C3 DIFFERENTIATION — Worksheet H

1 Differentiate with respect to x

a $\cos x$ **b** $5 \sin x$ **c** $\cos 3x$ **d** $\sin \frac{1}{4}x$

e $\sin (x + 1)$ **f** $\cos (3x - 2)$ **g** $4 \sin (\frac{\pi}{3} - x)$ **h** $\cos (\frac{1}{2}x + \frac{\pi}{6})$

i $\sin^2 x$ **j** $2 \cos^3 x$ **k** $\cos^2 (x - 1)$ **l** $\sin^4 2x$

2 Use the derivatives of $\sin x$ and $\cos x$ to show that

a $\frac{\mathrm{d}}{\mathrm{d}x}(\tan x) = \sec^2 x$ **b** $\frac{\mathrm{d}}{\mathrm{d}x}(\sec x) = \sec x \tan x$

c $\frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$ **d** $\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\operatorname{cosec}^2 x$

3 Differentiate with respect to t

a $\cot 2t$ **b** $\sec (t + 2)$ **c** $\tan (4t - 3)$ **d** $\operatorname{cosec} 3t$

e $\tan^2 t$ **f** $3 \operatorname{cosec} (t + \frac{\pi}{6})$ **g** $\cot^3 t$ **h** $4 \sec \frac{1}{2}t$

i $\cot (2t - 3)$ **j** $\sec^2 2t$ **k** $\frac{1}{2}\tan (\pi - 4t)$ **l** $\operatorname{cosec}^2 (3t + 1)$

4 Differentiate with respect to x

a $\ln (\sin x)$ **b** $6e^{\tan x}$ **c** $\sqrt{\cos 2x}$ **d** $e^{\sin 3x}$

e $2 \cot x^2$ **f** $\sqrt{\sec x}$ **g** $3e^{-\operatorname{cosec} 2x}$ **h** $\ln (\tan 4x)$

5 Find the coordinates of any stationary points on each curve in the interval $0 \le x \le 2\pi$.

a $y = x + 2 \sin x$ **b** $y = 2 \sec x - \tan x$ **c** $y = \sin x + \cos 2x$

6 Find an equation for the tangent to each curve at the point on the curve with the given x-coordinate.

a $y = 1 + \sin 2x$, $x = 0$ **b** $y = \cos x$, $x = \frac{\pi}{3}$

c $y = \tan 3x$, $x = \frac{\pi}{4}$ **d** $y = \operatorname{cosec} x - 2 \sin x$, $x = \frac{\pi}{6}$

7 Differentiate with respect to x

a $x \sin x$ **b** $\frac{\cos 2x}{x}$ **c** $e^x \cos x$ **d** $\sin x \cos x$

e $x^2 \operatorname{cosec} x$ **f** $\sec x \tan x$ **g** $\frac{x}{\tan x}$ **h** $\frac{\sin 2x}{e^{3x}}$

i $\cos^2 x \cot x$ **j** $\frac{\sec 2x}{x^2}$ **k** $x \tan^2 4x$ **l** $\frac{\sin x}{\cos 2x}$

8 Find the value of $f'(x)$ at the value of x indicated in each case.

a $f(x) = \sin 3x \cos 5x$, $x = \frac{\pi}{4}$ **b** $f(x) = \tan 2x \sin x$, $x = \frac{\pi}{3}$

c $f(x) = \frac{\ln(2\cos x)}{\sin x}$, $x = \frac{\pi}{3}$ **d** $f(x) = \sin^2 x \cos^3 x$, $x = \frac{\pi}{6}$

9 Find an equation for the normal to the curve $y = 3 + x \cos 2x$ at the point where it crosses the y-axis.

10 A curve has the equation $y = \frac{2 + \sin x}{1 - \sin x}$, $0 \le x \le 2\pi$, $x \ne \frac{\pi}{2}$.

a Find and simplify an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

b Find the coordinates of the turning point of the curve.

c Show that the tangent to the curve at the point P, with x-coordinate $\frac{\pi}{6}$, has equation

$$y = 6\sqrt{3}\,x + 5 - \sqrt{3}\,\pi.$$

11 A curve has the equation $y = \mathrm{e}^{-x} \sin x$.

a Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$.

b Find the exact coordinates of the stationary points of the curve in the interval $-\pi \le x \le \pi$ and determine their nature.

12 The curve C has the equation $y = x \sec x$.

a Show that the x-coordinate of any stationary point of C must satisfy the equation

$$1 + x \tan x = 0.$$

b By sketching two suitable graphs on the same set of axes, deduce the number of stationary points C has in the interval $0 \le x \le 2\pi$.

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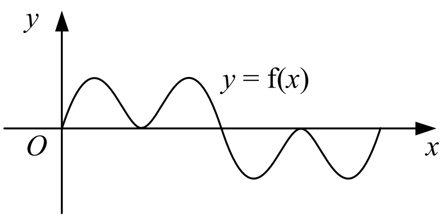

The diagram shows the curve $y = \mathrm{f}(x)$ in the interval $0 \le x \le 2\pi$, where

$$\mathrm{f}(x) \equiv \cos x \sin 2x.$$

a Show that $\mathrm{f}'(x) = 2 \cos x\,(1 - 3 \sin^2 x)$.

b Find the x-coordinates of the stationary points of the curve in the interval $0 \le x \le 2\pi$.

c Show that the maximum value of f(x) in the interval $0 \le x \le 2\pi$ is $\frac{4}{9}\sqrt{3}$.

d Explain why this is the maximum value of f(x) for all real values of x.

14 A curve has the equation $y = \operatorname{cosec}\,(x - \frac{\pi}{6})$ and crosses the y-axis at the point P.

a Find an equation for the normal to the curve at P.

The point Q on the curve has x-coordinate $\frac{\pi}{3}$.

b Find an equation for the tangent to the curve at Q.

The normal to the curve at P and the tangent to the curve at Q intersect at the point R.

c Show that the x-coordinate of R is given by $\frac{8\sqrt{3} + 4\pi}{13}$.