Edexcel GCE

Core Mathematics M1

Vectors

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A particle P moves with constant acceleration $(2\mathbf{i} - 5\mathbf{j})$ m s^{-2}. At time $t = 0$, P has speed u m s^{-1}. At time $t = 3$ s, P has velocity $(-6\mathbf{i} + \mathbf{j})$ m s^{-1}.

Find the value of u.

(5)

2. A particle is acted upon by two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, given by

$\mathbf{F}_1 = (\mathbf{i} - 3\mathbf{j})$ N,

$\mathbf{F}_2 = (p\mathbf{i} + 2p\mathbf{j})$ N, where p is a positive constant.

(*a*) Find the angle between $\mathbf{F}_2$ and $\mathbf{j}$.

(2)

The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$. Given that $\mathbf{R}$ is parallel to $\mathbf{i}$,

(*b*) find the value of p.

(4)

3. A particle P of mass 2 kg is moving under the action of a constant force $\mathbf{F}$ newtons. When $t = 0$, P has velocity $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1} and at time $t = 4$ s, P has velocity $(15\mathbf{i} - 4\mathbf{j})$ m s^{-1}. Find

(*a*) the acceleration of P in terms of $\mathbf{i}$ and $\mathbf{j}$,

(2)

(*b*) the magnitude of $\mathbf{F}$,

(4)

(*c*) the velocity of P at time $t = 6$ s.

(3)

3. A particle P of mass 0.4 kg moves under the action of a single constant force $\mathbf{F}$ newtons. The acceleration of P is $(6\mathbf{i} + 8\mathbf{j})$ m s^{-2}. Find

(*a*) the angle between the acceleration and $\mathbf{i}$,

(2)

(*b*) the magnitude of $\mathbf{F}$.

(3)

At time t seconds the velocity of P is $\mathbf{v}$ m s^{-1}. Given that when $t = 0$, $\mathbf{v} = 9\mathbf{i} - 10\mathbf{j}$,

(*c*) find the velocity of P when $t = 5$.

(3)

4. A boat B is moving with constant velocity. At noon, B is at the point with position vector $(3\mathbf{i} - 4\mathbf{j})$ km with respect to a fixed origin O. At 1430 on the same day, B is at the point with position vector $(8\mathbf{i} + 11\mathbf{j})$ km.

(*a*) Find the velocity of B, giving your answer in the form $p\mathbf{i} + q\mathbf{j}$.

(3)

At time t hours after noon, the position vector of B is $\mathbf{b}$ km.

(*b*) Find, in terms of t, an expression for $\mathbf{b}$.

(3)

Another boat C is also moving with constant velocity. The position vector of C, $\mathbf{c}$ km, at time t hours after noon, is given by

$$\mathbf{c} = (-9\mathbf{i} + 20\mathbf{j}) + t(6\mathbf{i} + \lambda\mathbf{j}),$$

where λ is a constant.

Given that C intercepts B,

(*c*) find the value of λ,

(5)

(*d*) show that, before C intercepts B, the boats are moving with the same speed.

(3)

5. [*In this question, the unit vectors* **i** *and* **j** *are due east and due north respectively.*]

A particle P is moving with constant velocity $(-5\mathbf{i} + 8\mathbf{j})$ m s^{-1}. Find

(*a*) the speed of P,

(2)

(*b*) the direction of motion of P, giving your answer as a bearing.

(3)

At time $t = 0$, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})$ m relative to a fixed origin O. When $t = 3$ s, the velocity of P changes and it moves with velocity $(u\mathbf{i} + v\mathbf{j})$ m s^{-1}, where u and v are constants. After a further 4 s, it passes through O and continues to move with velocity $(u\mathbf{i} + v\mathbf{j})$ m s^{-1}.

(*c*) Find the values of u and v.

(5)

(*d*) Find the total time taken for P to move from A to a position which is due south of A.

(3)

6. [*In this question the horizontal unit vectors* **i** *and* **j** *are due east and due north respectively.*]

A model boat A moves on a lake with constant velocity $(-\mathbf{i} + 6\mathbf{j})$ m s^{-1}. At time $t = 0$, A is at the point with position vector $(2\mathbf{i} - 10\mathbf{j})$ m. Find

(*a*) the speed of A,

(2)

(*b*) the direction in which A is moving, giving your answer as a bearing.

(3)

At time $t = 0$, a second boat B is at the point with position vector $(-26\mathbf{i} + 4\mathbf{j})$ m.

Given that the velocity of B is $(3\mathbf{i} + 4\mathbf{j})$ m s^{-1},

(*c*) show that A and B will collide at a point P and find the position vector of P.

(5)

Given instead that B has speed 8 m s^{-1} and moves in the direction of the vector $(3\mathbf{i} + 4\mathbf{j})$,

(*d*) find the distance of B from P when $t = 7$ s.

(6)

7. [*In this question the unit vectors* **i** *and* **j** *are due east and north respectively*]

A ship S is moving with constant velocity $(-2.5\mathbf{i} + 6\mathbf{j})$ km h^{-1}. At time 1200, the position vector of S relative to a fixed origin O is $(16\mathbf{i} + 5\mathbf{j})$ km. Find

(*a*) the speed of S,

(2)

(*b*) the bearing on which S is moving.

(2)

The ship is heading directly towards a submerged rock R. A radar tracking station calculates that, if S continues on the same course with the same speed, it will hit R at the time 1500.

(*c*) Find the position vector of R.

(2)

The tracking station warns the ship's captain of the situation. The captain maintains S on its course with the same speed until the time is 1400. He then changes course so that S moves due north at a constant speed of 5 km h^{-1}. Assuming that S continues to move with this new constant velocity, find

(*d*) an expression for the position vector of the ship t hours after 1400,

(4)

(*e*) the time when S will be due east of R,

(2)

(*f*) the distance of S from R at the time 1600.

(3)

8. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively.*]

A hiker H is walking with constant velocity $(1.2\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

(*a*) Find the speed of H.

(2)

Figure 3

A horizontal field $OABC$ is rectangular with OA due east and OC due north, as shown in Figure 3. At twelve noon hiker H is at the point Y with position vector $100\mathbf{j}$ m, relative to the fixed origin O.

(*b*) Write down the position vector of H at time t seconds after noon.

(2)

At noon, another hiker K is at the point with position vector $(9\mathbf{i} + 46\mathbf{j})$ m. Hiker K is moving with constant velocity $(0.75\mathbf{i} + 1.8\mathbf{j})$ m s^{-1}.

(*c*) Show that, at time t seconds after noon,

$$\overrightarrow{HK} = [(9 - 0.45t)\mathbf{i} + (2.7t - 54)\mathbf{j}] \text{ metres.}$$

(4)

Hence,

(*d*) show that the two hikers meet and find the position vector of the point where they meet.

(5)

9. Two forces, $(4\mathbf{i} - 5\mathbf{j})$ N and $(p\mathbf{i} + q\mathbf{j})$ N, act on a particle P of mass m kg. The resultant of the two forces is $\mathbf{R}$. Given that $\mathbf{R}$ acts in a direction which is parallel to the vector $(\mathbf{i} - 2\mathbf{j})$,

(*a*) find the angle between $\mathbf{R}$ and the vector $\mathbf{j}$,

(3)

(*b*) show that $2p + q + 3 = 0$.

(4)

Given also that $q = 1$ and that P moves with an acceleration of magnitude $8\sqrt{5}$ m s^{-2},

(*c*) find the value of m.

(7)