

C3 Numerical Methods

$$1a) \quad x_{n+1} = \frac{2}{(x_n)^2} + 2$$

$$x_0 = 2.5$$

$$x_1 = 2.32$$

$$x_2 = 2.372$$

$$x_3 = 2.356$$

$$x_4 = 2.360$$

$$b) \quad \text{Upper Bound: } 2.3595$$

$$\text{Lower Bound: } 2.3585$$

$$-(2.3595)^3 + 2(2.3595)^2 + 2 = -1.42 \times 10^{-3}$$

$$-(2.3585)^3 + 2(2.3585)^2 + 2 = 5.84 \times 10^{-3}$$

change of sign $\therefore x=2.359$ is correct to 3dp

$$2a) \quad f(x) = \ln(x+2) - x + 1$$

$$f(2) = \ln(4) - 2 + 1 = 0.386 \text{ (3dp)}$$

$$f(3) = \ln(5) - 3 + 1 = -0.391 \text{ (3dp)}$$

change of sign \therefore there is a root of $f(x)=0$
in the interval $2 < x < 3$

$$b) \quad x_{n+1} = \ln(x_n + 2) + 1$$

$$x_0 = 2.5$$

$$x_1 = 2.50408$$

$$x_2 = 2.50498$$

$$x_3 = 2.50518$$

$$c) \quad f(x) = \ln(x+2) - x + 1$$

$$\text{upper bound} = 2.5055$$

$$\text{lower bound} = 2.5045$$

$$f(2.5055) = \ln(4.5055) - 2.5055 + 1 = -2.01 \times 10^{-4} \text{ (2dp)}$$

$$f(2.5045) = \ln(4.5045) - 2.5045 + 1 = 5.77 \times 10^{-4} \text{ (2dp)}$$

change of sign $\therefore x = 2.505$ is a root of $f(x) = 0$

correct to 3dp.

