

- 1 a $\sin(A+B) \equiv \sin A \cos B + \cos A \sin B$ (1)
 $\sin(A-B) \equiv \sin A \cos B - \cos A \sin B$ (2)
- b (1) + (2) $\sin(A+B) + \sin(A-B) \equiv \sin A \cos B + \cos A \sin B + \sin A \cos B - \cos A \sin B$
 $\Rightarrow 2 \sin A \cos B \equiv \sin(A+B) + \sin(A-B)$
- c $\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$ (3)
 $\cos(A-B) \equiv \cos A \cos B + \sin A \sin B$ (4)
- (3) + (4) $2 \cos A \cos B \equiv \cos(A+B) + \cos(A-B)$
(4) - (3) $2 \sin A \sin B \equiv \cos(A-B) - \cos(A+B)$
- 2 a $= \sin(30+10)^\circ + \sin(30-10)^\circ$ b $= \cos(36+18)^\circ + \cos(36-18)^\circ$
 $= \sin 40^\circ + \sin 20^\circ$ $= \cos 54^\circ + \cos 18^\circ$
- c $= \frac{1}{2} [\sin(49+25)^\circ - \sin(49-25)^\circ]$ d $= \cos(3A-A) - \cos(3A+A)$
 $= \frac{1}{2} \sin 74^\circ - \frac{1}{2} \sin 24^\circ$ $= \cos 2A - \cos 4A$
- e $= \sin(5A+2A) - \sin(5A-2A)$ f $= 2[\cos(3A+B) + \cos(3A-B)]$
 $= \sin 7A - \sin 3A$ $= 2 \cos(3A+B) + 2 \cos(3A-B)$
- g $= \frac{1}{2} [\sin(A+6B) + \sin(A-6B)]$ h $= \sin[A+(A+40^\circ)] - \sin[A-(A+40^\circ)]$
 $= \frac{1}{2} \sin(A+6B) + \frac{1}{2} \sin(A-6B)$ $= \sin(2A+40^\circ) - \sin(-40^\circ)$
 $= \sin(2A+40^\circ) + \sin 40^\circ$
- 3 a $2 \sin A \cos B \equiv \sin(A+B) + \sin(A-B)$
let $P = A+B$ (1) and $Q = A-B$ (2)
- (1) + (2) $\Rightarrow 2A = P+Q \Rightarrow A = \frac{P+Q}{2}$, (1) - (2) $\Rightarrow 2B = P-Q \Rightarrow B = \frac{P-Q}{2}$
- $\therefore \sin P + \sin Q \equiv 2 \sin \frac{P+Q}{2} \cos \frac{P-Q}{2}$
- b let $P = A+B$ and $Q = A-B$ in each part
- i $2 \cos A \sin B \equiv \sin(A+B) - \sin(A-B) \Rightarrow \sin P - \sin Q \equiv 2 \cos \frac{P+Q}{2} \sin \frac{P-Q}{2}$
- ii $2 \cos A \cos B \equiv \cos(A+B) + \cos(A-B) \Rightarrow \cos P + \cos Q \equiv 2 \cos \frac{P+Q}{2} \cos \frac{P-Q}{2}$
- iii $2 \sin A \sin B \equiv \cos(A-B) - \cos(A+B) \Rightarrow \cos Q - \cos P \equiv 2 \sin \frac{P+Q}{2} \sin \frac{P-Q}{2}$
 $\Rightarrow \cos P - \cos Q \equiv -2 \sin \frac{P+Q}{2} \sin \frac{P-Q}{2}$
- 4 a $= 2 \cos \frac{25+15}{2} \cos \frac{25-15}{2}$ b $= 2 \cos \frac{84+30}{2} \sin \frac{84-30}{2}$
 $= 2 \cos 20^\circ \cos 5^\circ$ $= 2 \cos 57^\circ \sin 27^\circ$
- c $= 2 \sin \frac{5A+A}{2} \cos \frac{5A-A}{2}$ d $= -2 \sin \frac{A+2A}{2} \sin \frac{A-2A}{2}$
 $= 2 \sin 3A \cos 2A$ $= -2 \sin \frac{3A}{2} \sin(-\frac{A}{2}) = 2 \sin \frac{3A}{2} \sin \frac{A}{2}$
- e $= -2 \sin \frac{2A+4B}{2} \sin \frac{2A-4B}{2}$ f $= 2 \sin \frac{2A+90}{2} \cos(\frac{-30}{2})$
 $= -2 \sin(A+2B) \sin(A-2B)$ $= 2 \sin(A+45) \cos(-15) = 2 \sin(A+45^\circ) \cos 15^\circ$
- g $= 4 \cos \frac{A+3A}{2} \cos \frac{A-3A}{2}$ h $= 2 \cos \frac{4A+B}{2} \sin \frac{3B-2A}{2}$
 $= 4 \cos 2A \cos(-A) = 4 \cos 2A \cos A$ $= 2 \cos(2A + \frac{1}{2}B) \sin(\frac{3}{2}B - A)$

