1) $S_n = a + ar + ar^2 + \dots + ar^{n-3} + ar^{n-2} + ar^{n-1}$
$rS_n = ar + ar^2 + ar^3 + \dots + ar^{n-2} + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ [Top line - Bottom Line]
$S_n(1-r) = a(1-r^n)$
$$S_n = \frac{a(1-r^n)}{1-r}$$

2a/ $U_n = ar^{n-1}$ $U_5 = 12$
$U_8 = 96$

① $ar^4 = 12$
② $ar^7 = 96$
② ÷ ① $r^3 = 8$
$r = 2$

b/ $a(2)^4 = 12$
$a(16) = 12$
$a = 3/4$

c/ $$S_{20} = \frac{\frac{3}{4}(1 - 2^{20})}{1-2}$$

~~= 767.25~~
= 786431

3a) $u_3 = 135 \qquad u_6 = 40$

$ar^2 = 135$

$ar^5 = 40$

$r^3 = \frac{8}{27}$

$\underline{\underline{r = 2/3}}$

b/ $a\left(\frac{2}{3}\right)^2 = 135$

$a = 303.75$

c/ $S_n = \dfrac{a(1-r^n)}{1-r}$

$S_{10} = \dfrac{303.75\left(1-\left(\frac{2}{3}\right)^{10}\right)}{1-\frac{2}{3}}$

$= 895$ (nearest whole number)

d/ $S_\infty = \dfrac{a}{1-r}$

$= \dfrac{303.75}{1-\frac{2}{3}}$

$= \underline{\underline{911.25}}$

4a) $u_2 = 3.75$ $\quad S_\infty = 20$

$ar = 3.75$ $\quad \frac{a}{1-r} = 20$

$a = 20(1-r)$

$a = 20 - 20r$

$r(20 - 20r) = 3.75$

$20r - 20r^2 = 3.75$

$80r - 80r^2 = 15$

$0 = 80r^2 - 80r + 15$

$0 = 16r^2 - 16r + 3$

$0 = (4r-3)(4r-1)$

$\underline{\underline{r = 3/4}}$ $\quad \underline{\underline{r = \frac{1}{4}}}$

b) $a = \frac{3.75}{0.75}$ $\quad a = \frac{3.75}{0.25}$

$= \underline{\underline{5}}$ $\quad = \underline{\underline{15}}$

c) ~~$r = \frac{1}{4}$ $\quad a = 15$~~

$\underline{\underline{r = \frac{3}{4}}}$ $\quad \underline{\underline{a = 5}}$

$$\frac{5\left(1 - \left(\frac{3}{4}\right)^n\right)}{1 - 3/4} > 19$$

$$5\left(1 - \left(\frac{3}{4}\right)^n\right) > \frac{19}{4}$$

$$1 - \left(\frac{3}{4}\right)^n > \frac{19}{20}$$

$$\frac{1}{20} > \left(\frac{3}{4}\right)^n$$

$\underline{\underline{n = 11}}$ $\quad \log \frac{1}{20} > n \log \frac{3}{4}$

$10.4 < n$

5a)

$$\frac{u_3}{u_2} = \frac{u_2}{u_1}$$

$$\frac{k}{k+3} = \frac{k+3}{2k-2}$$

$$k(2k-2) = (k+3)(k+3)$$

$$2k^2 - 2k = k^2 + 6k + 9$$

$$k^2 - 8k - 9 = 0$$

$$(k-9)(k+1) = 0$$

b) $k = 9 \quad k = -1$

k is a positive constant $\therefore$ $\underline{\underline{k = 9}}$

c)

$$r = \frac{k}{k+3}$$

$$= \frac{9}{9+3}$$

$$= \underline{\underline{\frac{3}{4}}}$$

d)

$$a = 2(9) - 2$$

$$= 16$$

$$S_\infty = \frac{16}{1 - \frac{3}{4}}$$

$$= \underline{\underline{64}}$$

6a $a = 35000 \qquad r = 1.03$

$u_3 = 35000(1.03)^2$

$= £37100$ (nearest £100)

b)

$$S_{20} = \frac{35000(1 - 1.03^{20})}{1 - 1.03}$$

$= £940500$ (nearest £100)