Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Differentiation and Integration

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink or Green)	Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. Find the coordinates of the stationary point on the curve with equation $y = 2x^2 - 12x$.

(4)

$$\frac{dy}{dx} = 4x - 12$$

$$4x - 12 = 0$$
$$4x = 12$$
$$x = 3$$

$$y = 2(3)^2 - 12(3)$$
$$= -18$$

$$(3, -18)$$

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2. $$f(x) = x^3 + 3x^2 + 5.$$

Find

(*a*) $f''(x)$,

(3)

(*b*) $\int_1^2 f(x)\ dx$.

(4)

a) $f'(x) = 3x^2 + 6x$

$f''(x) = 6x + 6$

b) $\int_1^2 x^3 + 3x^2 + 5 \ dx$

$$\left[\frac{x^4}{4} + \frac{3x^3}{3} + 5x + c \right]_1^2$$

$$\left[\frac{x^4}{4} + x^3 + 5x \right]_1^2$$

$$\left(\frac{(2)^4}{4} + (2)^3 + 5(2) \right) - \left(\frac{(1)^4}{4} + (1)^3 + 5(1) \right)$$

$$(22) - \left(\frac{25}{4}\right)$$

$$= \frac{63}{4} \text{ units}^2$$

3. Evaluate $\int_1^8 \frac{1}{\sqrt{x}}\, dx$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers.

(4)

$\int_1^8 x^{-1/2}\, dx$

$\left[2x^{1/2}\right]_1^8$

$\left[2(8)^{1/2}\right] - \left[2(1)^{1/2}\right]$

$\underline{\underline{4\sqrt{2} - 2}}$

$a = -2$
$b = 4$

4. Use calculus to find the value of

$$\int_1^4 (2x + 3\sqrt{x})\, dx.$$

(5)

$\int_1^4 2x + 3x^{1/2}\, dx$

$\left[\frac{2x^2}{2} + \frac{3x^{3/2}}{3/2} + c\right]_1^4$

$\left[x^2 + 2x^{3/2}\right]_1^4$

$(4^2 + 2(4)^{3/2}) - ((1)^2 + 2(1)^{3/2})$

$(32) - (3)$

$\underline{\underline{= 29}}$

5.

Figure 1

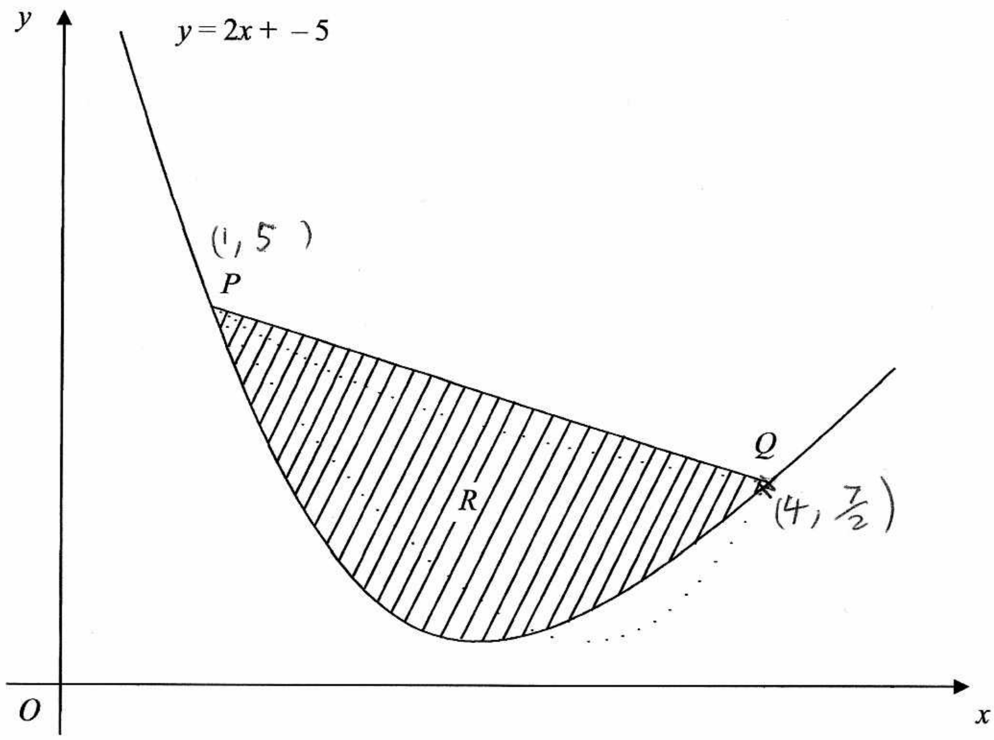

Figure 1 shows part of a curve C with equation $y = 2x + \dfrac{8}{x^2} - 5, x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 4 respectively. The region R, shaded in Figure 1, is bounded by C and the straight line joining P and Q.

