1a) $\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$1 + \tan^2\theta = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

b/ $\tan^2\theta + \sec^2\theta + 5\sec\theta = 2$

$\sec^2\theta - 1 + \sec^2\theta + 5\sec\theta = 2$

$2\sec^2\theta + 5\sec\theta - 3 = 0$

$(2\sec\theta - 1)(\sec\theta + 3) = 0$

$\sec\theta = \frac{1}{2}$ $\quad$ $\sec\theta = -3$

$\cos\theta = 2$ $\quad$ $\cos\theta = -\frac{1}{3}$

$\times$ $\quad$ $\theta = \underline{109.5}, \underline{250.5}$

2a/ $\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

b/ $\operatorname{cosec}^2\theta + \cot^2\theta = 3$

$1 + \cot^2\theta + \cot^2\theta = 3$

$2\cot^2\theta = 2$

$\cot^2\theta = 1$

$\tan^2\theta = 1$

$\tan\theta = \pm 1$

$\tan\theta = 1$ $\quad$ $\tan\theta = -1$

$\theta = \frac{\pi}{4}, \frac{5}{4}\pi$ $\quad$ $\theta = -\frac{1}{4}\pi, \frac{3}{4}\pi, \frac{7}{4}\pi$

$\underline{\frac{\pi}{4}, \frac{3}{4}\pi, \frac{5}{4}\pi, \frac{7}{4}\pi}$

3/ $\tan^2 x + 4\sec x - 2 = 0$

$\sec^2 x - 1 + 4\sec x - 2 = 0$

$\sec^2 x + 4\sec x - 3 = 0$

$(\sec x + 2)^2 - 4 - 3 = 0$

$(\sec x + 2)^2 = 7$

$\sec(x) + 2 = \pm\sqrt{7}$

$\sec x = -2 \pm \sqrt{7}$

$\sec x = -2 + \sqrt{7}$ $\quad$ $\sec x = -2 - \sqrt{7}$

$\cos x = \frac{2+\sqrt{7}}{3}$ $\quad$ $\cos x = \frac{2-\sqrt{7}}{3}$

X

$x = 102.4, 257.6$

4/ $2\cot^2 x - \operatorname{cosec}^2 x + \operatorname{cosec} x = 4$

$2(\operatorname{cosec}^2 x - 1) - \operatorname{cosec}^2 x + \operatorname{cosec} x = 4$

$2\operatorname{cosec}^2 x - 2 - \operatorname{cosec}^2 x + \operatorname{cosec} x = 4$

$\operatorname{cosec}^2 x + \operatorname{cosec} x - 6 = 0$

$(\operatorname{cosec} x + 3)(\operatorname{cosec} x - 2) = 0$

$\operatorname{cosec} x = -3$ $\quad$ $\operatorname{cosec} x = 2$

$\sin x = -\frac{1}{3}$ $\quad$ $\sin x = \frac{1}{2}$

$x = -19.5, -160.5$ $\quad$ $x = 30, 150$

5a) $\sec^2 x - \operatorname{cosec}^2 x$

$(1 + \tan^2 x) - (1 + \cot^2 x)$

$1 + \tan^2 x - 1 - \cot^2 x$

$\tan^2 x - \cot^2 x$

b) $(\sec x - \cos x)^2$

$(\sec x - \cos x)(\sec x - \cos x)$

$\sec^2 x - 2\cos x \sec x + \cos^2 x$

$\sec^2 x - 2 + \cos^2 x$

$1 + \tan^2 x - 2 + 1 - \sin^2 x$

$\tan^2 x - \sin^2 x$

8a) $\sec^4 x - \tan^4 x$

$(\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x)$

$(1 + \tan^2 x + \tan^2 x)(1 + \tan^2 x - \tan^2 x)$

$(1 + 2\tan^2 x)(1)$

$1 + 2\tan^2 x$

b) $1 + 2\tan^2 x = 3$

$2\tan^2 x = 2$

$\tan^2 x = 1$

$\tan x = \pm 1$

$\tan x = 1$ $\qquad$ $\tan x = -1$

$x = 45, 225$ $\qquad$ $x = -45, 135, 315$

$x = 45, 135, 225, 315$