Name: ____________________

Maths Genie Stage 14

Test B

Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**
- **Calculators may be used.**

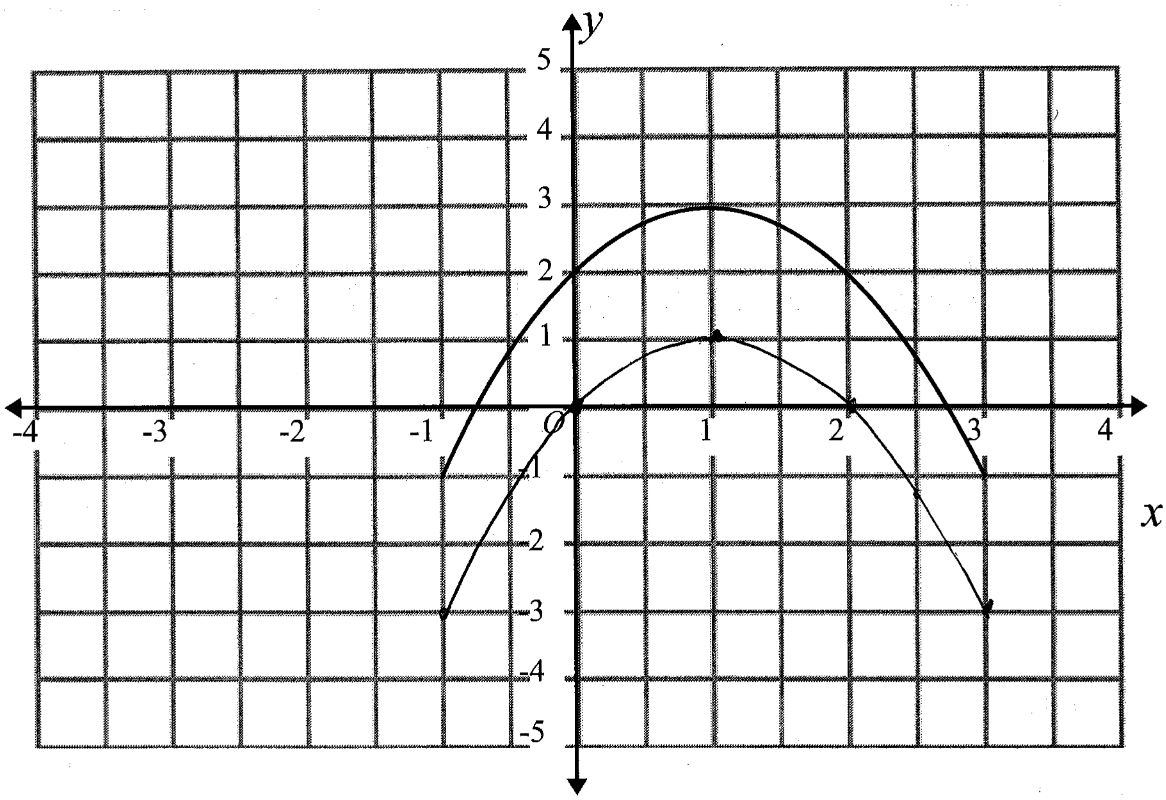
Information

- The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1 The graph of $y = f(x)$ is shown on the grid.



(a) On the grid above, sketch the graph of $y = f(x) - 2$

(1)

The graph of $y = f(x)$ has a turning point at (1, 3).

(b) Write down the coordinates of the turning point of $y = -f(x + 3)$

(-2, -3)

(1)

(Total for Question 1 is 2 marks)

2 Solve $36 - 9x \leq x^2$

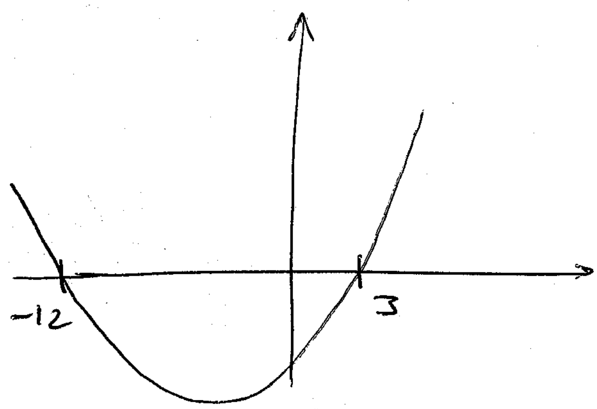
$$0 \leqslant x^2 + 9x - 36$$

$$x^2 + 9x - 36 \geqslant 0$$

$$(x+12)(x-3) \geqslant 0$$

Roots at :

