Name: ______________________________

Maths Genie Stage 14

Test C

Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**
- **Calculators may be used.**

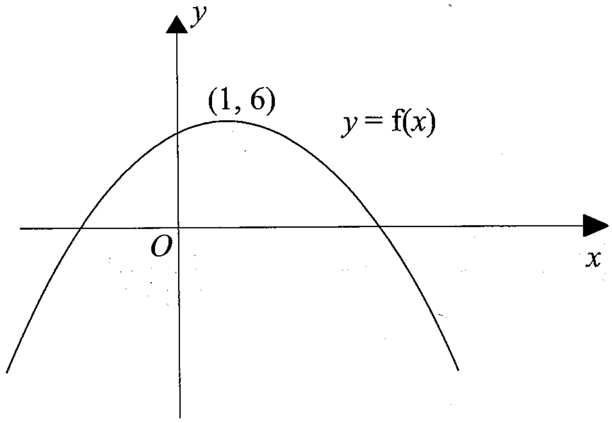
Information

- The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1 The graph of $y = f(x)$ is shown below.



The coordinates of the maximum point of this curve are (1, 6).

Write down the coordinates of the maximum point of the curve with equation

(a) $y = f(x + 4)$

(−3, 6)
(1)

(b) $y = -f(x)$

(1, −6)
(1)

(c) $y = f(x) + 2$

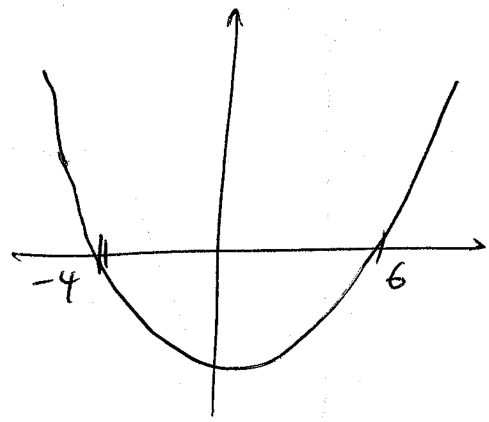
(1, 8)
(1)

(Total for Question 1 is 3 marks)

