

Write your name here

Surname

Other Names

Mathematics

2019 Practice Paper Paper 3 (Calculator) Higher Tier

Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – there may be more space than you need.
- **Calculators may be used.**
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working.**



Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

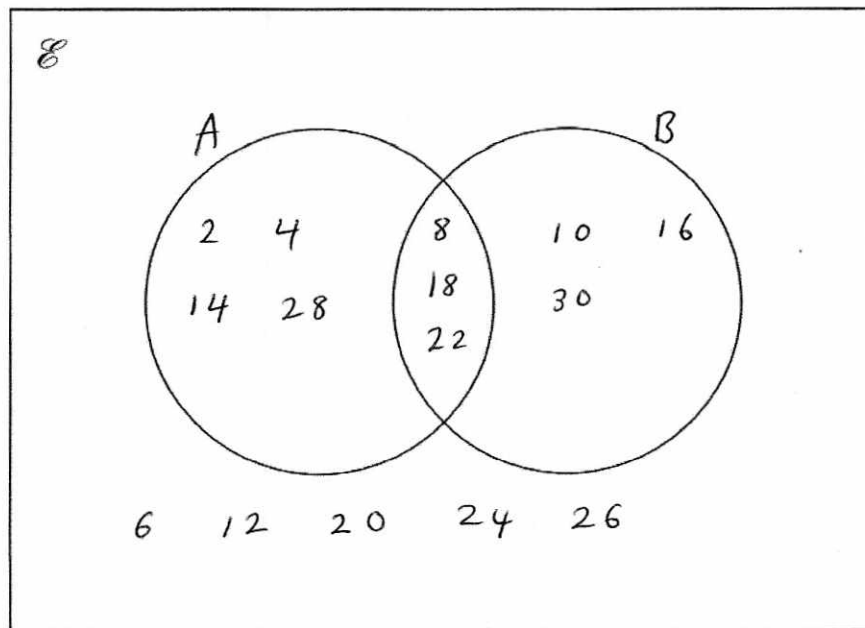
- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

1 $\mathcal{E} = \{\text{even numbers between 1 and 31}\}$

$A = \{2, 4, 8, 14, 18, 22, 28\}$

$B = \{8, 10, 16, 18, 22, 30\}$

(a) Complete the Venn diagram to represent this information.



(4)

A number is chosen at random from the universal set, \mathcal{E}

(b) What is the probability that the number is in the set $A \cup B$?

$$\frac{10}{15} \left[\text{OR} \frac{2}{3} \right]$$

.....

(2)

(Total for question 1 is 6 marks)

2 The frequency table shows the time taken for 100 people to travel to an event.

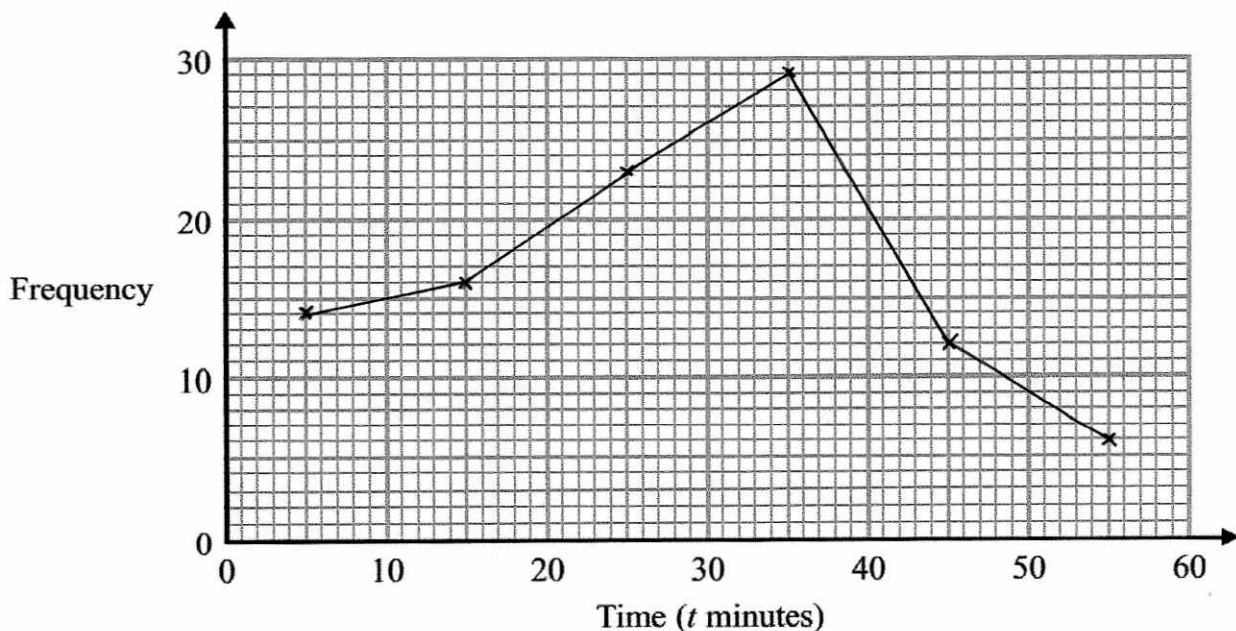
Time (minutes)	Frequency
$0 < t \leq 10$	14
$10 < t \leq 20$	16
$20 < t \leq 30$	23
$30 < t \leq 40$	29
$40 < t \leq 50$	12
$50 < t \leq 60$	6