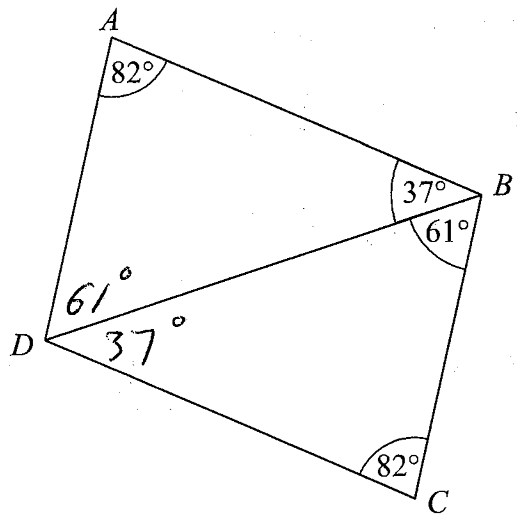
$x = -12$ and $x = 3$



$x \leqslant -12$ or $x \geqslant 3$

(Total for Question 2 is 4 marks)

3 The diagram shows two triangles, *ABD* and *BCD*.



Prove that triangle *ABD* is congruent to triangle *BCD*.

ADB = 180 − 37 − 82

= 61°

Angles in a triangle add to 180°

BDC = 180 − 82 − 61

= 37°

BD is common to both triangles

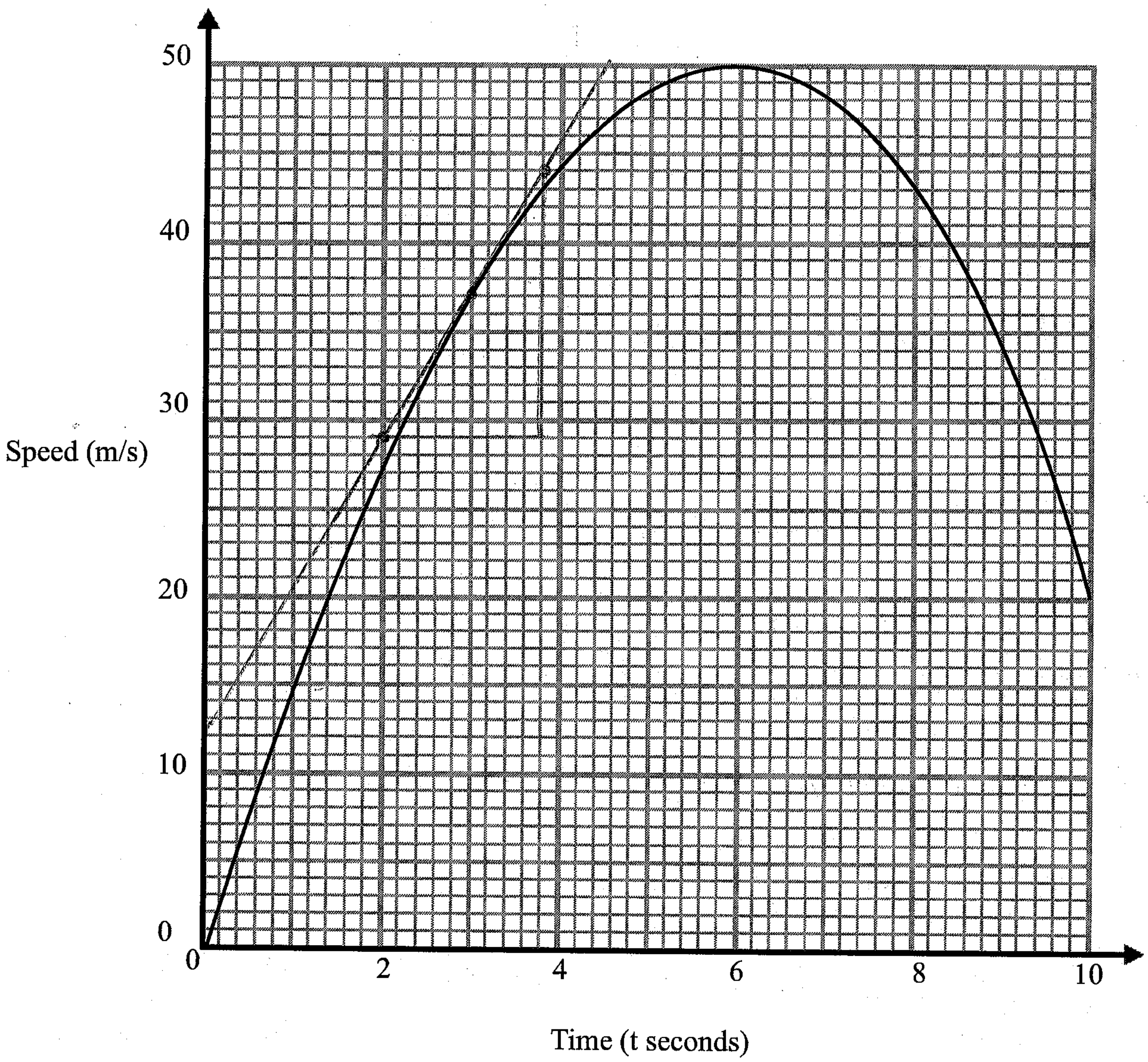
ABD = BDC

ADB = CBD

ASA

(Total for Question 3 is 3 marks)

4 Here is a speed-time graph.



Work out an estimate for the acceleration when t = 3.

$$\frac{44-29}{3.8-2} = \frac{15}{1.8} = \frac{25}{3}$$

$\frac{25}{3}$ .... ms^{-2}

(Total for Question 4 is 2 marks)

[7 – 9]

5 Solve the simultaneous equations
Give your answers to 3 significant figures

$$x^2 + y^2 = 20$$

$$2x + 3y = 7$$

$$2x = 7 - 3y$$

$$x = \frac{7-3y}{2}$$

