# Partial Fractions

## Splitting Fractions

$$\frac{6x-2}{(x-3)(x+1)}=\frac{A}{(x-3)}+\frac{B}{(x+1)}$$

$$\frac{6x-2}{(x-3)(x+1)}=\frac{A(x+1)}{(x-3)(x+1)}+\frac{B(x-3)}{(x-3)(x+1)}$$

$$let\ x=-1 \qquad 6(-1)-2=A((-1)+1)+B((-1)-3)$$

$$B=2$$

$$let\ x=3 \qquad 6(3)-2=A((3)+1)+B((3)-3)$$

$$A=4$$

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When there is a square in the denominator, it is included twice:

$$\frac{6x-2}{(x-3)(x+1)^2}=\frac{A}{(x-3)}+\frac{B}{(x+1)}+\frac{C}{(x+1)^2}$$

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## Top Heavy Fractions

For top heavy fractions divide the numerator by the denominator algebraic long division).

# Parametric Equations

## Cartesian Equation: Eliminate t

$$x = 2\mathrm{t} \quad y = t^2$$

$$t = \frac{x}{2}$$

$$y = \left(\frac{x}{2}\right)^2$$

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## Differentiating: Differentiate both separately

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

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## Integrating: We can change to t

$$\int y\, dx = \int y \frac{dx}{dt}\, dt$$

remember to change the limits

# The Binomial Expansion

We can expand negative and fractional powers using the formula:

$$(1+kx)^n = 1 + n(kx) + \frac{n(n-1)(kx)^2}{2\times 1} + \frac{n(n-1)(n-2)(kx)^3}{3\times 2\times 1}$$

If the number at the front is not 1, factorise out first

$$(4-3x)^{-\frac{1}{2}}$$

$$4^{-\frac{1}{2}}(1+\frac{3}{4}x)^{-\frac{1}{2}}$$

$$\frac{1}{2}(1+\frac{3}{4}x)^{-\frac{1}{2}}$$

$$\frac{1}{2}(1+(\frac{-1}{2})(\frac{3}{4}x)+\frac{(\frac{-1}{2})(\frac{-3}{2})(\frac{3}{4}x)^2}{2\times 1}+\frac{(\frac{-1}{2})(\frac{-3}{2})(\frac{-5}{2})(\frac{3}{4}x)^3}{3\times 2\times 1})$$

$$\frac{1}{2}(1-\frac{3}{8}x+\frac{27}{128}x^2-\frac{135}{1024}x^3)$$

$$\frac{1}{2}-\frac{3}{16}x+\frac{27}{256}x^2-\frac{135}{2048}x^3$$

# Differentiation

$$y = a^x$$
$$\frac{dy}{dx} = a^x \ln a$$

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# Implicit Differentiation

$$2\mathrm{x}^2 + y = x^3 + 3\mathrm{xy}$$
$$4\mathrm{x} + \frac{dy}{dx} = 3\mathrm{x}^2 + 3\mathrm{x}\frac{dy}{dx} + 3\mathrm{y}$$

# Vectors

$$a.b = |a||b|\cos\theta$$

Lines are perpendicular if a.b = 0

Intersection is where i, j and k are equal

Length of line is Pythagoras

the direction of $\overrightarrow{AB}$ is the position of B minus the position of A

Draw a diagram to help

# Integration

## Integration by Inspection: reversing the chain rule

$$y=24(4x+2)^2$$
$$\int y\,dx=2(4x+2)^3$$

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## Integration by substitution: substitute into the expression eliminating x. Remember to change the limits.

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## Integration by parts: the product rule for integration

$$\int u\frac{dv}{dx}\,dx=uv-\int v\frac{du}{dx}\,dx$$

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## Volumes of Revolution

$$\pi\int_a^b y^2\,dx$$

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## Differential Equations: split to have f(y) dy = g(x) dx