(a) Find the percentage of people that travelled for more than 30 minutes to the event

$$29 + 12 + 6$$

$$\frac{47}{100} \times 100\%$$

(b) Draw a frequency polygon for the information on the table.



(2)

(Total for question 2 is 4 marks)

3 (a) Find the reciprocal of 8

$$\frac{1}{8}$$

(1)

(b) Use your calculator to work out $(2 \cos 40^\circ + 3 \sin 25^\circ)^3$
Write down all the figures on your calculator display.

$$21.95067518$$

(2)

(Total for question 3 is 3 marks)

4 Solve the simultaneous equations

$$\begin{array}{r} 2x + 5y = 2 \quad \times 7 \\ 7x - 4y = -1 \quad \times 2 \end{array}$$

$$14x + 35y = 14$$

$$14x - 8y = -2$$

$$43y = 16$$

$$y = \frac{16}{43}$$

$$2x + 5\left(\frac{16}{43}\right) = 2$$

$$2x + \frac{80}{43} = 2$$

$$2x = \frac{6}{43}$$

$$x = \frac{3}{43}$$

$$x = \frac{3}{43}$$

$$y = \frac{16}{43}$$

(Total for question 4 is 3 marks)

- 5 A is the point with coordinates (3, 8)
 B is the point with coordinates (x, 13)

The gradient of AB is 2.5
 Work out the value of x

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$2.5 = \frac{13 - 8}{x - 3}$$

$$2.5 = \frac{5}{x - 3}$$

$$x - 3 = 2$$

$$x = 5$$

5

(Total for question 5 is 2 marks)

- 6 (a) Olivia is going to invest some money for 5 years.

She can choose from two options:

Investment A: 2.7% compound interest per annum

Investment B: 2.8% simple interest per annum

Which investment should Olivia choose

You must show your working.

$$\begin{aligned} & \underline{A} \\ 100 \times 1.027^5 \\ & = 114.2489502 \end{aligned}$$

14.25% increase

$$\begin{aligned} & \underline{B} \\ 2.8 \times 5 = 14 \end{aligned}$$

14% increase

Investment A

(Total for question 6 is 4 marks)

7 The exchange rate in London is £1 = \$1.31

The exchange rate in New York is \$1 = £0.79

Bernie wants to change some pounds into dollars.

In which of these cities would Bernie get the most dollars?

You must show your working.

$$\begin{array}{l} \text{New York} \quad \$1 = \pounds 0.79 \\ \qquad \qquad \qquad \div 0.79 \qquad \div 0.79 \end{array}$$

$$\$1.27 = \pounds 1$$

$$\text{London} \quad \pounds 1 = \$1.31$$

LONDON

(Total for question 7 is ³ marks)

8 Each year Rose buys an annual ticket for his train journey to work.

The price of Rose's ticket increased by 2% in 2017 and 3% in 2018.

