C4 Differential Equations

1a) $$\frac{2}{(2+y)(2-y)} = \frac{A}{2+y} + \frac{B}{2-y}$$

$$2 = A(2-y) + B(2+y)$$

Let $y = 2$

$$2 = 4B$$
$$B = 1/2$$

Let $y = -2$

$$2 = 4A$$
$$A = \frac{1}{2}$$

$$\frac{1}{2(2+y)} + \frac{1}{2(2-y)}$$

b/ $$2\cot x \frac{dy}{dx} = (4 - y^2)$$

$$\frac{2}{4-y^2}\, dy = \frac{1}{\cancel{2}\cot x}\, dx$$

$$\int \frac{1}{2(2+y)} + \frac{1}{2(2-y)}\, dy = \int \frac{\cancel{2}}{\cancel{2}} \tan x \, dx$$

$$\int \frac{1/2}{2+y} + \frac{1/2}{2-y}\, dy = \int \tan x \, dx$$

$$1/2 \ln(2+y) - 1/2 \ln(2-y) = \ln|\sec x| + c$$

$(\frac{\pi}{3}, 0)$ $$0 = \ln 2 + c$$

$$c = -\ln 2$$

$$\frac{1}{2}(\ln(2+y) - \ln(2-y)) = \ln(|\sec x|) - \ln 2$$

$$\frac{1}{2}\ln\left(\frac{2+y}{2-y}\right) = 2\ln\left(\frac{\sec x|}{2}\right)$$

$$\frac{2+y}{2-y} = \left(\frac{\sec x|}{2}\right)^2$$

$$\frac{\cancel{2}(2+y)}{2-y} = \left(\frac{\sec^2 x}{4}\right)$$

$$4\frac{(2+y)}{2-y} \quad \cancel{\frac{4+2y}{2-y}} = \left(\sec^2 x\right)$$

$$\cancel{\left(\frac{4+2y}{2-y}\right)^2} = \cancel{\sec^2 x}$$

$$\frac{8+4y}{2-y} = \sec^2 x$$

2

$$\frac{dP}{dt} = kP$$

$$\int \frac{1}{P}\,dP = \int k\,dt$$

$$\ln p = kt + c$$

$$P = e^{kt+c}$$

$$P = e^c e^{kt}$$

$$P = P_0 e^{kt}$$

b/

$$P = P_0 e^{2.5t}$$

$$2P_0 = P_0 e^{2.5t}$$

$$2 = e^{2.5t}$$

$$\ln 2 = 2.5t$$

$$t = \frac{\ln 2}{2.5} = 0.277 \text{ Days!!!}$$

$\times 24 \times 60$

$= \cancel{16 \text{ mins } 38 \text{ secs}}$ 399 minutes

$\therefore$ ~~17 mins (nearest min)~~

c/

$$\int \frac{1}{\cancel{\lambda}P}\,dP = \lambda \int \cos \lambda t \, dt$$

$$\ln P = \lambda\left(\frac{1}{\lambda}\sin \lambda t\right)$$

$$\ln P = \sin \lambda t + c$$

$$P = e^{\sin(\lambda t) + c}$$

$$P = e^c e^{\sin(\lambda t)}$$

$$P = P_0 e^{\sin(\lambda t)}$$

d/ $$P = P_0 e^{\sin(2.5t)}$$

$$2P_0 = P_0 e^{\sin(2.5t)}$$

$$2 = e^{\sin(2.5t)}$$

$$\ln 2 = \sin(2.5t)$$

$$t = \frac{\sin^{-1}(\ln 2)}{2.5}$$

$$= 0.306\ldots \quad \text{Days}$$

$$= \cancel{18 \text{ mins}, 2\overset{3}{\cancel{2}} \text{ secs}} \qquad \underline{\underline{441 \text{ mins}}}$$