5 a $2 \cos \frac{3x+x}{2} \sin \frac{3x-x}{2} = 0$

$$\cos 2x \sin x = 0$$

$$\cos 2x = 0 \text{ or } \sin x = 0$$

$$2x = \frac{\pi}{2}, 2\pi - \frac{\pi}{2} \text{ or } x = 0, \pi$$

$$2x = \frac{\pi}{2}, \frac{3\pi}{2} \text{ or } x = 0, \pi$$

$$x = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi$$

c $\cos(x - 5x) - \cos(x + 5x) = \cos 4x$

$$\cos(-4x) - \cos 6x = \cos 4x$$

$$\cos 4x - \cos 6x = \cos 4x$$

$$\cos 6x = 0$$

$$6x = \frac{\pi}{2}, 2\pi - \frac{\pi}{2}, 2\pi + \frac{\pi}{2},$$

$$4\pi - \frac{\pi}{2}, 4\pi + \frac{\pi}{2}, 6\pi - \frac{\pi}{2}$$

$$= \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}$$

$$x = \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}, \frac{11\pi}{12}$$

e $2 \sin \frac{x+\frac{x}{2}}{2} \cos \frac{x-\frac{x}{2}}{2} = 0$

$$\sin \frac{3}{4}x \cos \frac{1}{4}x = 0$$

$$\sin \frac{3}{4}x = 0 \text{ or } \cos \frac{1}{4}x = 0$$

$$\frac{3}{4}x = 0 \text{ or (none in interval)}$$

$$x = 0$$

6 a $\cos(2x + 3x) + \cos(2x - 3x) - \cos x = 0$

$$\cos 5x + \cos(-x) - \cos x = 0$$

$$\cos 5x + \cos x - \cos x = 0$$

$$\cos 5x = 0$$

$$5x = 90, 360 - 90, 360 + 90,$$

$$720 - 90, 720 + 90$$

$$= 90, 270, 450, 630, 810$$

$$x = 18^\circ, 54^\circ, 90^\circ, 126^\circ, 162^\circ$$

b $\cos 4x - \cos x = 0$

$$-2 \sin \frac{4x+x}{2} \sin \frac{4x-x}{2} = 0$$

$$\sin \frac{5}{2}x \sin \frac{3}{2}x = 0$$

$$\sin \frac{5}{2}x = 0 \text{ or } \sin \frac{3}{2}x = 0$$

$$\frac{5}{2}x = 0, \pi, 2\pi \text{ or } \sin \frac{3}{2}x = 0, \pi$$

$$x = 0, \frac{2\pi}{5}, \frac{2\pi}{3}, \frac{4\pi}{5}$$

d $4[\sin(2x + \frac{\pi}{2}) - \sin \frac{\pi}{6}] = 1$

$$\sin(2x + \frac{\pi}{2}) - \frac{1}{2} = \frac{1}{4}$$

$$\sin(2x + \frac{\pi}{2}) = \frac{3}{4}$$

$$2x + \frac{\pi}{2} = \pi - 0.8481, 2\pi + 0.8481$$

$$= 2.2935, 7.1312$$

$$2x = 0.7227, 5.5605$$

$$x = 0.36, 2.78$$

f $2 \cos \frac{3x+x}{2} \cos \frac{3x-x}{2} = \cos 2x$

$$2 \cos 2x \cos x = \cos 2x$$

$$\cos 2x(2 \cos x - 1) = 0$$

$$\cos 2x = 0 \text{ or } \cos x = \frac{1}{2}$$

$$2x = \frac{\pi}{2}, 2\pi - \frac{\pi}{2} \text{ or } x = \frac{\pi}{3}$$

$$2x = \frac{\pi}{2}, \frac{3\pi}{2} \text{ or } x = \frac{\pi}{3}$$

$$x = \frac{\pi}{4}, \frac{\pi}{3}, \frac{3\pi}{4}$$

b $2 \cos \frac{3x+2x}{2} \sin \frac{3x-2x}{2} = 0$

$$\cos \frac{5}{2}x \sin \frac{1}{2}x = 0$$