The ticket cost £2534 in 2018.

What was the price of the ticket in 2016?

$$x \times 1.02 \times 1.03 = 2534$$

$$x = \frac{2534}{1.02 \times 1.03}$$

£ 2411.96

(Total for question 8 is 3 marks)

9 Last year Patrick paid £2534 for his annual train ticket.
This year he has to pay £2612 for his annual train ticket.

Work out the percentage increase in the cost of his train ticket.

Give your answer correct to 3 significant figures

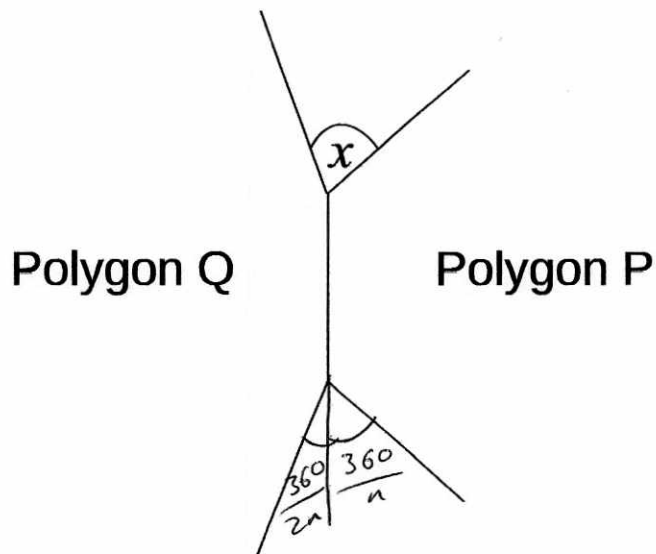
$$\frac{\text{change}}{\text{original}} \times 100$$

$$\frac{2612 - 2534}{2534} \times 100$$

3.08%

(Total for question 9 is 3 marks)

- 10 Two regular polygons P and Q have a common side as shown in the diagram.



Polygon P has n sides. Polygon Q has twice as many sides as Polygon P.

Find the size of angle x in terms of n . *2n sides*

$$\text{Exterior Angle of P} = \frac{360}{n}$$

$$\text{Exterior Angle of Q} = \frac{360}{2n}$$

$$x = \frac{360}{n} + \frac{360}{2n}$$

$$= \frac{360}{n} + \frac{180}{n}$$

$$= \frac{540}{n}$$

$$\frac{540}{n}$$

.....
(Total for question 10 is 3 marks)

11 Liquid A has a density of 1.2 g/cm³

150 cm³ of Liquid A is mixed with some of Liquid B to make Liquid C.

Liquid C has a mass of 210 g and a density of 1.12 g/cm³

Find the density of Liquid B.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

~~Give your answer to 2 decimal places.~~

$$\begin{aligned} A// \text{ mass} &= \text{density} \times \text{volume} \\ &= 1.2 \times 150 \\ &= 180\text{g} \end{aligned}$$

$$\begin{aligned} C// \text{ volume} &= \frac{\text{mass}}{\text{density}} \\ &= \frac{210}{1.12} \\ &= 187.5 \end{aligned}$$

$$\begin{aligned} B// \text{ mass} &= 210 - 180 = 30\text{g} \\ \text{volume} &= 187.5 - 150 = 37.5 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{density} &= \frac{30}{37.5} \\ &= 0.8 \end{aligned}$$

.....0.8.....g/cm³

(Total for question 11 is 3 marks)

- 12 Emma has a bag containing a large number of beads.
She wants to find an estimate for the number of beads in the bag.

Emma takes a sample of 50 beads from the bag.
She marks each bead with a black cross and then puts the beads back in the bag.

Emma shakes the bag.
She now takes another sample of 50 beads from the bag.

6 of these beads have been marked with a black cross.