$$= \cancel{\underline{\underline{18 \text{ mins}}} \ (\text{nearest min})}$$

3 $$\frac{2x-1}{(x-1)(2x-3)} = \frac{A}{x-1} + \frac{B}{2x-3}$$

$$2x-1 = A(2x-3) + B(x-1)$$

Let $x = 1$

$$1 = -A$$

$$A = -1$$

Let $x = 3/2$

$$2 = 0.5B$$

$$B = 4$$

$$\frac{-1}{x-1} + \frac{4}{2x-3}$$

b/ $$(2x-3)(x-1)\frac{dy}{dx} = (2x-1)y$$

$$\frac{1}{y}\,dy = \frac{2x-1}{(2x-3)(x-1)}\,dx$$

$$\int \frac{1}{y}\,dy = \int \frac{-1}{x-1} + \frac{4}{2x-3}\,dx$$

$$\ln y = -\ln(x-1) + 2\ln(2x-3) + C$$

c/ $\ln(10) = -\ln(2-1) + 2(\ln(2(2)-3)) + c$

$\ln(10) = -\ln(1) + 2\ln(1) + c$

$\ln(10) = c$

$\ln y = -\ln(x-1) + 2\ln(2x-3) + \ln 10$

$\ln y = \ln\left(\frac{10(2x-3)^2}{x-1}\right)$

$y = \frac{10(2x-3)^2}{x-1}$

$\cancel{= \frac{10(4x^2 - 12x + 9)}{x-1}}$

4a/ $\frac{dS}{dt} = 8$

$S = 6x^2$ $\qquad V = x^3$

$\frac{dS}{dx} = 12x$ $\qquad \frac{dV}{dx} = 3x^2$

$\frac{dx}{dt} = \frac{dS}{dt} \times \frac{dx}{dS}$

$= 8 \times \frac{1}{12x}$

$= \frac{8}{12x}$

$= \frac{2}{3x}$ $\qquad k = 2/3$

b/ $\frac{dV}{dt} = \frac{dx}{dt} \times \frac{dV}{dx}$

$= \frac{2}{3x} \times 3x^2$

$= 2x$ $\qquad V = x^3$

$= 2V^{\frac{1}{3}}$ $\qquad V^{\frac{1}{3}} = x$

c/ $$\frac{dv}{dt} = 2v^{\frac{1}{3}}$$

$$\int \frac{1}{v^{\frac{1}{3}}}\,dv = \int 2\,dt$$

$$\int v^{-\frac{1}{3}}\,dv = \int 2\,dt$$

$$\frac{3}{2}v^{\frac{2}{3}} = 2t + c$$

$\overset{v}{(8},\overset{t}{0)}$ $\therefore$ $c = 6$

$$\frac{3}{2}v^{\frac{2}{3}} = 2t + 6$$

$$\frac{3}{2}(16\sqrt{2})^{\frac{2}{3}} = 2t + 6$$

$$12 = 2t + 6$$

$$6 = 2t$$

$$\underline{\underline{t = 3}}$$

5/ $$V = \frac{4}{3}\pi r^3$$

$$\frac{dv}{dr} = 4\pi r^2$$

b/ $$\frac{dr}{dt} = \frac{dr}{dv} \times \frac{dv}{dt}$$

$$= \frac{1}{4\pi r^2} \times \frac{1000}{(2t+1)^2}$$

$$= \frac{1000}{4\pi r^2(2t+1)^2}$$

$$= \frac{250}{\pi r^2(2t+1)^2}$$

c/ $$\frac{dv}{dt} = \frac{1000}{(2t+1)^2}$$

$$dv = \frac{1000}{(2t+1)^2}\,dt$$

$$\int 1 \, dv = \int 1000(2t+1)^{-2} \, dt$$

$$v = -\frac{1}{2} \times 1000(2t+1)^{-1} + c$$

$$v = -500(2t+1)^{-1} + c$$

$(0,0)$

$$0 = -500 + c$$

$$c = 500$$

$$v = -500(2t+1)^{-1} + 500$$

d i/ when $t = 5$

$$v = \frac{-500}{2(5)+1} + 500$$

$$= \cancel{\frac{4000}{9}} \quad \frac{5000}{11}$$

$$\frac{5000}{11} \cancel{\frac{4000}{9}} = \frac{4}{3}\pi r^3$$

$$\frac{3750}{11\pi} \cancel{\frac{1000}{3\pi}} = r^3$$

$$r = 4.7\cancel{3}7 \text{ cm} \quad 3sf$$

ii
$$\frac{dr}{dt} = \frac{250}{\pi(r)^2(2(5)+1)^2}$$

$$= 0.0289076\ldots$$

$$= \underline{2.90 \times 10^{-2} \ (3sf)}$$