(*a*) Find the exact area of R.

(8)

(*b*) Use calculus to show that y is increasing for $x > 2$.

(4)

a) Area of trapezium $= \frac{1}{2}(5 + \frac{7}{2}) \times 3$

$= 12.75$ units^2

$$\int_1^4 2x + 8x^{-2} - 5 \; dx$$

$$\left[\frac{2x^2}{\cancel{2}} - \frac{8x^{-1}}{\cancel{-1}} - 5x + c\right]_1^4$$

$$((4)^2 - 8(4)^{-1} - 5(4)) - ((1)^2 - 8(1)^{-1} - 5(1))$$

$$(-6) - (-12)$$

$= 6$ units^2

$R = 12.\cancel{6}75 - 6 = \underline{6.75 \text{ units}^2}$

b/ $y = 2x + 8x^{-2} - 5$

$\frac{dy}{dx} = 2 - 16x^{-3}$

decreasing where $\frac{dy}{dx} < 0$

$2 - 16x^{-3} < 0$

$2 < 16x^{-3}$

$\frac{1}{8} < x^{-3}$

$\frac{1}{8} < \frac{1}{x^3}$

$8 < x^3$

$\underline{\underline{2 < x}}$

6. The curve C has equation

$$y = 2x^3 - 5x^2 - 4x + 2.$$

(*a*) Find $\dfrac{dy}{dx}$.

(2)

(*b*) Using the result from part (*a*), find the coordinates of the turning points of C.

(4)

(*c*) Find $\dfrac{d^2y}{dx^2}$.

(2)

(*d*) Hence, or otherwise, determine the nature of the turning points of C.

(2)

a) $\dfrac{dy}{dx} = 6x^2 - 10x - 4$

b/ $6x^2 - 10x - 4 = 0$

$3x^2 - 5x - 2 = 0$

$(3x + 1)(x - 2) = 0$

$x = -\tfrac{1}{3} \quad x = 2$

$y = 2(-\tfrac{1}{3})^3 - 5(-\tfrac{1}{3})^2 - 4(-\tfrac{1}{3}) + 2$

$= \tfrac{73}{27}$

$y = 2(2)^3 - 5(2)^2 - 4(2) + 2$

$= -10$

$(-\tfrac{1}{3}, \tfrac{73}{27})$ and $(2, -10)$

c/ $\dfrac{d^2y}{dx^2} = 12x - 10$

when $x = 2$ $\dfrac{d^2y}{dx^2} = 14$ positive $\therefore$ MINIMUM

when $x = -\tfrac{1}{3}$ $\dfrac{d^2y}{dx^2} = -14$ negative $\therefore$ MAXIMUM

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7. Use calculus to find the exact value of $\displaystyle\int_1^2 \left(3x^2 + 5 + \frac{4}{x^2}\right) dx$.

(5)

$\int_1^2 3x^2 + 5 + 4x^{-2}\, dx$

$\left[\dfrac{\cancel{3}x^3}{\cancel{3}} + 5x \mp \dfrac{4x^{-1}}{-1} + c\right]_1^2$

$((2)^3 + 5(2) - 4(2)^{-1}) - ((1)^3 + 5(1) - 4(1)^{-1})$

$(16) - (2)$

$= 14$

8.

Figure 3

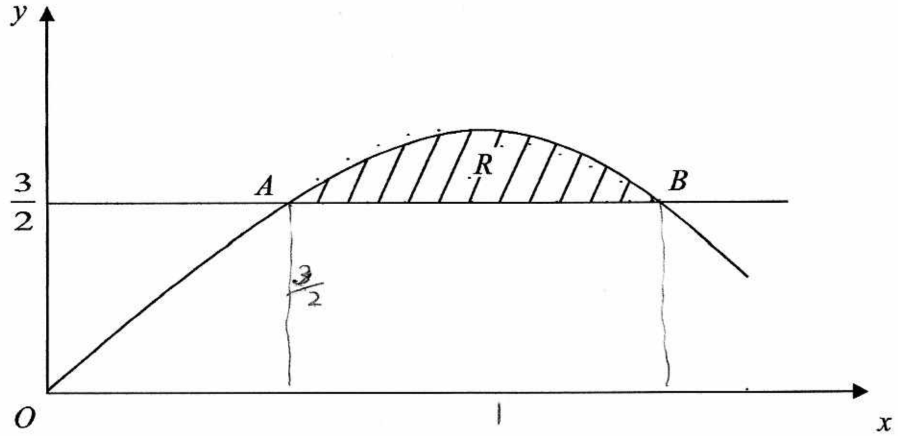


Figure 3 shows the shaded region R which is bounded by the curve $y = -2x^2 + 4x$ and the line $y = \frac{3}{2}$. The points A and B are the points of intersection of the line and the curve.