$$3a) \quad f(x) = 3xe^x - 1$$

turning point where $f'(x) = 0$

$$f'(x) = 3e^x + 3xe^x \quad u = 3x \quad v = e^x$$

$$\frac{du}{dx} = 3$$

$$\frac{dv}{dx} = e^x$$

$$f'(x) = 3e^x(1+x)$$

Turning point is where $f'(x) = 0$

$$3e^x(1+x) = 0$$

No solution $x = -1$

$$\text{when } x = -1 \quad y = 3(-1)e^{(-1)} - 1$$

$$= -3e^{-1} - 1$$

$$(-1, -3e^{-1} - 1)$$

$$b) \quad x_{n+1} = \frac{1}{3} e^{-x_n}$$

$$x_0 = 0.25$$

$$x_1 = 0.2596$$

$$x_2 = 0.2571$$

$$x_3 = 0.2578$$

$$c) \quad \text{upper bound} = 0.25765 \quad \text{lower bound} = 0.25755$$

$$f(0.25765) = 3(0.25765)e^{(0.25765)} - 1 = 1.09 \times 10^{-4}$$

$$f(0.25755) = 3(0.25755)e^{(0.25755)} - 1 = -3.79 \times 10^{-4}$$

change of sign $\therefore x = 0.2576$ is correct to 4dp

$$4a) \quad f(x) = 3x^3 - 2x - 6$$

$$f(1.4) = 3(1.4)^3 - 2(1.4) - 6 = -0.568$$

$$f(1.45) = 3(1.45)^3 - 2(1.45) - 6 = 0.245875$$

change of sign $\therefore a$ lies between 1.4 and 1.45

$$b) \quad 3x^3 - 2x - 6 = 0$$

$$3x^3 - 2x = 6$$

$$x(3x^2 - 2) = 6$$

$$3x^2 - 2 = \frac{6}{x}$$

$$3x^2 = \frac{6}{x} + 2$$

$$x^2 = \frac{2}{x} + \frac{2}{3}$$

$$x = \sqrt{\frac{2}{x} + \frac{2}{3}}$$

$$c) \quad x_{n+1} = \sqrt{\frac{2}{x_n} + \frac{2}{3}}$$

$$x_0 = 1.43$$

$$x_1 = 1.4371$$

$$x_2 = 1.4347$$

$$x_3 = 1.4355$$

$$d) \quad \text{upper bound} = 1.4355 \quad \text{Lower bound} = 1.4345$$

$$f(1.4355) = 3(1.4355)^3 - 2(1.4355) - 6 = 3.23 \times 10^{-3} \text{ (2dp)}$$

$$f(1.4345) = 3(1.4345)^3 - 2(1.4345) - 6 = -0.01 \text{ (2dp)}$$

change of sign $\therefore a = 1.435$ (3dp)

5a)

$$f(x) = -x^3 + 3x^2 - 1$$

$$0 = -x^3 + 3x^2 - 1$$

$$0 = x^2(-x + 3) - 1$$

$$1 = x^2(3 - x)$$

$$\frac{1}{3-x} = x^2$$

$$x = \sqrt{\frac{1}{3-x}}$$

b)

$$x_{n+1} = \sqrt{\frac{1}{3-x_n}}$$

$$x_1 = 0.6$$

$$x_2 = 0.6455$$

$$x_3 = 0.6517$$

$$x_4 = 0.6526$$

c)

$$\text{upper bound} = 0.6535$$

$$\text{lower bound} = 0.6525$$

$$f(0.6535) = -(0.6535)^3 + 3(0.6535)^2 - 1 = 2.10 \times 10^{-3} \quad (2 \text{ dp})$$

$$f(0.6525) = -(0.6525)^3 + 3(0.6525)^2 - 1 = -5.37 \times 10^{-4}$$

change of sign $\therefore x = 0.653$ is a root of

$$f(x) = 0 \quad (\text{to 3 dp})$$

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6a)

$$f(x) = 2x^3 - x - 4$$

$$0 = 2x^3 - x - 4$$

$$0 = x(2x^2 - 1) - 4$$

$$4 = x(2x^2 - 1)$$

$$\frac{4}{x} = 2x^2 - 1$$

$$\frac{4}{x} + 1 = 2x^2$$

$$\frac{2}{x} + \frac{1}{2} = x^2$$

$$x = \sqrt{\frac{2}{x} + \frac{1}{2}}$$

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b)

$$x_{n+1} = \sqrt{\frac{2}{x_n} + \frac{1}{2}}$$

$$x_0 = 1.35$$

$$x_1 = 1.41$$

$$x_2 = 1.39$$

$$x_3 = 1.39$$

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c)

$$a = 1.392$$

$$\text{upper bound} = 1.3925$$

$$\text{lower bound} = 1.3915$$

$$f(1.3925) = 2(1.3925)^3 - (1.3925) - 4 = 7.77 \times 10^{-3} \text{ (2dp)}$$

$$f(1.3915) = 2(1.3915)^3 - (1.3915) - 4 = -2.85 \times 10^{-3} \text{ (2dp)}$$

change of sign $\therefore a = 1.392$ to 3dp

$$7a) \quad y = \ln(4 - 2x)$$

$$e^y = 4 - 2x$$

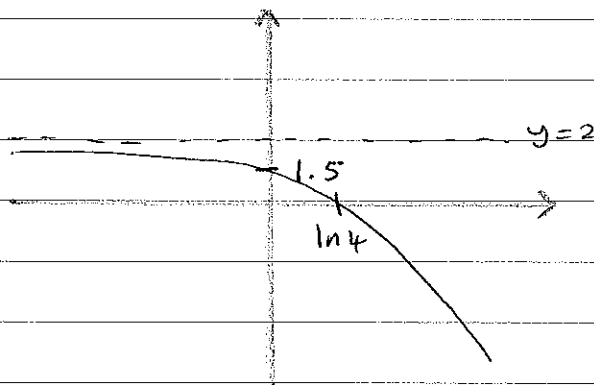
$$2x = 4 - e^y$$

$$x = 2 - \frac{1}{2}e^y$$

$$\underline{f^{-1}(x) = 2 - \frac{1}{2}e^x} \quad x \in \mathbb{R}$$

$$b) \quad f^{-1}(x) < 2$$

c)