Work out an estimate for the total number of beads in the bag.

$$\frac{50}{x} = \frac{6}{50}$$

$$\frac{50}{6} = 8.3$$

$$50 \times 8.3 = 416.6$$

417

(Total for question 12 is 2 marks)

- 13 A radioactive substance decays by $x\%$ each day. After 8 days half of the substance has decayed.
Find the value of x .
Give your answer to 1 decimal place.

$$y^8 = 0.5$$

$$y = \sqrt[8]{0.5}$$

$$y = 0.917$$

$$(1 - 0.917) \times 100 = 8.3\%$$

8.3

(Total for question 13 is 3 marks)

14 (a) Expand and simplify $(x+5)(x+3)(x-4)$

$$(x+5)(x+3)(x-4)$$

$$(x+5)(x^2 - 4x + 3x - 12)$$

$$(x+5)(x^2 - x - 12)$$

$$x^3 - x^2 - 12x + 5x^2 - 5x - 60$$

$$x^3 + 4x^2 - 17x - 60$$

$$\underline{\underline{x^3 + 4x^2 - 17x - 60}}$$

(3)

(b) Solve $3x^2 - 5x - 7 = 0$

Give your solutions correct to 3 significant figures

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

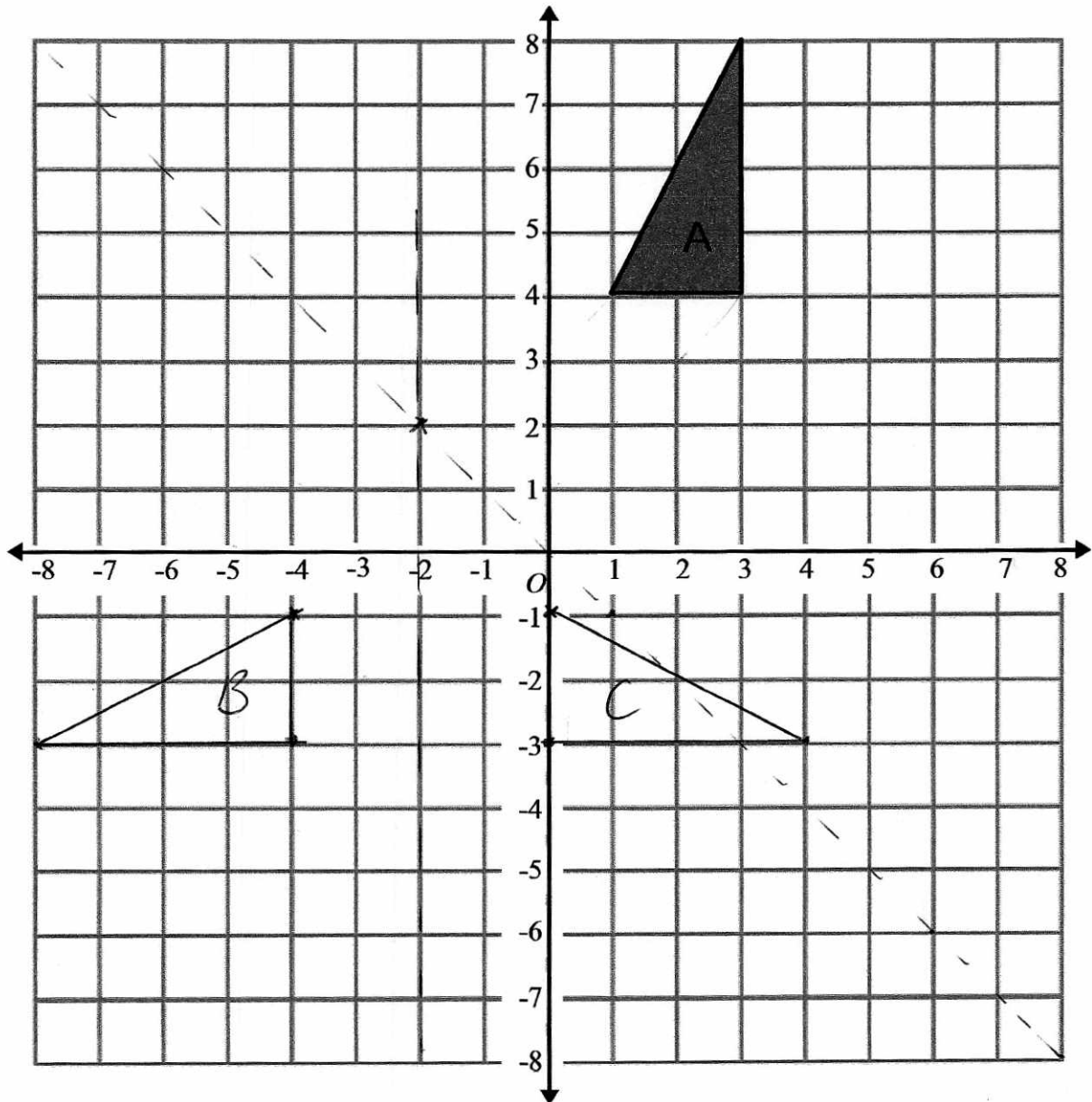
$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-7)}}{2(3)}$$

