

$$\frac{dy}{dx} = -3 + 12x^{-4}$$

Turning points where $\frac{dy}{dx} = 0$

$$-3 + \frac{12}{x^4} = 0$$

$$\frac{12}{x^4} = 3$$

$$12 = 3x^4$$

$$4 = x^4$$

$$x = \pm\sqrt{2}$$

$$b/ \quad \underline{-\sqrt{2}}$$

$$c/ \quad \frac{d^2y}{dx^2} = -48x^{-5}$$

$$\text{when } x = \sqrt{2} \quad \frac{d^2y}{dx^2} = -8.48528\dots$$

negative \therefore maximum

$$\text{when } x = -\sqrt{2} \quad \frac{d^2y}{dx^2} = 8.48528\dots$$

positive \therefore minimum



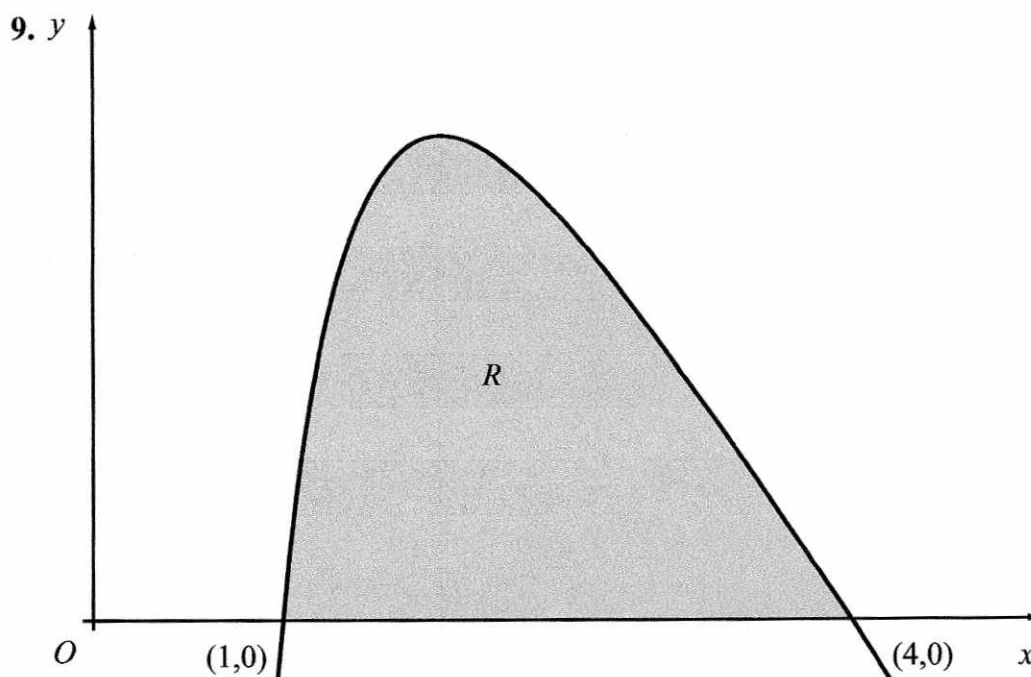


Figure 2

The finite region R , as shown in Figure 2, is bounded by the x -axis and the curve with equation

$$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}, \quad x > 0$$

The curve crosses the x -axis at the points $(1, 0)$ and $(4, 0)$.

(a) Complete the table below, by giving your values of y to 3 decimal places.

x	1	1.5	2	2.5	3	3.5	4
y	0	5.866	6.272	5.210	3.634	1.856	0

(2)

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R , giving your answer to 2 decimal places.

(4)

(c) Use integration to find the exact value for the area of R .

(6)

b/ $0.5(5.866 + 6.272 + 5.210 + 3.634 + 1.856)$

$= 11.419 \text{ units}^2$



Question 9 continued

$$\int_1^4 27 - 2x - 9x^{1/2} - 16x^{-2} dx$$

$$\left[27x - \frac{2x^2}{2} - \frac{9x^{3/2}}{3/2} - \frac{16x^{-1}}{-1} + C \right]_1^4$$

$$\left[27x - x^2 - 6x^{3/2} + 16x^{-1} \right]_1^4$$

$$\left[27(4) - (4)^2 - 6(4)^{3/2} + \frac{16}{4} \right] - \left[27(1) - (1)^2 - 6(1)^{3/2} + \frac{16}{1} \right]$$

$$[48] - [36]$$

$$\underline{\underline{12 \text{ units}^2}}$$

