Edexcel GCE

Core Mathematics C3

Logarithms

Materials required for examination   **Items included with question papers**
Mathematical Formulae (Green)   Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers
without working may gain no credit.

1. Rabbits were introduced onto an island. The number of rabbits, P, t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \quad t \in \mathbb{R}, \quad t \geq 0.$$

(a) Write down the number of rabbits that were introduced to the island.

(1)

(b) Find the number of years it would take for the number of rabbits to first exceed 1000.

(2)

(c) Find $\dfrac{dP}{dt}$.

(2)

(d) Find P when $\dfrac{dP}{dt} = 50$.

(3)

2. The radioactive decay of a substance is given by

$$R = 1000e^{-ct}, \quad t \geq 0.$$

where R is the number of atoms at time t years and c is a positive constant.

(a) Find the number of atoms when the substance started to decay.

(1)

It takes 5730 years for half of the substance to decay.

(b) Find the value of c to 3 significant figures.

(4)

(c) Calculate the number of atoms that will be left when $t = 22\,920$.

(2)

(d) Sketch the graph of R against t.

(2)

3. Find the exact solutions to the equations

(a) $\ln x + \ln 3 = \ln 6$,

(2)

(b) $e^x + 3e^{-x} = 4$.

(4)

2

4. The amount of a certain type of drug in the bloodstream t hours after it has been taken is given by the formula

$$x = De^{-\frac{1}{8}t},$$

where x is the amount of the drug in the bloodstream in milligrams and D is the dose given in milligrams.

A dose of 10 mg of the drug is given.

(*a*) Find the amount of the drug in the bloodstream 5 hours after the dose is given.
Give your answer in mg to 3 decimal places.

(2)

A second dose of 10 mg is given after 5 hours.

(*b*) Show that the amount of the drug in the bloodstream 1 hour after the second dose is 13.549 mg to 3 decimal places.

(2)

No more doses of the drug are given. At time T hours after the second dose is given, the amount of the drug in the bloodstream is 3 mg.

(*c*) Find the value of T.

(3)

5. The point P lies on the curve with equation

$$y = 4e^{2x+1}.$$

The y-coordinate of P is 8.

(*a*) Find, in terms of ln 2, the x-coordinate of P.

(2)

(*b*) Find the equation of the tangent to the curve at the point P in the form $y = ax + b$, where a and b are exact constants to be found.

(4)

6. A heated metal ball is dropped into a liquid. As the ball cools, its temperature, $T\,°C$, t minutes after it enters the liquid, is given by

$$T = 400\mathrm{e}^{-0.05t} + 25, \quad t \geq 0.$$

(a) Find the temperature of the ball as it enters the liquid.

(1)

(b) Find the value of t for which $T = 300$, giving your answer to 3 significant figures.

(4)

(c) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$. Give your answer in $°C$ per minute to 3 significant figures.

(3)

(d) From the equation for temperature T in terms of t, given above, explain why the temperature of the ball can never fall to $20\,°C$.

(1)

7. A particular species of orchid is being studied. The population p at time t years after the study started is assumed to be

$$p = \frac{2800a\mathrm{e}^{0.2t}}{1 + a\mathrm{e}^{0.2t}}, \text{ where } a \text{ is a constant.}$$

Given that there were 300 orchids when the study started,

(a) show that $a = 0.12$,

(3)

(b) use the equation with $a = 0.12$ to predict the number of years before the population of orchids reaches 1850.

(4)

(c) Show that $p = \dfrac{336}{0.12 + \mathrm{e}^{-0.2t}}$.

(1)

(d) Hence show that the population cannot exceed 2800.

(2)