Find

(a) the x-coordinates of the points A and B, **(4)**

(b) the exact area of R. **(6)**

a) $\frac{3}{2} = -2x^2 + 4x$

$3 = -4x^2 + 8x$

$4x^2 - 8x + 3 = 0$

$(2x - 1)(2x - 3) = 0$

$x = 1/2 \quad x = 3/2$

b/ area under curve: $\int_{1/2}^{3/2} -2x^2 + 4x \, dx$

$\left[\frac{-2x^3}{3} + \frac{\overset{2}{\cancel{4}}x^2}{\cancel{2}} \right]_{1/2}^{3/2}$

$\left(-\frac{2}{3}\left(\frac{3}{2}\right)^3 + 2\left(\frac{3}{2}\right)^2\right) - \left(-\frac{2}{3}\left(\frac{1}{2}\right)^3 + 2\left(\frac{1}{2}\right)^2\right)$

$\frac{9}{4} - \frac{5}{12} = \frac{11}{6}$

Area of rectangle $= 1 \times \frac{3}{2} = \frac{3}{2}$

$\frac{11}{6} - \frac{3}{2} = \underline{\underline{\frac{1}{3}}}$ units^2

9. **Figure 3**

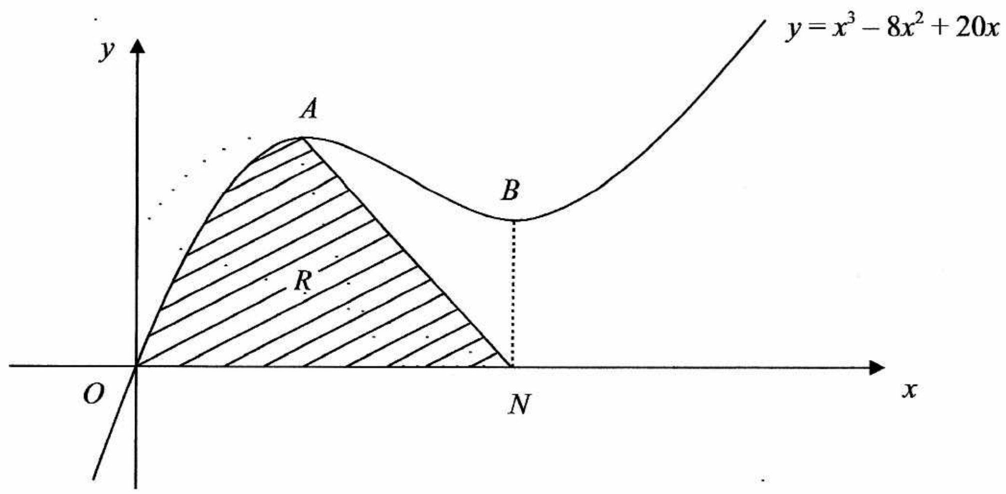


Figure 3 shows a sketch of part of the curve with equation $y = x^3 - 8x^2 + 20x$. The curve has stationary points A and B.

(*a*) Use calculus to find the x-coordinates of A and B.

(4)

(*b*) Find the value of $\dfrac{d^2y}{dx^2}$ at A, and hence verify that A is a maximum.

(2)

The line through B parallel to the y-axis meets the x-axis at the point N. The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line from A to N.

(*c*) Find $\displaystyle\int (x^3 - 8x^2 + 20x)\ dx$.

(3)

(*d*) Hence calculate the exact area of R.