2 Solve $x^2 - 2x + 24 \geq 0$

$$(x-6)(x+4) \geqslant 0$$

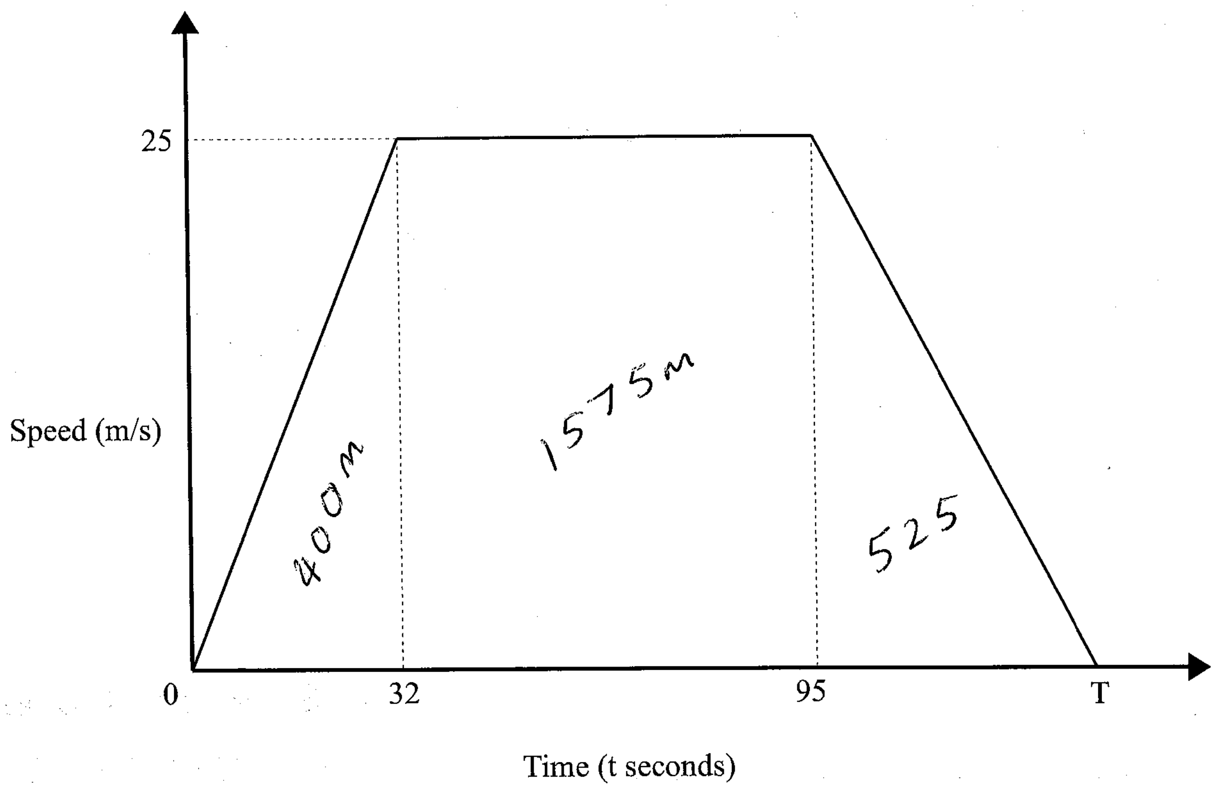
roots at $x = 6$ and $x = -4$



$x \leqslant -4$ or $x \geqslant 6$

(Total for Question 2 is 3 marks)

3 Here is a speed-time graph for a train journey between 2 stations.



The train travelled 2.5 km in T seconds.

Work out the value of T.

$\frac{1}{2}(32)(25) = 400$

$(95-32)(25) = 1575$

$2500 - 1575 - 400 = 525\text{m}$

$\frac{1}{2}x(25) = 525$

$\frac{1}{2}x = 21$

$x = 42$

$95 + 42$

$= 137$ seconds

137

(Total for Question 3 is 3 marks)

$x_1\ y_1$

4 The point A has the coordinates (9,2)
The point B has the coordinates (3,4)

$x_2\ y_2$

Find the equation of the perpendicular bisector to AB.

midpoint of $AB = \left(\frac{9+3}{2}, \frac{2+4}{2}\right)$

$= (6, 3)$

Gradient of $AB = \frac{4-2}{3-9}$

$= \frac{2}{-6}$

$= -\frac{1}{3}$

perpendicular gradient $= 3$

$y = 3x + c$

$3 = 3(6) + c$

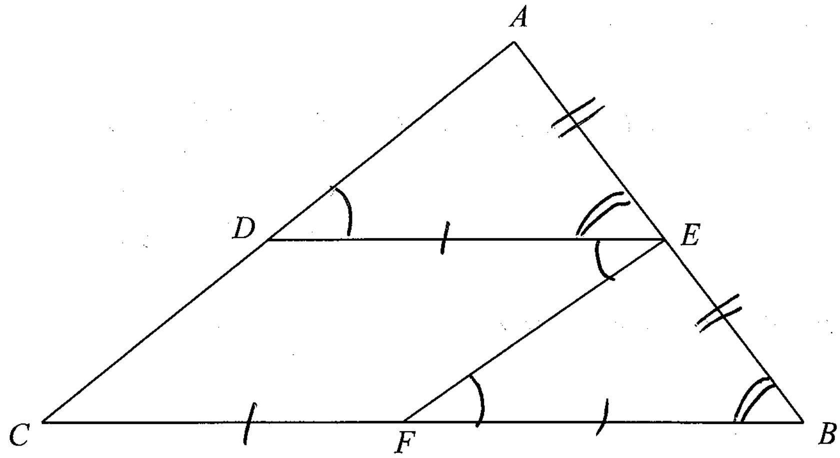
$3 = 18 + c$

$c = -15$

$y = 3x - 15$

(Total for Question 4 is 4 marks)

5 *ABC* is a triangle.



CDEF is a parallelogram such that:
D is the midpoint of *AC*
E is the midpoint of *AB*
F is the midpoint of *BC*

Prove that triangle *ADE* is congruent to triangle *BEF*.

DE = CF opposite sides of a parallelogram are equal

CF = FB F is midpoint of BC

∴ DE = FB S

A

AED = EBF corresponding angles are equal

AE = EB E is the midpoint of AB

S

SAS

(Total for Question 5 is 4 marks)

