



1. Given that  $y = x^4 + 6x^{\frac{1}{2}}$ , find in their simplest form

(a)  $\frac{dy}{dx}$

(3)

(b)  $\int y dx$

(3)

$$a/ \quad \frac{dy}{dx} = \underline{4x^3 + 3x^{-1/2}}$$

$$b/ \quad \int y dx = \frac{x^5}{5} + \frac{6x^{3/2}}{3/2} + C$$

$$= \underline{\frac{1}{5}x^5 + 4x^{3/2} + C}$$

2. (a) Simplify

$$\sqrt{32} + \sqrt{18}$$

giving your answer in the form  $a\sqrt{2}$ , where  $a$  is an integer.

(2)

(b) Simplify

$$\frac{\sqrt{32} + \sqrt{18}}{3 + \sqrt{2}}$$

giving your answer in the form  $b\sqrt{2} + c$ , where  $b$  and  $c$  are integers.

(4)

$$\begin{aligned} \text{a)} \quad & \sqrt{16}\sqrt{2} + \sqrt{9}\sqrt{2} \\ & 4\sqrt{2} + 3\sqrt{2} \\ & \underline{7\sqrt{2}} \end{aligned}$$

$$\begin{aligned} \text{b)} \quad & \frac{7\sqrt{2}(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})} \end{aligned}$$

$$\frac{21\sqrt{2} - 14}{7}$$

$$\underline{3\sqrt{2} - 2}$$

3. Find the set of values of  $x$  for which

(a)  $4x - 5 > 15 - x$

(2)

(b)  $x(x - 4) > 12$

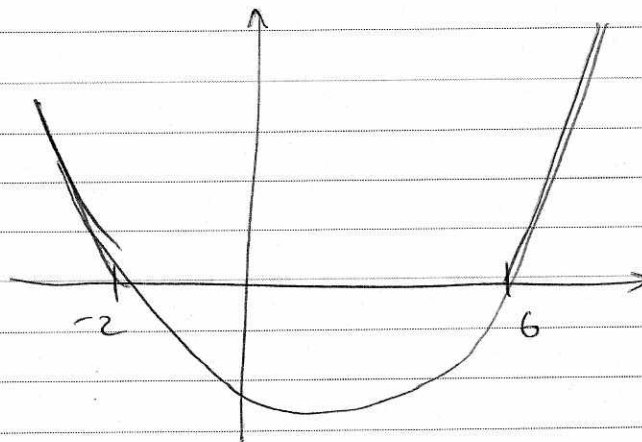
(4)

$$\begin{aligned} \text{a) } & 4x - 5 > 15 - x \\ & 5x - 5 > 15 \\ & 5x > 20 \\ & \underline{\underline{x > 4}} \end{aligned}$$

$$\begin{aligned} \text{b) } & x(x - 4) > 12 \\ & x^2 - 4x > 12 \\ & x^2 - 4x - 12 > 0 \end{aligned}$$

$$(x - 6)(x + 2) > 0$$

$$x = 6 \quad x = -2$$



$$\underline{\underline{x < -2 \quad \text{or} \quad x > 6}}$$

4. A sequence  $x_1, x_2, x_3, \dots$  is defined by

$$x_1 = 1$$

$$x_{n+1} = ax_n + 5, \quad n \geq 1$$

where  $a$  is a constant.

(a) Write down an expression for  $x_2$  in terms of  $a$ .

(1)

(b) Show that  $x_3 = a^2 + 5a + 5$

(2)

Given that  $x_3 = 41$

(c) find the possible values of  $a$ .

(3)

$$a) \quad x_2 = a(x_1) + 5$$

$$x_1 = 1 \quad = a(1) + 5 \\ = a + 5$$

$$b) \quad x_3 = a(x_2) + 5 \\ = a(a + 5) + 5 \\ = \underline{a^2 + 5a + 5}$$

$$c) \quad a^2 + 5a + 5 = 41 \\ a^2 + 5a - 36 = 0 \\ (a + 9)(a - 4) = 0 \\ \underline{a = -9} \quad \underline{a = 4}$$

5. The curve  $C$  has equation  $y = x(5-x)$  and the line  $L$  has equation  $2y = 5x + 4$

$$2y = 2x(5-x)$$

$$y = \frac{5}{2}x + 2$$

- (a) Use algebra to show that  $C$  and  $L$  do not intersect.