(5)

a) $y = x^3 - 8x^2 + 20x$

$\frac{dy}{dx} = 3x^2 - 16x + 20$

$3x^2 - 16x + 20 = 0$

$(3x - 10)(x - 2) = 0$

$x = 10/3$ ↑ B $\qquad$ $x = 2$ ↑ A

b/ $\frac{d^2y}{dx^2} = 6x - 16$

when $x = 2$ $\quad \frac{d^2y}{dx^2} = 6(2) - 16 = -4$

negative $\therefore$ maximum

c/ $\int x^3 - 8x^2 + 20x \; dx$

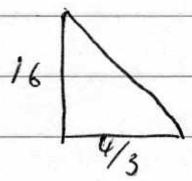
$\left[\frac{x^4}{4} - \frac{8x^3}{3} + \frac{20x^2}{2} + c\right]$

$\underline{\underline{\frac{1}{4}x^4 - \frac{8}{3}x^3 + 10x^2 + c}}$

d/ $\left[\frac{1}{4}x^4 - \frac{8}{3}x^3 + 10x^2\right]_0^2$

$\left(\frac{1}{4}(2)^4 - \frac{8}{3}(2)^3 + 10(2)^2\right) - (0)$

$= \frac{68}{3}$ units^2



$\frac{1}{2} \cdot 16 \times \frac{4}{3} = \frac{32}{3}$

$\frac{68}{3} + \frac{32}{3} = \underline{\underline{\frac{100}{3}}}$ units^2

10.

Figure 1

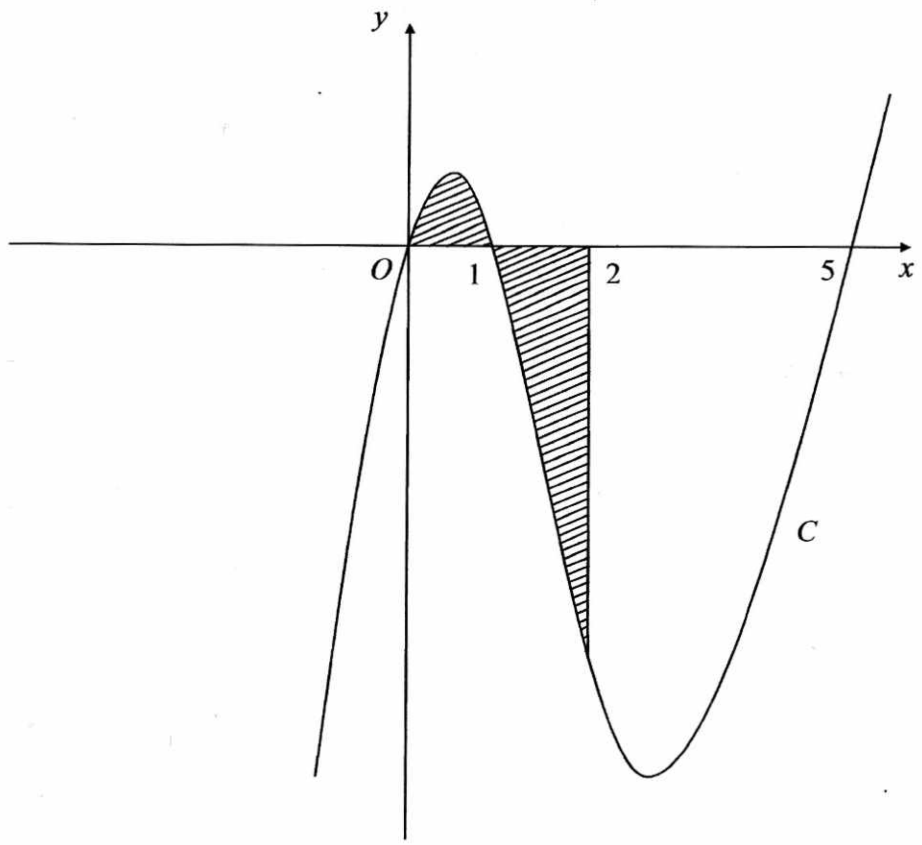


Figure 1 shows a sketch of part of the curve C with equation

$$y = x(x-1)(x-5).$$

Use calculus to find the total area of the finite region, shown shaded in Figure 1, that is between $x = 0$ and $x = 2$ and is bounded by C, the x-axis and the line $x = 2$.