$$x = \underline{\underline{2.57}} \quad \text{and} \quad x = \underline{\underline{-0.907}}$$

$$\underline{\underline{2.57 \quad \text{and} \quad -0.907}}$$

(3)

(Total for question 14 is 6 marks)



Triangle A is reflected in the line $y = -x$ to give triangle B

Triangle B is reflected in the line $x = -2$ to give triangle C

Describe the single transformation which maps triangle A onto triangle C.

Rotation 90° clockwise centre $(-2, 2)$

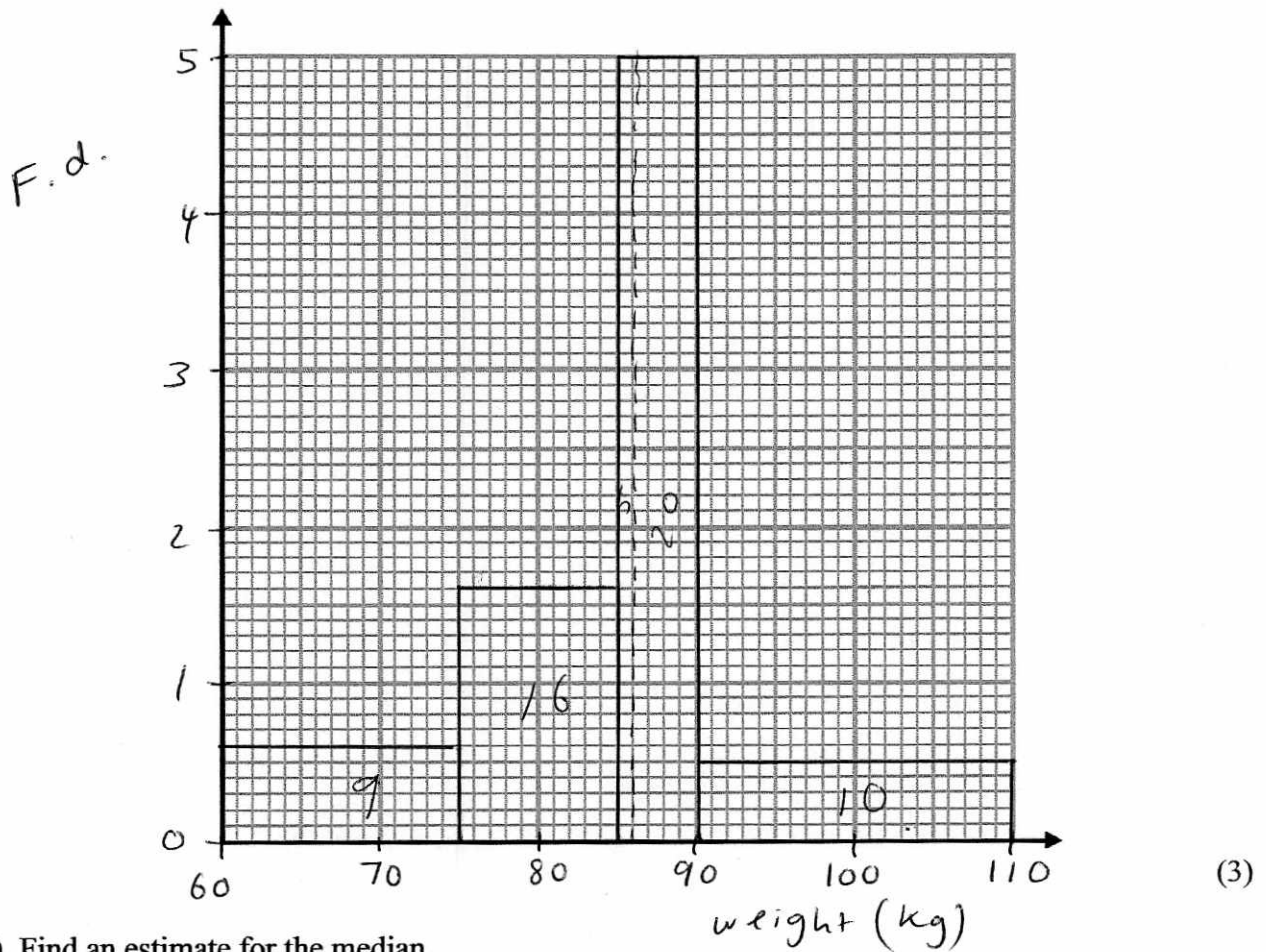
(Total for question 15 is 3 marks)