(4)

- (b) In the space on page 11, sketch  $C$  and  $L$  on the same diagram, showing the coordinates of the points at which  $C$  and  $L$  meet the axes.

(4)

$$\begin{aligned} \text{a) } 2x(5-x) &= 5x + 4 \\ 10x - 2x^2 &= 5x + 4 \\ 10x &= 2x^2 + 5x + 4 \\ 0 &= 2x^2 - 5x + 4 \end{aligned}$$

no solutions  $\therefore b^2 - 4ac$  will be negative

$$a=2 \quad b=-5 \quad c=4$$

$$\begin{aligned} &(-5)^2 - 4(2)(4) \\ &25 - 32 \\ &\underline{\underline{-7}} \end{aligned}$$

b/

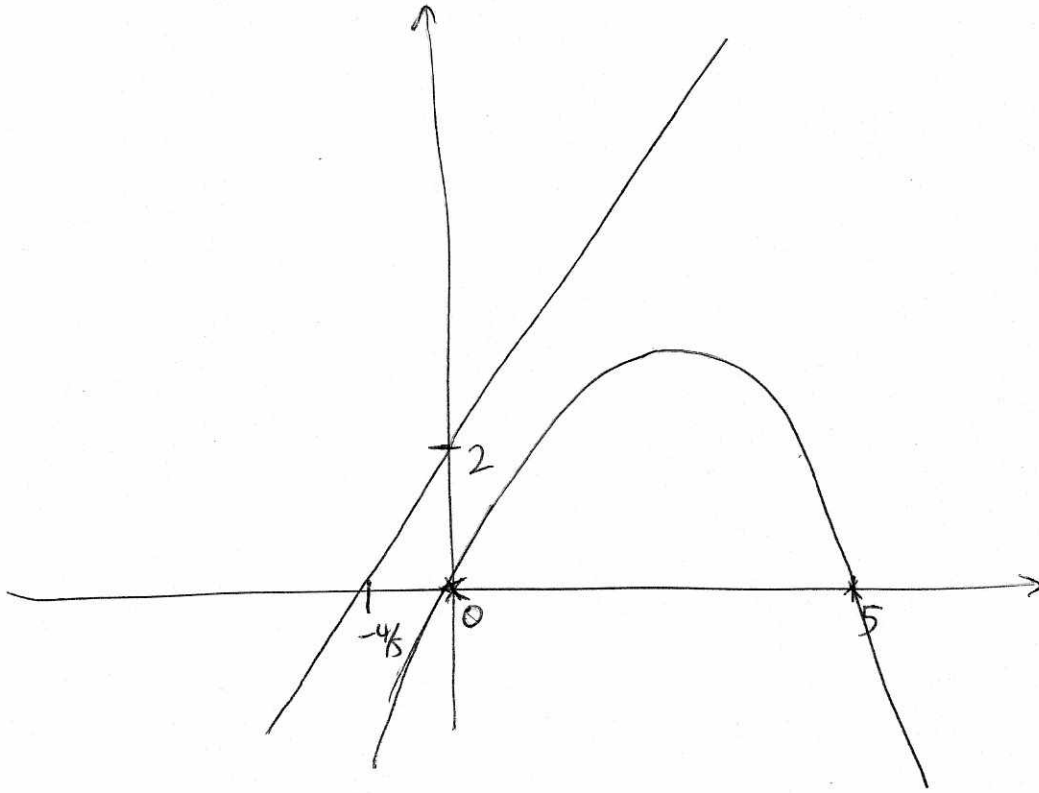
$$2y = 5x + 4 \quad \text{crosses } x \text{ when } y=0$$

$$0 = 5x + 4$$

$$-4 = 5x$$

$$x = -\frac{4}{5}$$

Question 5 continued



Q5

(Total 8 marks)



6.