(9)

$$y = x(x^2 - 6x + 5)$$
$$= x^3 - 6x^2 + 5x$$

$$\int_1^2 x^3 - 6x^2 + 5x \; dx \qquad \int_0^1 x^3 - 6x^2 + 5x \; dx$$

$$\left[\tfrac{1}{4}x^4 - 2x^3 + \tfrac{5}{2}x^2\right]_1^2 \qquad \left[\tfrac{1}{4}x^4 - 2x^3 + \tfrac{5}{2}x^2\right]_0^1$$

$$\left(\tfrac{1}{4}(2)^4 - 2(2)^3 + \tfrac{5}{2}(2)^2\right) - \left(\tfrac{1}{4}(1)^4 - 2(1)^3 + \tfrac{5}{2}(1)^2\right) \qquad \left(\tfrac{3}{4}\right) - 0$$

$$(-2) - \tfrac{3}{4} = -\tfrac{11}{4}$$

$$\tfrac{11}{4} + \tfrac{3}{4} = \underline{\underline{\tfrac{7}{2}}} \text{ units}^2$$

8

11. A diesel lorry is driven from Birmingham to Bury at a steady speed of v kilometres per hour. The total cost of the journey, £C, is given by

$$C = \frac{1400}{v} + \frac{2v}{7}.$$

(*a*) Find the value of v for which C is a minimum.

(5)

(*b*) Find $\frac{d^2C}{dv^2}$ and hence verify that C is a minimum for this value of v.

(2)

(*c*) Calculate the minimum total cost of the journey.

(2)

a) $C = 1400v^{-1} + \frac{2}{7}v$

$\frac{dC}{dv} = -1400v^{-2} + \frac{2}{7}$

min where $\frac{dC}{dv} = 0$

$\frac{-1400}{v^2} + \frac{2}{7} = 0$

$\frac{1400}{v^2} = \frac{2}{7}$

$9800 = 2v^2$

$4900 = v^2$

$v = 70$

b) $\frac{d^2C}{dv^2} = 2800v^{-3}$

$= \frac{2800}{(70)^3} = \frac{2}{245}$ (positive ∴ minimum)

c) $C = \frac{1400}{70} + \frac{2(70)}{7} = £40$

12.

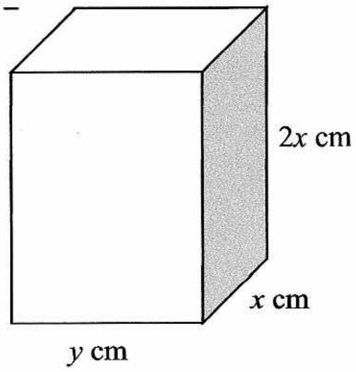


Figure 4

Figure 4 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2.

(*a*) Show that the volume, V cm^3, of the brick is given by

$$V = 200x - \frac{4x^3}{3}.$$

(4)

Given that x can vary,

(*b*) use calculus to find the maximum value of V, giving your answer to the nearest cm^3. **(5)**

(*c*) Justify that the value of V you have found is a maximum. **(2)**

a) Surface area $= 2(xy) + 2(2x^2) + 2(2xy)$

$= 2xy + 4x^2 + 4xy$

$600 = 6xy + 4x^2$

$V = y \times x \times 2x$ $\qquad 600 - 4x^2 = 6xy$

$V = 2x^2 y$ $\qquad \dfrac{600 - 4x^2}{6x} = y$

$V = \cancel{2}x^{\cancel{2}}\left(\dfrac{600-4x^2}{3\cancel{6}\cancel{x}}\right)$

