1 Simplify

a $\dfrac{3x-1}{18x-6}$ b $\dfrac{6x+15}{8x+20}$ c $\dfrac{3y+3}{y^2+7y+6}$ d $\dfrac{x^2-25}{x^2-7x+10}$

e $\dfrac{a^2-a-6}{a^2+3a-18}$ f $\dfrac{x^2+3x}{2x^2+5x-3}$ g $\dfrac{3t^2-11t-4}{t^2-16}$ h $\dfrac{6x^2-13x+6}{12x^2+x-6}$

2 Express as simply as possible

a $\dfrac{3x^2}{9x-9}\times\dfrac{4x-4}{2x}$

b $\dfrac{x^2-36}{x^2+7x+10}\div\dfrac{x-6}{x+2}$

c $\dfrac{n^2+2n}{n^2+6n+8}\times\dfrac{n+4}{n^2}$

d $\dfrac{4x-12}{x^2-4}\times\dfrac{x^2+2x}{x^2-2x-3}$

e $\dfrac{4y^2}{2y^2+y}\div\dfrac{y^2+2y-15}{2y^2+11y+5}$

f $\dfrac{x^2-1}{2x^2+7x-4}\times\dfrac{6x^2-5x+1}{3x^2-4x+1}$

g $\dfrac{10x-10}{5x+15}\div\dfrac{4-3x-x^2}{x^2+7x+12}$

h $\dfrac{a^3-3a^2}{8a^2-4a}\div\dfrac{a^2-9}{2a^2+5a-3}$

3 Express as a single fraction in its simplest form

a $\dfrac{2}{y}+\dfrac{7}{y+4}$ b $\dfrac{2x}{x-5}-\dfrac{1}{x+3}$ c $\dfrac{7}{x(x+2)}-\dfrac{3x}{x+2}$

d $\dfrac{x}{(x-3)(x-1)}+\dfrac{5}{2(x-1)}$ e $\dfrac{2}{q^2+3q}+\dfrac{5q}{4q+12}$ f $\dfrac{4}{3x-3}+\dfrac{x+2}{x^2-x}$

g $\dfrac{4}{x+5}+\dfrac{x}{x^2+8x+15}$ h $\dfrac{6x}{x^2-4}-\dfrac{3}{x+2}$ i $\dfrac{5t+12}{2t^2+7t+3}-\dfrac{4}{2t+1}$

4 Simplify

a $\dfrac{x^2-5x}{6x-30}$ b $\dfrac{16-x^2}{x^2+2x-8}$ c $\dfrac{2x^2-4x-6}{3x^2-12x+9}$ d $\dfrac{x^3-x}{2x^2-x-1}$

e $\dfrac{3x-x^2}{2x^2-18}$ f $\dfrac{x^3+x^2-2x}{3x^2+4x-4}$ g $\dfrac{2+5x-3x^2}{2x^2+x-10}$ h $\dfrac{x^4-5x^2+4}{x^2-x-2}$

5 Express as simply as possible

a $\dfrac{10x^2-10}{5x+10}\times\dfrac{x^2+6x+8}{x^2+5x+4}$

b $\dfrac{t^2-2t}{2t^2-t-6}\div\dfrac{9t^2-4}{6t^2+13t+6}$

c $\dfrac{2x^2+12x+10}{4x^2-7x+3}\div\dfrac{4x^2+20x}{4x^2-3x}$

d $\dfrac{8x^2+6x-9}{4x^2+12x+9}\times\dfrac{2x^2+3x}{6-8x}$

e $\dfrac{x^4+6x^2+5}{x^2-9}\times\dfrac{2x^2-6x}{4x^2+4}$

f $\dfrac{y^4-16}{5y^2+9y-2}\div\dfrac{y^2+4}{25y^2-10y+1}$

6 Express as a single fraction in its simplest form

a $\dfrac{5}{x^2-1} - \dfrac{1}{2x+2}$ 　　　　　　　　**b** $\dfrac{3x}{x^2-4} - \dfrac{4}{2x^2+3x-2}$

c $\dfrac{4}{x^2+2x-3} + \dfrac{1}{x^2-3x+2}$ 　　　　**d** $\dfrac{x+1}{x^2-25} + \dfrac{2}{x^2+5x}$

e $\dfrac{2x-1}{x^2+4x+4} + \dfrac{x}{3x+6}$ 　　　　　**f** $\dfrac{1}{x-3} + \dfrac{3}{x^2-3x} + \dfrac{x}{x^2-6x+9}$

g $\dfrac{x}{x^2-x-6} + \dfrac{2}{2x^2+3x-2}$ 　　　**h** $\dfrac{1}{x^2} - \dfrac{1}{3x^2-2x} + \dfrac{3}{6x-4}$

7 Solve

a $1 - \dfrac{2}{x} = \dfrac{3}{2x-5}$ 　　　　　　　　　**b** $\dfrac{2}{x^2-1} + \dfrac{3}{x+1} = 1$

c $\dfrac{20}{2x^2+5x+2} + 1 = \dfrac{10}{2x+1}$ 　　　**d** $\dfrac{y+3}{y+5} - \dfrac{1}{2} = \dfrac{2y-1}{y}$

e $5 + \dfrac{1}{x^2+5x+6} = \dfrac{11}{x+3}$ 　　　　**f** $\dfrac{3}{1-4x+4x^2} - \dfrac{10}{1-4x^2} = \dfrac{5}{1+2x}$

8 $\qquad\qquad \mathrm{f}(x) \equiv \dfrac{7x-15}{x^2-5x} - \dfrac{4}{x-5}, \quad 0 < x < 5.$

Show that $\mathrm{f}(x) = \dfrac{k}{x}$, where k is an integer to be found.

9 $\qquad\qquad \mathrm{f}(x) \equiv \dfrac{x-5}{3x^2+5x-2} + \dfrac{2}{3x-1}, \quad x > 1.$

Show that $\mathrm{f}(x) = \dfrac{1}{x+2}$.

10 Given that $\mathrm{f}(x) \equiv \dfrac{x+2}{x-2}$, $x \neq \pm 2$, show that $\mathrm{f}(x) - \dfrac{1}{\mathrm{f}(x)} = \dfrac{8x}{x^2-4}$.

11 **a** Express $\dfrac{2}{x+5} + \dfrac{3}{(x+2)(x+5)}$ as a single fraction in its simplest form.

b Hence solve the equation

$$\dfrac{2}{x+5} + \dfrac{3}{(x+2)(x+5)} = \dfrac{1}{3},$$

giving your answers to 2 decimal places.

12 Show that the equation $\dfrac{3}{4x+2} - \dfrac{5}{4x^2+4x+1} = 2$ has no real roots.

13 Express $\left(\dfrac{6}{x+5} - \dfrac{1}{x}\right) \div \dfrac{x-1}{x^2-25}$ as a single fraction in its simplest form.