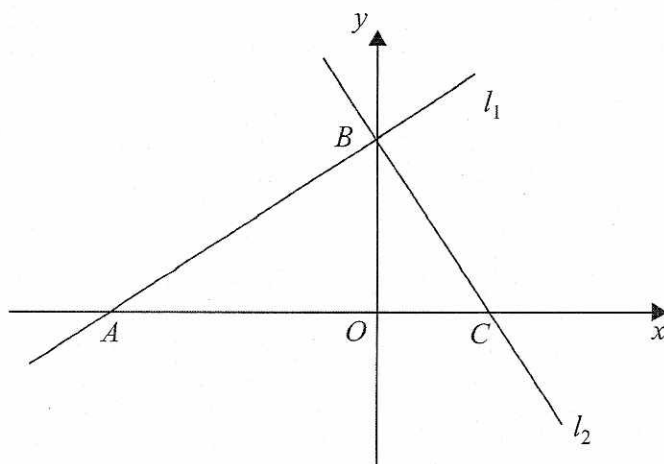


Figure 1

The line  $l_1$  has equation  $2x - 3y + 12 = 0$

(a) Find the gradient of  $l_1$ .

(1)

The line  $l_1$  crosses the  $x$ -axis at the point  $A$  and the  $y$ -axis at the point  $B$ , as shown in Figure 1.

The line  $l_2$  is perpendicular to  $l_1$  and passes through  $B$ .

(b) Find an equation of  $l_2$ .

(3)

The line  $l_2$  crosses the  $x$ -axis at the point  $C$ .

(c) Find the area of triangle  $ABC$ .

(4)

$$\begin{aligned} \text{a) } 2x - 3y + 12 &= 0 \\ 2x + 12 &= 3y \\ \frac{2}{3}x + 4 &= y \end{aligned}$$

$$m = \frac{2}{3}$$

$$\text{b/ perpendicular gradient} = -\frac{3}{2}$$

$$\text{crosses } y \text{ when } x=0 \quad y=4 \quad \underline{\underline{(0,4)}}$$



## Question 6 continued

$$y = -\frac{3}{2}x + c$$

$$y = -\frac{3}{2}x + 4$$

c/ B: (0, 4)

A:  $2x - 3y + 12 = 0$

crosses x when  $y = 0$

$$2x - 3(0) + 12 = 0$$

$$2x + 12 = 0$$

$$2x = -12$$

$$x = -6$$

A: (-6, 0)

c:  $y = -\frac{3}{2}x + 4$

$$0 = -\frac{3}{2}x + 4$$

$$-4 = -\frac{3}{2}x$$

$$-8 = -3x$$

$$x = \frac{8}{3}$$

c:  $(\frac{8}{3}, 0)$

Area of triangle =  $\frac{1}{2}$  base  $\times$  height

base =  $6 + \frac{8}{3}$

$$= \frac{18}{3} + \frac{8}{3} = \frac{26}{3}$$

$$\text{Area} = \frac{1}{2} \cdot \frac{26}{3} \cdot 4$$

$$= 2 \times \frac{26}{3}$$

$$= \frac{52}{3} \text{ units}^2$$

7. A curve with equation  $y = f(x)$  passes through the point  $(2, 10)$ . Given that

$$f'(x) = 3x^2 - 3x + 5$$

$$\frac{dy}{dx}$$

find the value of  $f(1)$ .

(5)