$= x\left(\dfrac{600-4x^2}{3}\right)$

$= \dfrac{600x - 4x^3}{3}$

$= 200x - \dfrac{4x^3}{3}$

b) $$V = 200x - \frac{4}{3}x^3$$

$$\frac{dV}{dx} = 200 - 4x^2$$

~~M1A~~ · Max when $\frac{dV}{dx} = 0$ $\qquad 200 - 4x^2 = 0$

$$200 = 4x^2$$

$$50 = x^2$$

$$x = \sqrt{50}$$

$$V = 200\sqrt{50} - \frac{4(\sqrt{50})^3}{3}$$

$$= \underline{943 \text{ cm}^2}$$

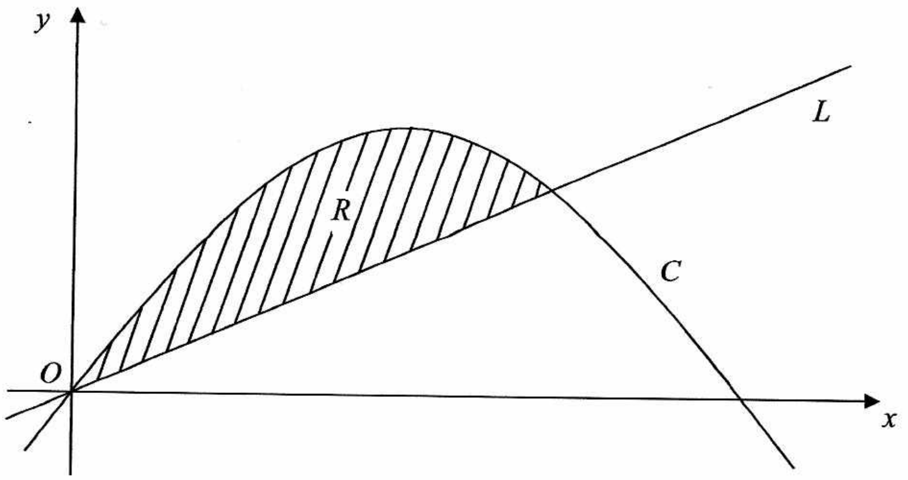
c) $$\frac{d^2V}{dx^2} = -8x$$

$$= -8(\sqrt{50})$$

<u>negative ∴ maximum</u>

13.

Figure 2



In Figure 2 the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects with the x-axis at $x = 0$ and $x = 6$. **(1)**

(b) Show that the line L intersects the curve C at the points (0, 0) and (4, 8). **(3)**

The region R, bounded by the curve C and the line L, is shown shaded in Figure 2.

(c) Use calculus to find the area of R. **(6)**

a) crosses x when $y = 0$

$0 = 6x - x^2$

$0 = x(6 - x)$

$\underline{x = 0}$ $\underline{x = 6}$

b/ $2x = 6x - x^2$

$0 = 4x - x^2$

$0 = x(4 - x)$

$x = 0$ $x = 4$

$y = 2(0)$ $y = 2(4)$

$= 0$ $= 8$

$(0, 0)$ $(4, 8)$

c/ ~~Area under curve:~~ $\cancel{\int_0^4 6x - x^2 \, dx}$

$\int_0^4 4x - x^2 \, dx$

$\left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^4$

$\left(2(4)^2 - \frac{(4)^3}{3}\right) - (0)$

$\underline{\frac{32}{3}}$ units^2

14. **Figure 4**

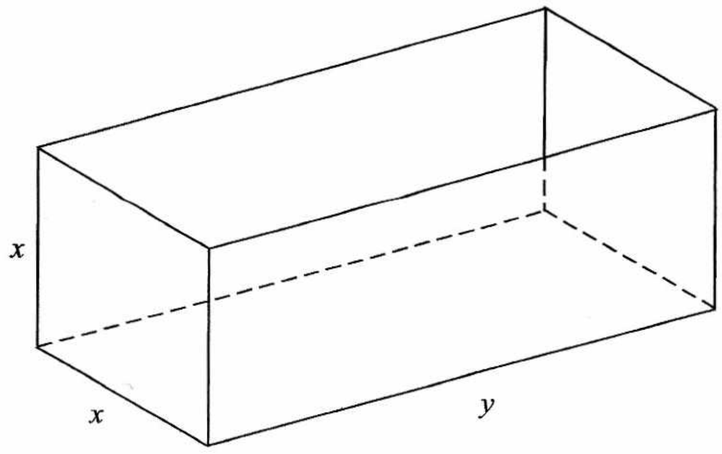


Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is 100 m^3.

(*a*) Show that the area A m^2 of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2.$$

(4)

(*b*) Use calculus to find the value of x for which A is stationary.

(4)

(*c*) Prove that this value of x gives a minimum value of A.

(2)

(*d*) Calculate the minimum area of sheet metal needed to make the tank.

(2)

a) $V = 100$

$100 = x^2 y$ $\qquad \left[y = \frac{100}{x^2} \right]$

$A = 2(x^2) + 2(xy) + xy$

$= 2x^2 + 3xy$

$= 2x^2 + 3x\left(\frac{100}{x^2}\right)$

$= 2x^2 + \frac{300}{x}$

$= \frac{300}{x} + 2x^2$

b/ $A = 300x^{-1} + 2x^2$

$\frac{dA}{dx} = -300x^{-2} + 4x$

stationary where $\frac{dA}{dx} = 0$

$-\frac{300}{x^2} + 4x = 0$

$4x = \frac{300}{x^2}$

$4x^3 = 300$

$x^3 = \frac{300}{4}$

$x^3 = 75$

$x = \sqrt[3]{75}$

$x = 4.217163327$

c/ $\frac{d^2A}{dx^2} = 600x^{-3} + 4$

$= 600(4.217)^{-3} + 4$

$= 12$

positive $\therefore$ minimum

d/ $A = \frac{300}{\sqrt[3]{75}} + 2(\sqrt[3]{75})^2$

$= \underline{106.71\ m^2}$ (2dp)

15.