18 The table shows information about the weight of 60 pigs.

Weight (kg)	Frequency
$60 < w \leq 75$	9
$75 < w \leq 85$	16
$85 < w \leq 90$	25
$90 < w \leq 110$	10

F.d
 0.6
 1.6
 5
 0.5

(a) On the grid, draw a histogram for the information in the table.



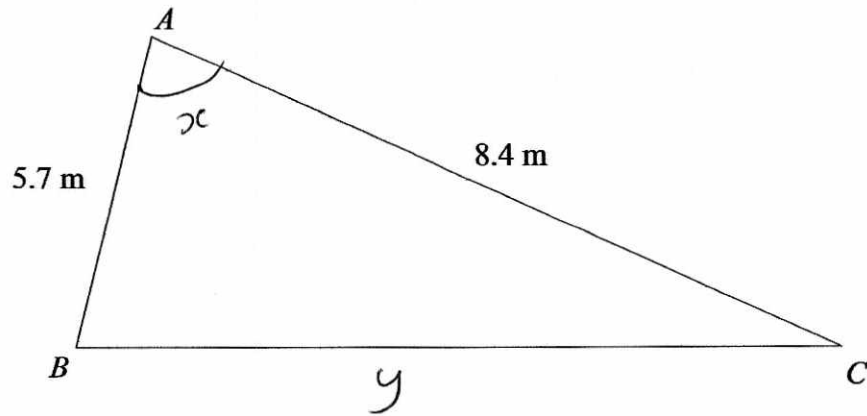
(b) Find an estimate for the median.

30 on each side

..... 86 kg
 (2)

(Total for question 18 is 5 marks)

19



The area of the triangle is 21m^2
 Calculate the perimeter of triangle ABC.
 Give your answer to 1 decimal place.

$$\frac{1}{2} a b \sin C = 21$$

$$\frac{1}{2} (5.7)(8.4) \sin x = 21$$

$$\sin x = \frac{50}{57}$$

$$x = 61.30558647$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$y^2 = (5.7)^2 + (8.4)^2 - 2(5.7)(8.4) \cos (61.3)$$

$$y^2 = 57.07198786$$

$$y = 7.5546 \dots$$

$$\begin{aligned} \text{Perimeter} &= 5.7 + 8.4 + 7.5546 \\ &= 21.7 \text{ m} \end{aligned}$$

21.7.....m

(Total for question 19 is 5 marks)

20 (a) Show that the equation $x^3 - 4x^2 + 1 = 0$ has a solution between $x = 3$ and $x = 4$

$$(3)^3 - 4(3)^2 + 1 = -8$$

$$(4)^3 - 4(4)^2 + 1 = 1$$

change of sign \therefore solution between $x = 3$ and $x = 4$

(2)