$$f(x) = \frac{3x^3}{3} - \frac{3x^2}{2} + 5x + C$$

$$= x^3 - \frac{3}{2}x^2 + 5x + C$$

$$(2, 10) \quad 10 = (2)^3 - \frac{3}{2}(2)^2 + 5(2) + C$$

$$10 = 8 - \frac{3}{2}(4) + 10 + C$$

$$10 = 8 - 6 + 10 + C$$

$$10 = 12 + C$$

$$C = -2$$

$$f(x) = x^3 - \frac{3}{2}x^2 + 5x - 2$$

$$f(1) = (1)^3 - \frac{3}{2}(1)^2 + 5(1) - 2$$

$$= 1 - \frac{3}{2} + 5 - 2$$

$$= 4 - \frac{3}{2}$$

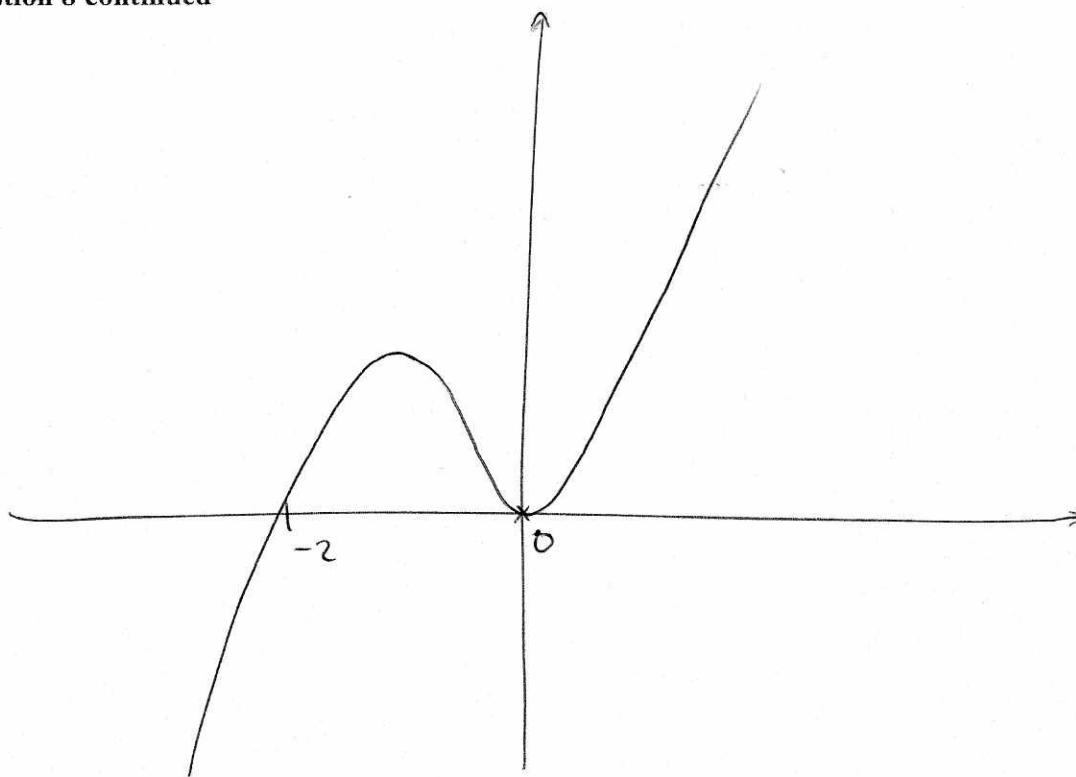
$$= \frac{8}{2} - \frac{3}{2}$$

$$= \frac{5}{2}$$





## Question 8 continued



$$c/ \quad \frac{dy}{dx} = x^3 + 2x^2 + 3x^2 + 4x$$

$$\text{when } x = -2$$

$$\frac{dy}{dx} = \frac{(-2)^3}{-8} + 2(-2)^2$$

$$\frac{dy}{dx} = 3(-2)^2 + 4(-2)$$

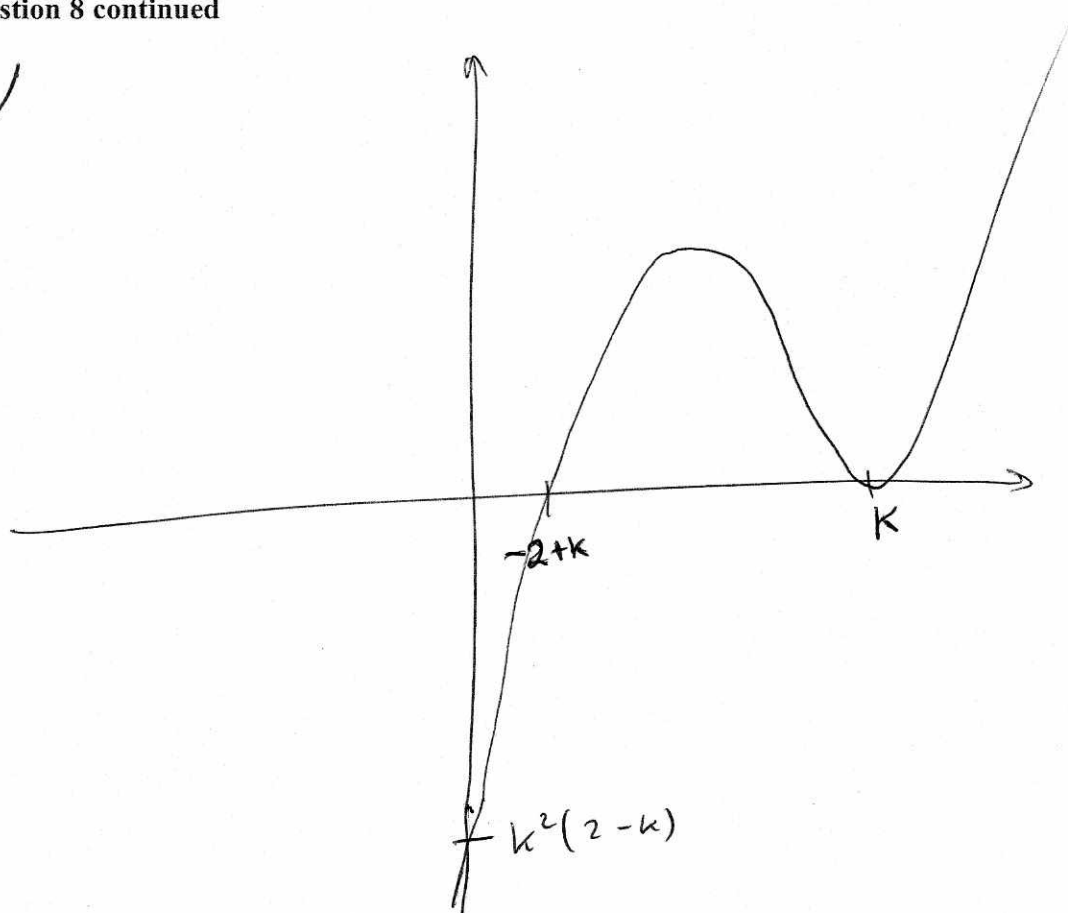
$$= 12 - 8$$

$$= \underline{\underline{4}}$$

$$\text{when } x = 0 \quad \frac{dy}{dx} = 0$$

Question 8 continued

d/



crosses y when  $x=0$

$$y = (k)^2(-k+2)$$

$$= k^2(2-k)$$



9. A company offers two salary schemes for a 10-year period, Year 1 to Year 10 inclusive.

Scheme 1: Salary in Year 1 is  $\pounds P$ .  