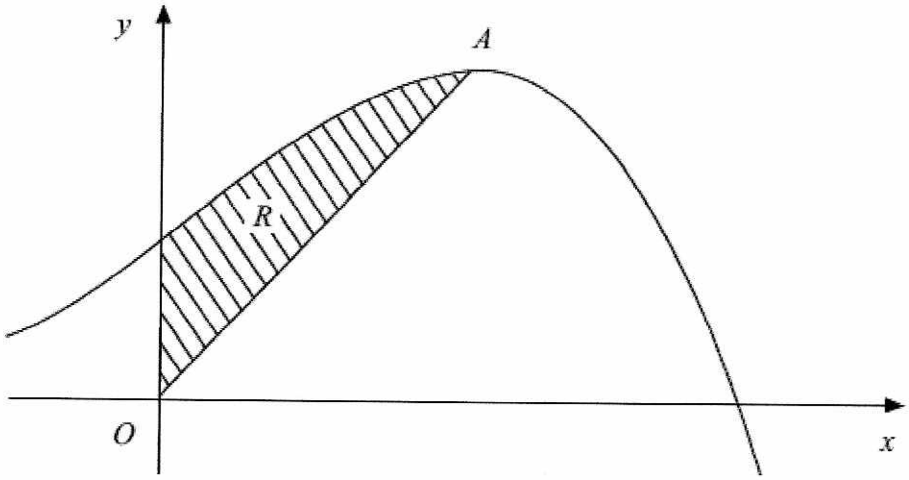


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = 10 + 8x + x^2 - x^3$.

The curve has a maximum turning point A.

(a) Using calculus, show that the x-coordinate of A is 2.

(3)

The region R, shown shaded in Figure 2, is bounded by the curve, the y-axis and the line from O to A, where O is the origin.

(b) Using calculus, find the exact area of R.

(8)

a) $\frac{dy}{dx} = 8 + 2x - 3x^2$

min where $\frac{dy}{dx} = 0$ $\quad 8 + 2x - 3x^2 = 0$

$3x^2 - 2x - 8 = 0$

$(3x + 4)(x - 2) = 0$

$x = -\frac{4}{3} \quad \underline{x = 2}$

b/ $\int_0^2 10 + 8x + x^2 - x^3 \, dx$

$\left[10x + 4x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4\right]_0^2$

$\left[10(2) + 4(2)^2 + \frac{1}{3}(2)^3 - \frac{1}{4}(2)^4\right] - (0)$

$= \frac{104}{3}$

Area of triangle $= \frac{1}{2} \times 2 \times 22 = 22$

$\frac{104}{3} - 22 = \underline{\frac{38}{3}}$ units^2

16.

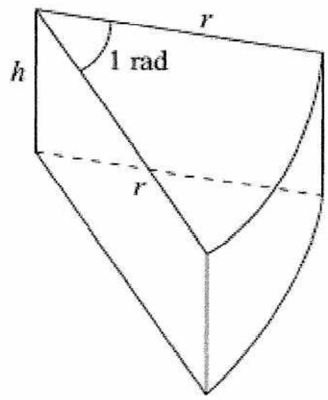


Figure 2

Figure 2 shows a closed box used by a shop for packing pieces of cake. The box is a right prism of height h cm. The cross section is a sector of a circle. The sector has radius r cm and angle 1 radian.

The volume of the box is 300 cm^3.

(*a*) Show that the surface area of the box, S cm^2, is given by

$$S = r^2 + \frac{1800}{r}.$$

(5)

(*b*) Use calculus to find the value of r for which S is stationary.

(4)

(*c*) Prove that this value of r gives a minimum value of S.

(2)

(*d*) Find, to the nearest cm^2, this minimum value of S.

(2)

a) $V = \frac{\theta}{2} r^2 h$

$300 = \frac{1}{2} r^2 h \longrightarrow 600 = r^2 h$

$h = \frac{600}{r^2}$

$S = 2\left(\frac{1}{2}r^2\right) + 2hr + hr$

$= r^2 + 3hr$

$= r^2 + 3\cancel{r}\left(\frac{600}{r^{\cancel{2}}}\right)$

$= r^2 + \frac{1800}{r}$

b) $\frac{dS}{dr} = 2r - 1800r^{-2}$

$2r - \frac{1800}{r^2} = 0$

$2r = \frac{1800}{r^2}$

$2r^3 = 1800$

$r^3 = 900$

$r = 9.65$ cm 3sf

c) $\frac{d^2S}{dr^2} = 2 + 3600r^{-3}$

$= 2 + \frac{3600}{(9.65)^3}$

$= 6$ (positive ∴ minimum)

d) $S = (9.65)^2 + \frac{1800}{9.65}$

$= \underline{\underline{280 \text{ cm}^2}}$ (nearest cm²)

17.