(b) Show that the equation $x^3 - 4x^2 + 1 = 0$ can be rearranged to give: $x = \sqrt[3]{4x^2 - 1}$

$$x^3 - 4x^2 + 1 = 0$$

$$x^3 + 1 = 4x^2$$

$$x^3 = 4x^2 - 1$$

$$x = \sqrt[3]{4x^2 - 1}$$

(1)

(c) Starting with $x_0 = 4$, use the iteration formula $x_{n+1} = \sqrt[3]{4x_n^2 - 1}$ to find the value of x_2

Give your answer to 3 decimal places.

$$\begin{aligned} x_1 &= \sqrt[3]{4(4)^2 - 1} \\ &= 3.979057208 \end{aligned}$$

$$\begin{aligned} x_2 &= \sqrt[3]{4(\text{Ans})^2 - 1} \\ &= 3.964934863 \end{aligned}$$

3.965

(3)

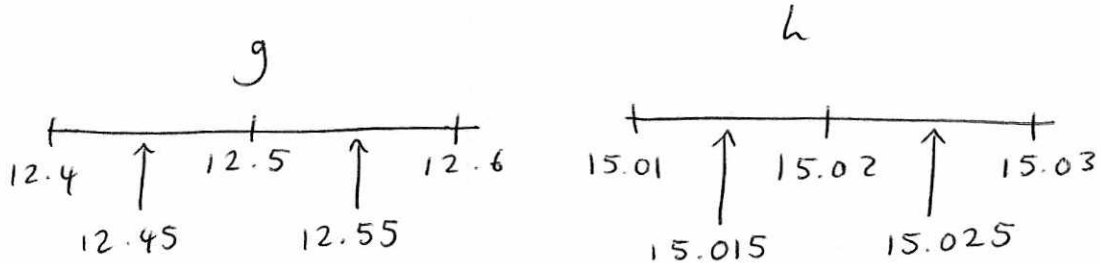
(Total for question 20 is 6 marks)

21

$$f = \frac{\sqrt{g}}{h}$$

$g = 12.5$ correct to 3 significant figures
 $h = 15.02$ correct to 4 significant figures

By considering bounds, work out the value of f to a suitable degree of accuracy.
 Give a reason for your answer.



$$\begin{aligned} \text{upper } f &= \frac{\sqrt{\text{upper } g}}{\text{lower } h} \\ &= \frac{\sqrt{12.55}}{15.015} \end{aligned}$$

$$= 0.235937\dots$$

$$\begin{aligned} \text{lower } f &= \frac{\sqrt{\text{lower } g}}{\text{upper } h} \\ &= \frac{\sqrt{12.45}}{15.025} \end{aligned}$$