Salary increases by  $\pounds(2T)$  each year, forming an arithmetic sequence.

Scheme 2: Salary in Year 1 is  $\pounds(P + 1800)$ .  
Salary increases by  $\pounds T$  each year, forming an arithmetic sequence.

- (a) Show that the **total** earned under Salary Scheme 1 for the 10-year period is

$$\pounds(10P + 90T) \quad (2)$$

For the 10-year period, the **total** earned is the same for both salary schemes.

- (b) Find the value of  $T$ . (4)

For this value of  $T$ , the salary in Year 10 under Salary Scheme 2 is  $\pounds 29\,850$

- (c) Find the value of  $P$ . (3)

$$a) \quad A = P \quad d = 2T$$

$$S_n = \frac{n}{2} (2a + (n-1)d)$$

$$S_{10} = \frac{10}{2} (2P + (10-1)2T)$$

$$= 5(2P + 9(2T))$$

$$= 5(2P + 18T)$$

$$= \underline{10P + 90T}$$

$$b/ \text{ Scheme 2: } \quad A = P + 1800$$

$$D = T$$

$$S_{10} = \frac{10}{2} (2(P + 1800) + (10-1)T)$$

$$= 5(2P + \overset{3600}{1800} + 9T)$$

$$= \cancel{5P} + 9000 + 45T$$

$$10P + 18000 + 45T$$

Amount earned is the same.