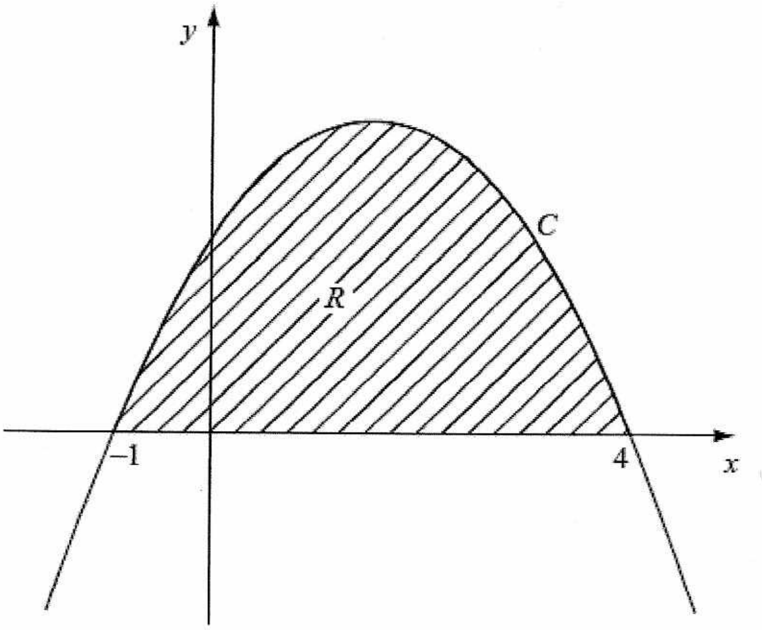


Figure 1

Figure 1 shows part of the curve C with equation $y = (1 + x)(4 - x)$.

The curve intersects the x-axis at $x = -1$ and $x = 4$. The region R, shown shaded in Figure 1, is bounded by C and the x-axis.

Use calculus to find the exact area of R.

(5)

$y = 4 - x + 4x - x^2$

$= 4 + 3x - x^2$

$\int_{-1}^{4} 4 + 3x - x^2 \; dx$

$\left[4x + \frac{3}{2}x^2 - \frac{1}{3}x^3 + c \right]_{-1}^{4}$

$\left(4(4) + \frac{3}{2}(4)^2 - \frac{1}{3}(4)^3 \right) - \left(4(-1) + \frac{3}{2}(-1)^2 - \frac{1}{3}(-1)^3 \right)$

$\frac{56}{3} - -\frac{13}{6} = \underline{\underline{\frac{125}{6} \text{ units}^2}}$

18. A solid right circular cylinder has radius r cm and height h cm.

The total surface area of the cylinder is 800 cm^2 .

(*a*) Show that the volume, V cm^3 , of the cylinder is given by

$$V = 400r - \pi r^3.$$

(4)

Given that r varies,

(*b*) use calculus to find the maximum value of V, to the nearest cm^3.

(6)

(*c*) Justify that the value of V you have found is a maximum.

(2)

a) $\pi r^2 h = \overset{V}{\cancel{800}}$

$S = 2\pi r^2 + 2\pi r h$

$800 = 2\pi r^2 + 2\pi r h$

$\frac{800 - 2\pi r^2}{2\pi r} = h$

$V = \cancel{\pi} r^{\cancel{2}} \left(\frac{\overset{4}{\cancel{8}}00 - \cancel{2}\pi r^2}{\cancel{2}\cancel{\pi}\cancel{r}} \right)$

$= r(400 - \pi r^2)$

$= \underline{400r - \pi r^3}$

b) $\frac{dv}{dr} = 400 - 3\pi r^2$

$0 = 400 - 3\pi r^2$

$3\pi r^2 = 400$

$r^2 = \frac{400}{3\pi}$

$r = \sqrt{\frac{400}{3\pi}}$

$= 6.51$ cm 3sf

$V = 400(6.51) - \pi(6.51)^3$

$= \underline{1737 \text{ cm}^3}$

c) $\frac{d^2v}{dr^2} = -6\pi r$

$= -6\pi(6.51)$

<u>negative ∴ maximum</u>