1) $3^x = 13$

$x = \log_3 13$

$= 2.33$ (3sf)

2) $2^x = 32$

$x = \log_2 32$

$= 5$

3) $2\log_2 x - \log_2 5 = 1$

$\log_2 x^2 - \log_2 5 = 1$

$\log_2\left(\frac{x^2}{5}\right) = 1$

$\frac{x^2}{5} = 2$

$x^2 = 10$

$\underline{\underline{x = \sqrt{10}}}$

4) $\log_3 x + \log_3 4 = 2$

$\log_3 4x = 2$

$3^2 = 4x$

$9 = 4x$

$\underline{\underline{x = 9/4}}$

5) $2\log_a(x+1) - \log_a 4$

$\log_a(x+1)^2 - \log_a 4$

$\log_a\left(\frac{(x+1)^2}{4}\right)$

6) $e^{2y} = x+1$ ①

$\ln(x-2) = 2y-1$ ②

$x-2 = e^{2y-1}$ ②

$x-2 = \frac{e^{2y}}{e}$

$e(x-2) = e^{2y}$

$x+1 = e(x-2)$

$x+1 = ex-2e$

$1+2e = ex-x$

$1+2e = x(e-1)$

$x = \frac{1+2e}{e-1}$

$x = \underline{\underline{3.75}}$ (2dp)

$\ln(\text{"}3.75\text{"} - 2) = 2y-1$

$0.55728\ldots = 2y-1$

$1.557\ldots = 2y$

$y = \underline{\underline{0.78}}$ (2dp)

7) $\ln(2x+5) = 1$

$$2x + 5 = e^1$$
$$2x = e - 5$$
$$x = \frac{e-5}{2}$$

8) $y = \log_2 x$

a) $\log_2 x^2$

$2(\log_2 x)$

$\underline{\underline{2y}}$

b) $\log_2 2x$

$\log_2 x + \log_2 2$

$(\log_2 x) + 1$

$\underline{\underline{y + 1}}$

c) $\log_8 x = \dfrac{\log_2 x}{\log_2 8}$

$\dfrac{\log_2 x}{3}$

$\underline{\underline{\dfrac{y}{3}}}$

9)

$$2e^{y} + 15e^{-y} = 11$$

$$2e^{2y} + 15 = 11e^{y}$$

$$2e^{2y} - 11e^{y} + 15 = 0$$

$$(2e^{y} - 5)(e^{y} - 3) = 0$$

$$e^{y} = \tfrac{5}{2} \qquad e^{y} = 3$$

$$\underline{\underline{y = \ln\left(\tfrac{5}{2}\right)}} \qquad \underline{\underline{y = \ln(3)}}$$

10/ $P = 50e^{0.1t}$

a/ 50

b/ $\frac{dP}{dt} = 5e^{0.1t}$

when $t = 10$

$\frac{dP}{dt} = 13.6$ (3sf)

c/ $300 = 50e^{0.1t}$

$6 = e^{0.1t}$

$\ln 6 = 0.1t$

$t = 17.9$

It takes 18 weeks to exceed 300.

11) $N = 1000e^{-kt}$

a/ 1000

b/ when $t = 14.4$ $N = 500$

$$500 = 1000e^{-14.4k}$$

$$\frac{1}{2} = e^{-14.4k}$$

$$\ln\frac{1}{2} = -14.4k$$

$$k = 0.0481 \quad (3sf)$$

c/

$$N = 1000e^{-30(0.0481)}$$

$= 236$ nearest integer

12) $T = 75e^{-kt} + 22.$

a) $\frac{dT}{dt} = -75ke^{-kt}$

b) $t = 5 \quad T = 70$

$70 = 75e^{-5k} + 22$

$48 = 75e^{-5k}$

$\frac{16}{25} = e^{-5k}$

$\ln\left(\frac{16}{25}\right) = -5k$

$k = 0.0893 \quad (3sf)$

c) $55 = 75e^{-0.0893t} + 22$

$33 = 75e^{-0.0893t}$

$\frac{11}{25} = e^{-0.0893t}$

$\ln\left(\frac{11}{25}\right) = -0.0893t$

$t = 9.19...$

$\therefore$ 10 minutes

12/ $f(x) = e^{2x+1} - 3$

a/ $f(x) > -3$

b/ crosses y when $x = 0$

$$y = e^1 - 3$$
$$= \underline{e - 3}$$

$A: (0, e-3)$

crosses x when $y = 0$

$$0 = e^{2x+1} - 3$$
$$3 = e^{2x+1}$$
$$\ln 3 = 2x + 1$$
$$\ln(3) - 1 = 2x$$
$$x = \frac{\ln(3) - 1}{2}$$

$B: \left(\frac{\ln(3) - 1}{2}, 0\right)$

c/ $f'(x) = 2e^{2x+1}$

when $x = 0$ $f'(x) = 2e$

$$\underline{\underline{y = 2e\,x + e - 3}}$$