Question 9 continued

$$10P + 90T = \cancel{5P} + 18000 + 45T$$

$$90T = 18000 + 45T$$

$$45T = 18000$$

$$T = \frac{18000}{45} = \frac{36000}{90}$$

$$\underline{T = 400} \quad \pounds 400$$

$$c) \quad U_n = a + (n-1)d$$

$$29850 = P + 1800 + 9T$$

$$29850 = P + 1800 + 9(400)$$

$$29850 = P + 1800 + 3600$$

$$29850 = P + 5400$$

$$\underline{24450 = P}$$

$$\pounds 24450$$

10.

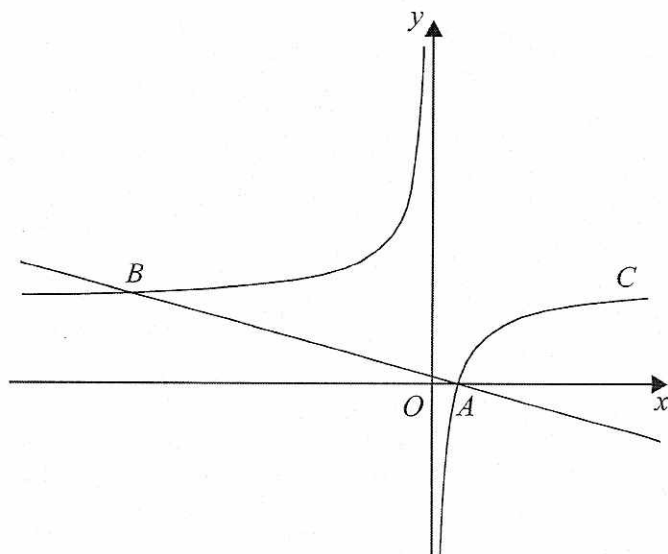


Figure 2

Figure 2 shows a sketch of the curve  $C$  with equation

$$y = 2 - \frac{1}{x}, \quad x \neq 0$$

The curve crosses the  $x$ -axis at the point  $A$ .

(a) Find the coordinates of  $A$ .

(1)

(b) Show that the equation of the normal to  $C$  at  $A$  can be written as

$$2x + 8y - 1 = 0$$

(6)

The normal to  $C$  at  $A$  meets  $C$  again at the point  $B$ , as shown in Figure 2.

(c) Find the coordinates of  $B$ .

(4)

a) Crosses  $x$  when  $y=0$

$$0 = 2 - \frac{1}{x}$$

$$\frac{1}{x} = 2$$

$$1 = 2x$$

$$x = \frac{1}{2}$$

$$\left(\frac{1}{2}, 0\right)$$



## Question 10 continued

$$b/ \quad y = 2 - x^{-1}$$

$$\frac{dy}{dx} = x^{-2}$$

$$\text{when } x = \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{1}{(\frac{1}{2})^2}$$

$$= \frac{1}{\frac{1}{4}}$$

$$= 4$$

$$\text{gradient of normal} = -\frac{1}{4}$$

$$y = -\frac{1}{4}x + c \quad (\frac{1}{2}, 0)$$

$$0 = -\frac{1}{4}(\frac{1}{2}) + c$$

$$0 = -\frac{1}{8} + c$$

$$c = \frac{1}{8}$$

$$y = -\frac{1}{4}x + \frac{1}{8}$$

$$8y = -2x + 1$$

$$\underline{2x + 8y - 1 = 0}$$

$$c/ \quad y = 2 - \frac{1}{x} \quad y = -\frac{1}{4}x + \frac{1}{8}$$

$$2 - \frac{1}{x} = -\frac{1}{4}x + \frac{1}{8}$$

$$16 - \frac{8}{x} = -2x + 1$$

$$16x - 8 = -2x^2 + x$$

## Question 10 continued

$$2x^2 + 16x - 8 = x$$

$$2x^2 + 15x - 8 = 0$$

$$(2x - 1)(x + 8) = 0$$

$$x = \frac{1}{2} \quad x = -8$$

when  $x = -8$

$$y = 2 - \frac{1}{-8}$$

$$= 2 + \frac{1}{8}$$

$$= \frac{16}{8} + \frac{1}{8}$$

$$= \frac{17}{8}$$

$$\underline{\underline{\left(-8, \frac{17}{8}\right)}}$$

Q10

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

