

Write your name here

Surname

Other Names

# AS/A Level Mathematics

## Exponentials and Logarithms

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled..
- Answer the questions in the spaces provided  
– there may be more space than you need.
- You should show sufficient working to make your methods clear.  
Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

### Information

- The marks for **each** question are shown in brackets  
– use this as a guide as to how much time to spend on each question.

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1 Solve  $3^x = 13$ , giving your answer to 3 significant figures.

(Total for question 1 is 3 marks)

2 Solve  $2^x = 32$

(Total for question 2 is 2 marks)

3 Solve the equation

$$2\log_2(x) - \log_2(5) = 1$$

(Total for question 3 is 4 marks)

4 Solve the equation

$$\log_3(x) + \log_3(4) = 2$$

(Total for question 4 is 4 marks)

5 Express as a single logarithm to base  $a$

$$2\log_a(x + 1) - \log_a(4)$$

(Total for question 5 is 4 marks)

6 Giving your answers to 2 decimal places, solve the simultaneous equations

$$\begin{aligned}e^{2y} &= x + 1 \\ \ln(x - 2) &= 2y - 1\end{aligned}$$

(Total for question 6 is 7 marks)

7 Solve the equation

$$\ln(2x + 5) = 1$$

(Total for question 7 is 3 marks)

8 Given that  $y = \log_2 x$ , find expressions in terms of  $y$  for

(a)  $\log_2 x^2$  (2)

(b)  $\log_2 2x$  (2)

(c)  $\log_8 x$  (2)

(Total for question 8 is 6 marks)

9 Solve the equation, giving your answers in exact form.

$$2e^y + 15e^{-y} = 11$$

(Total for question 9 is 5 marks)

**10** The population of a species of plant in a field is modelled using the formula  $P = 50e^{0.1t}$   
Where  $t$  is the number of weeks since the population was first recorded.

- (a) Write down the number of the plants when the population was first recorded. (1)
- (b) Find the rate of increase in the population 10 weeks after the population was first recorded. (2)
- (c) Find how many weeks it takes for the number of plants to exceed 300. (4)

**(Total for question 10 is 7 marks)**

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**11** The decay of a radioactive substance is modelled using the formula  $N = 1000e^{-kt}$   
Where  $N$  is the number of atoms after  $t$  years and  $k$  is a positive constant.

- (a) Write down the number of atoms when the substance started to decay. (1)
- Given it takes 14.4 years for half of the substance to decay.
- (b) Find the value of  $k$  to three significant figures. (4)
- (c) Calculate the number of atoms left when  $t=30$ . (1)

**(Total for question 11 is 6 marks)**

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**12** The temperature of water in a kettle is modelled using the formula  $T = 75e^{-kt} + 22$

Where  $T$  is the temperature  $t$  minutes after the kettle is turned off and  $k$  is a positive constant.

- (a) Find the rate of change of the temperature in terms of  $k$  (2)
- After 5 minutes the temperature of the water is  $70^\circ\text{C}$
- (b) Find the value of  $k$  (3)
- (c) Find how many minutes it takes for the water to cool to  $55^\circ\text{C}$  (4)

**(Total for question 12 is 9 marks)**

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**13**  $f(x) = e^{2x+1} - 3$

- (a) State the range of  $f$  (1)
- The curve  $y = f(x)$  meets the  $y$ -axis at  $A$  and the  $x$ -axis at  $B$ .
- (b) Find the exact coordinates of  $A$  and  $B$ . (4)
- (c) Find the equation of the tangent to the curve at  $A$ . (4)

**(Total for question 13 is 9 marks)**

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