6 Solve algebraically the simultaneous equations

$$x^2 - 2y^2 = 17$$
$$3x + 2y = 13$$

$$3x = 13 - 2y$$

$$x = \frac{13 - 2y}{3}$$

$$\left(\frac{13 - 2y}{3}\right)^2 - 2y^2 = 17$$

$$\frac{(13 - 2y)(13 - 2y)}{9} - 2y^2 = 17$$

$$(13 - 2y)(13 - 2y) - 18y^2 = 153$$

$$169 - 26y - 26y + 4y^2 - 18y^2 = 153$$

$$-14y^2 - 52y + 169 = 153$$

$$-14y^2 - 52y + 16 = 0$$

$$14y^2 + 52y + -16 = 0$$

$$7y^2 + 26y - 8 = 0$$

$$(7y - 2)(y + 4) = 0$$

$$y = \frac{2}{7} \qquad y = -4$$

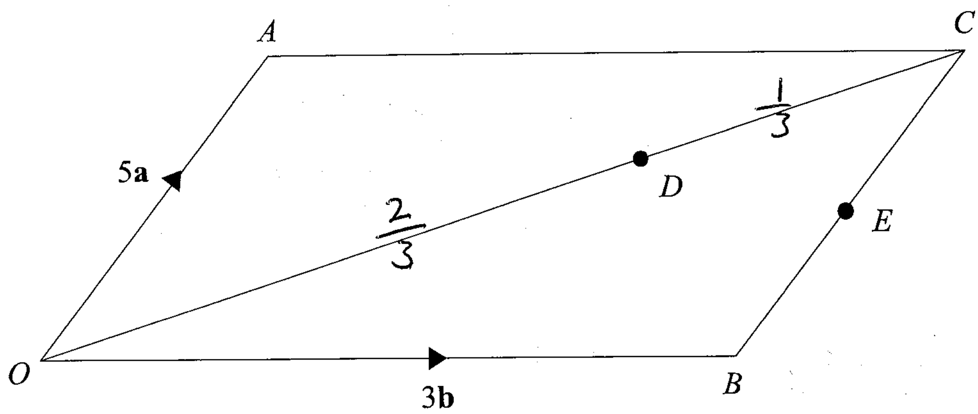
$$x = \frac{13 - 2\left(\frac{2}{7}\right)}{3} \qquad x = \frac{13 - 2(-4)}{3}$$

$$= \frac{29}{7} \qquad = 7$$

$x = \frac{29}{7}$, $y = \frac{2}{7}$ or $x = 7$, $y = -4$

(Total for Question 6 is 5 marks)

7 The diagram shows a parallelogram.



$\overrightarrow{OA} = 5a$

$\overrightarrow{OB} = 3b$

D is the point on OC such that OD:DC = 2:1

E is the midpoint of BC

Show that A, D and E are on the same straight line.

$$\overrightarrow{OC} = 5a + 3b$$

$$\overrightarrow{OD} = \frac{2}{3}(5a + 3b)$$

$$= \frac{10}{3}a + 2b$$

$$\overrightarrow{AD} = -5a + \frac{10}{3}a + 2b$$

$$= -\frac{5}{3}a + 2b$$

$$= \frac{1}{3}(-5a + 6b)$$

$$\overrightarrow{AE} = 3b - \frac{5}{2}a$$

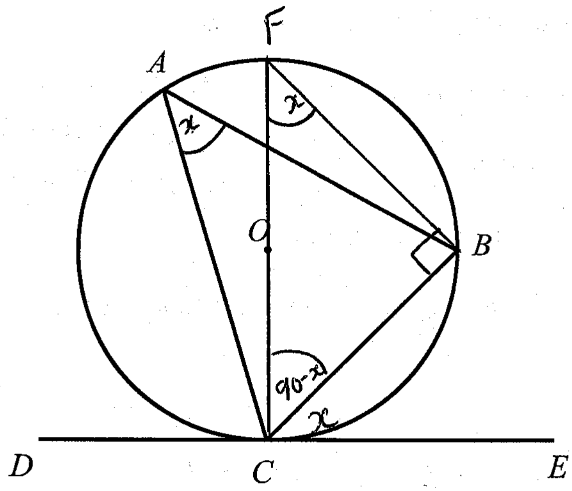
$$= -\frac{5}{2}a + 3b$$

$$= \frac{1}{2}(-5a + 6b)$$

$\overrightarrow{AD}$ and $\overrightarrow{AE}$ are both multiples of $(-5a + 6b)$ and both go through A.

(Total for Question 7 is 4 marks)

8



A, *B* and *C* are points on the circumference of a circle, centre *O*.
DCE is a tangent to the circle.

Prove that angle *BCE* and angle *BAC* are equal.

Let $BCE = x$

$OCE = 90 - x$ Tangent meets radius at $90°$

$FBC = 90$ Angle in a semicircle $= 90°$

$BFC = x$ $180 - 90 - (90 - x)$

$180 - 90 - 90 + x = x$

Angles in a triangle add to $180°$

$BAC = x$ Angles in the same segment (from the same points) are equal.

(Total for Question 8 is 4 marks)

9 There are some red counters and some blue counters in a bag.

The ratio of red counters to blue counters is 3:1

$3x$ Red
x Blue
$4x$ Total

Two counters are removed at random.

The probability that both the counters taken are blue is $\frac{2}{35}$

Work how many counters were in the bag before any counters were removed.

$$P(\text{Blue, Blue}) = \frac{1}{4} \times \frac{x-1}{4x-1}$$

$$\frac{1}{4} \times \frac{x-1}{4x-1} = \frac{2}{35}$$

$$\frac{x-1}{16x-4} = \frac{2}{35}$$

$$35(x-1) = 2(16x-4)$$

$$35x - 35 = 32x - 8$$

$$3x = 27$$

$$x = 9$$

$$4 \times 9 = \underline{\underline{36}}$$

36

(Total for Question 9 is 5 marks)