crosses x when $y=0$

$$0 = 2 - \frac{1}{2}e^x$$

$$\frac{1}{2}e^x = 2$$

$$e^x = 4$$

$$x = \ln 4$$

$$d) \quad x_{n+1} = -\frac{1}{2}e^{x_n}$$

$$x_0 = -0.3$$

$$x_1 = \underline{\cancel{-0.6749}} = -0.3704$$

$$x_2 = \underline{\cancel{-0.2546}} = -0.3452$$

$$e) \quad x_3 = -0.3540$$

$$x_4 = -0.3509$$

$$x_5 = -0.3520$$

$$x_6 = -0.3516$$

$$x_7 = -0.3518$$

$$x_8 = -0.3517$$

$$\underline{\underline{\kappa = -0.352}}$$

$$8a) \quad y = (2x-1) \tan 2x$$

$$u = 2x-1 \quad v = \tan 2x$$

$$\frac{du}{dx} = 2 \quad \frac{dv}{dx} = 2\sec^2 2x$$

$$\frac{dy}{dx} = 2 \tan 2x + (2x-1)(2)\sec^2 2x$$

$$0 = 2 \tan 2k + (2k-1)(2)\sec^2 2k$$

$$0 = \frac{\sin 2k}{\cos 2k} + (2k-1)\left(\frac{1}{\cos^2 2k}\right)$$

$$0 = \sin 2k \cos 2k + 2k-1$$

$$0 = 2 \sin 2k \cos 2k + 4k - 2$$

$$0 = \underline{\sin 4k + 4k - 2}$$

$$4k + \sin 4k - 2 = 0$$

$$b) \quad x_{n+1} = \frac{1}{4} (2 - \sin 4x_n)$$

$$x_0 = 0.3$$

$$x_1 = \underline{0.266} \quad 0.2670$$

$$x_2 = 0.2809$$

$$x_3 = 0.2746$$

$$x_4 = 0.2774$$

$$c) \quad k = 0.277 \quad \text{upper bound} = 0.2775$$

$$\text{lower bound} = 0.2765$$

$$4(0.2775) + \sin(4 \times 0.2775) - 2 = 5.699 \times 10^{-3} \quad 3dp$$

$$4(0.2765) + \sin(4 \times 0.2765) - 2 = -8.712 \times 10^{-5} \quad 3dp$$

change of sign $\therefore k = 0.277$ to (3 s.f.)

$$9a) \quad 3e^x - \frac{1}{2x} = 0$$

$$3e^x = \frac{1}{2x}$$

$$6xe^x = 1$$

$$6x = \frac{1}{e^x}$$

$$x = \frac{1}{6}e^{-x}$$

$$b) \quad x_{n+1} = \frac{1}{6}e^{-x_n}$$

$$x_0 = 1$$

$$x_1 = 0.0613$$

$$x_2 = 0.1568$$

$$x_3 = 0.1425$$

$$x_4 = 0.1445$$

$$c) \quad A = 0.1443 \text{ 4dp}$$

$$\text{upper bound} = 0.14435$$

$$\text{lower bound} = 0.14425$$

$$3e^{0.14435} - \frac{1}{2(0.14435)} = 2.06 \times 10^{-3} \text{ 2dp}$$

$$3e^{0.14425} - \frac{1}{2(0.14425)} = -6.86 \times 10^{-4} \text{ 2dp}$$

change of sign $\therefore x = 0.1443$ correct to 4dp.