Write your name here

Surname	Other Names

AS/A Level Mathematics

Kinematics with Vectors

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

• Use **black** ink or ball-point pen.
• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
• **Fill in the boxes** at the top of this page with your name.
• Answer **all** questions and ensure that your answers to parts of questions are clearly labelled..
• Answer the questions in the spaces provided
– there may be more space than you need.
• You should show sufficient working to make your methods clear.
Answers without working may not gain full credit.
• Answers should be given to three significant figures unless otherwise stated.

Information

• The marks for **each** question are shown in brackets
– use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.
• Try to answer every question.
• Check your answers if you have time at the end.

1 A particle P moves with constant acceleration. At time $t = 0$, P is moving in the direction $(4\mathbf{i} + 3\mathbf{j})$ with speed 15 ms^{-1}. 6 seconds later P is moving with velocity $(-12\mathbf{i} + 6\mathbf{j})$ ms^{-1}.

Find the acceleration of P.

(Total for question 1 is 5 marks)

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2 A particle P moves with constant acceleration $(2\mathbf{i} - 5\mathbf{j})$ ms^{-2}. At time $t = 0$, P has speed u ms^{-1}. At time $t = 3$ s, P has velocity $(-6\mathbf{i} + \mathbf{j})$ ms^{-1}.

Find the value of u.

(Total for question 2 is 5 marks)

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3 A particle P moves in a straight line with constant velocity. Initially P is at the point A with position vector $(4\mathbf{i} - 3\mathbf{j})$ m. At time $t = 3$, P is at the point B with position vector $(-5\mathbf{i} + 9\mathbf{j})$ m.

(a) Find the velocity of P. **(3)**

When t = 5 P is at the point C

(b) Find the distance AC. **(4)**

(Total for question 3 is 7 marks)

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4 A particle P moves with a constant velocity $(3\mathbf{i} + 2\mathbf{j})$ ms^{-1} with respect to a fixed origin O. It passes through the point A whose position vector is $(2\mathbf{i} + 11\mathbf{j})$ m at $t = 0$.

(a) Find the angle in degrees that the velocity vector of P makes with the vector $\mathbf{i}$. **(2)**

(b) Calculate the distance of P from O when $t = 2$. **(4)**

(Total for question 4 is 6 marks)

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5 A boat B moves with a constant velocity. At noon, B is at the point with position vector $(2\mathbf{i} - 5\mathbf{j})$ km with respect to a fixed origin O. At 1430 the boat is at the point with position vector $(-8\mathbf{i} + 10\mathbf{j})$ km.

(a) Find the velocity of B. **(2)**

(b) Find the bearing of the velocity vector **(2)**

(c) Find an expression, in terms of t, for the position of B t hours after noon. **(2)**

(Total for question 5 is 6 marks)

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6 A particle has an initial velocity of $(\mathbf{i} - 5\mathbf{j})$ ms^{-1} and is accelerating uniformly in the direction $(2\mathbf{i} + \mathbf{j})$ where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors.
Given that the magnitude of the acceleration is $3\sqrt{5}$ ms^{-2},

(a) show that, after t seconds, the velocity vector of the particle is $[(6t + 1)\mathbf{i} + (3t - 5)\mathbf{j}]$ ms^{-1}. **(6)**

(b) Using your answer to part (a), or otherwise, find the value of t for which the speed of the particle is at its minimum. **(5)**

(Total for question 6 is 11 marks)

The unit vectors **i** and **j** are due east and due north respectively.

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7 At midday a boat A is 5 km east of a fixed origin O and is moving with constant velocity $(-6\mathbf{i} + 5\mathbf{j})$ km h^{-1}. At the same time, another boat B is 10 km north of O and is moving with uniform velocity $(-4\mathbf{i} + \mathbf{j})$ km h^{-1}.

(a) Show that, at time t hours after midday, the position vector of A is $[(5 - 6t)\mathbf{i} + 5t\,\mathbf{j}]$ km and find a similar expression for the position vector of B at this time. **(5)**

(b) Hence show that, at time t, the position vector of B relative to A is $[(2t - 5)\mathbf{i} + (10 - 4t)\mathbf{j}]$ km **(2)**

(c) By using your answer to part (b), or otherwise, show that the boats would collide if they continued at the same velocities and find the time at which the collision would occur. **(3)**

(Total for question 7 is 10 marks)

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8 At 6 a.m. a boat A has position vector $(12\mathbf{i} - 11\mathbf{j})$ km relative to a fixed origin O and moves with constant velocity $(9\mathbf{i} - 6\mathbf{j})$ km h^{-1}.
Another boat B has position vector $(40\mathbf{i} - 39\mathbf{j})$ km relative to a fixed origin O and moves with constant velocity $(-12\mathbf{i} + 15\mathbf{j})$ km h^{-1}.

(a) Find expressions for the position vectors of A and B, in terms of t hours after 6 a.m. **(5)**

(b) Show that if both boats maintain their course and speed, they will collide and find the time and position vector at which this occurs. **(4)**

At 7 a.m. boast A realises that a collision is imminent and changes course so that it now has velocity $(-18\mathbf{i} + 21\mathbf{j})$ km h^{-1}.

(c) Find the distance between the two ships at the time when they would have collided. **(4)**

(Total for question 8 is 13 marks)

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9 At 10 a.m. plane A has position vector $(2\mathbf{i} - 5\mathbf{j})$ km and moves with constant velocity $(-4\mathbf{i} + 6\mathbf{j})$ km h^{-1}. Another plane B has position vector $(-3\mathbf{i} - 9\mathbf{j})$ km and moves with constant velocity $(\mathbf{i} + 8\mathbf{j})$ km h^{-1}.

(a) Find the relative displacement of plane A from plane B after t hours. **(6)**

(b) Find the time when A is due west of B **(2)**

(c) Find the time, after 10 a.m. when the planes are exactly 37 km apart. **(6)**

(Total for question 9 is 14 marks)