$$= 0.234838\dots$$

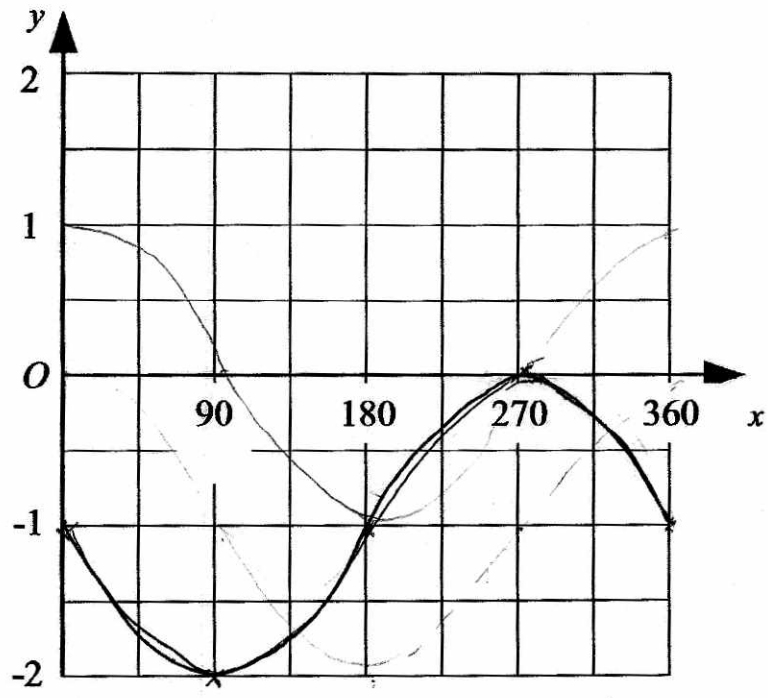
$$f = 0.2 \quad (1 \text{ dp})$$

Both round to 0.2 to 1 dp

.....0.2.....

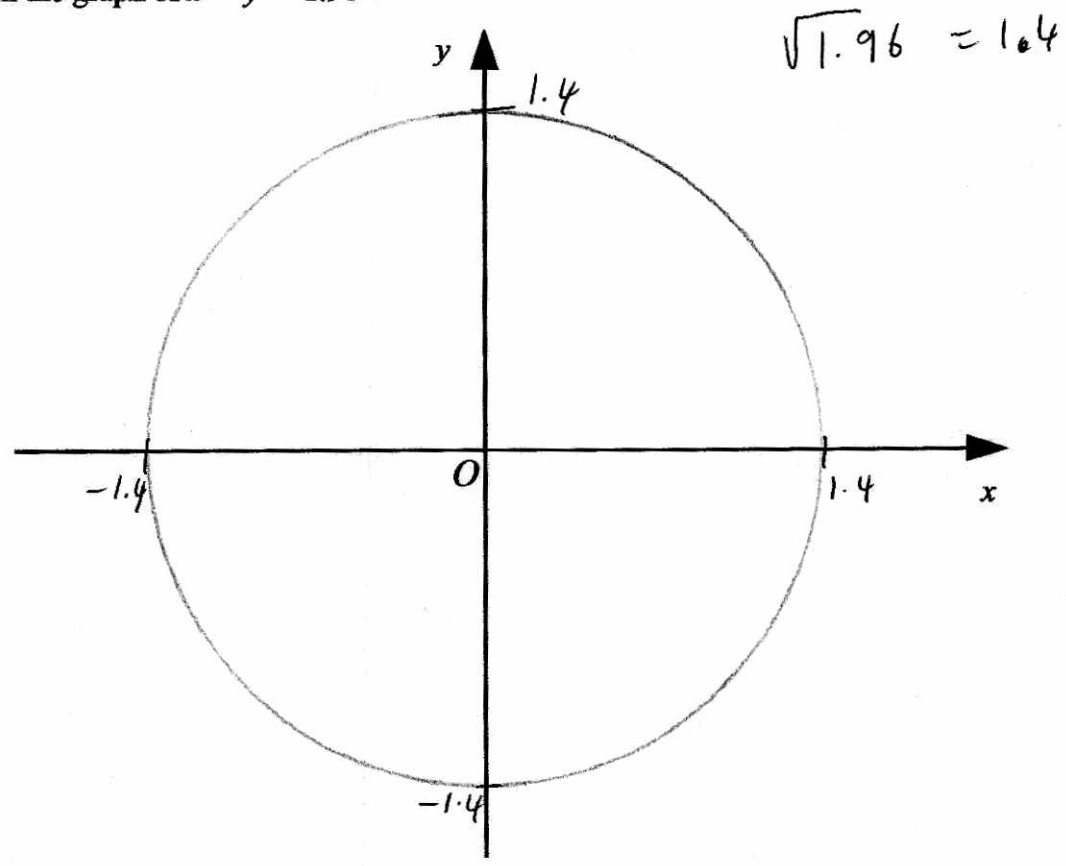
(Total for question 21 is 5 marks)

22 (a) On the graph draw a sketch of $y = \cos(x + 90) - 1$ from $0^\circ \leq x \leq 360^\circ$



(2)

(b) Sketch the graph of $x^2 + y^2 = 1.96$



(2)

(Total for question 22 is 4 marks)

TOTAL FOR PAPER IS 80 MARKS