$$\cos \frac{5}{2}x = 0 \text{ or } \sin \frac{1}{2}x = 0$$

$$\frac{5}{2}x = 90, 360 - 90, 360 + 90 \text{ or } \frac{1}{2}x = 0$$

$$\frac{5}{2}x = 90, 270, 450 \text{ or } \frac{1}{2}x = 0$$

$$x = 0, 36^\circ, 108^\circ, 180^\circ$$

$$\begin{aligned} \text{c } 2 \sin \frac{4x+2x}{2} \cos \frac{4x-2x}{2} &= \sin 3x \\ 2 \sin 3x \cos x &= \sin 3x \\ \sin 3x(2 \cos x - 1) &= 0 \\ \sin 3x = 0 \text{ or } \cos x &= \frac{1}{2} \\ 3x = 0, 180, 360, 540 \text{ or } x &= 60 \\ x = 0, 60^\circ, 120^\circ, 180^\circ \end{aligned}$$

$$\begin{aligned} \text{e } \frac{1}{2} [\sin (5x+x) - \sin (5x-x)] + \sin 4x &= 0 \\ \frac{1}{2} \sin 6x - \frac{1}{2} \sin 4x + \sin 4x &= 0 \\ \frac{1}{2} \sin 6x + \frac{1}{2} \sin 4x &= 0 \\ \sin \frac{6x+4x}{2} \cos \frac{6x-4x}{2} &= 0 \\ \sin 5x \cos x &= 0 \\ \sin 5x = 0 \text{ or } \cos x &= 0 \\ 5x = 0, 180, 360, 540, 720, 900 \text{ or } x &= 90 \\ x = 0, 36^\circ, 72^\circ, 90^\circ, 108^\circ, 144^\circ, 180^\circ \end{aligned}$$

$$\begin{aligned} 7 \quad \text{a } \text{LHS} &= 2 \sin \frac{x+3x}{2} \cos \frac{x-3x}{2} + \sin 2x \\ &= 2 \sin 2x \cos (-x) + \sin 2x \\ &= 2 \sin 2x \cos x + \sin 2x \\ &= \sin 2x(2 \cos x + 1) \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{d } \cos 2x - \cos (x-60) &= 0 \\ -2 \sin \frac{3x-60}{2} \sin \frac{x+60}{2} &= 0 \\ \sin \left(\frac{3}{2}x - 30\right) \sin \left(\frac{1}{2}x + 30\right) &= 0 \\ \sin \left(\frac{3}{2}x - 30\right) = 0 \text{ or } \sin \left(\frac{1}{2}x + 30\right) &= 0 \\ \frac{3}{2}x - 30 = 0, 180 \text{ or (none in interval)} \\ \frac{3}{2}x &= 30, 210 \\ x &= 20^\circ, 140^\circ \end{aligned}$$

$$\begin{aligned} \text{f } 2 \sin \frac{x+3x}{2} \cos \frac{x-3x}{2} &= 2 \cos \frac{x+3x}{2} \cos \frac{x-3x}{2} \\ \sin 2x \cos (-x) &= \cos 2x \cos (-x) \\ \sin 2x \cos x &= \cos 2x \cos x \\ \cos x(\sin 2x - \cos 2x) &= 0 \\ \cos x = 0 \text{ or } \sin 2x &= \cos 2x \\ \cos x = 0 \text{ or } \tan 2x &= 1 \\ x = 90 \text{ or } 2x = 45, 180 + 45 = 45, 225 \\ x &= 22.5^\circ, 90^\circ, 112.5^\circ \end{aligned}$$

$$\begin{aligned} \text{b } \text{LHS} &= \frac{-2 \sin \frac{x+3x}{2} \sin \frac{x-3x}{2}}{2 \cos \frac{x+3x}{2} \cos \frac{x-3x}{2}} \\ &= \frac{-\sin 2x \sin (-x)}{\cos 2x \cos (-x)} \\ &= \frac{\sin 2x \sin x}{\cos 2x \cos x} \\ &= \tan x \tan 2x \\ &= \text{RHS} \end{aligned}$$