$$\left(\frac{7-3y}{2}\right)^2 + y^2 = 20$$

$$\frac{(7-3y)(7-3y)}{4} + y^2 = 20$$

$$(7-3y)(7-3y) + 4y^2 = 80$$

$$49 - 21y - 21y + 9y^2 + 4y^2 = 80$$

$$13y^2 - 42y + 49 = 80$$

$$13y^2 - 42y - 31 = 0$$

$a = 13$
$b = -42$
$c = -31$

$$y = 3.85 \quad \text{or} \quad y = -0.619$$

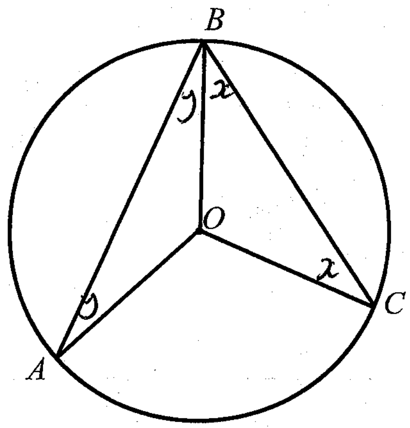
$$x = \frac{7-3(3.85)}{2} \qquad x = \frac{7-3(-0.619)}{2}$$

$$= -2.28 \qquad = 4.43$$

$x = -2.28$, $y = -3.85$ or $x = 4.43$, $y = -0.619$

(Total for Question 5 is 5 marks)

6



A, *B* and *C* are points on the circumference of a circle, centre *O*.

Prove that angle *AOC* is twice the size of angle *ABC*.
You must **not** use any circle theorems in your proof.

Let $OBC = x$ Let $OBA = y$

$ABC = \underline{\underline{x + y}}$

$OBC = OCB$ and $OBA = OAB$

Angles at the base of an isosceles triangle are equal

$AOB = 180 - 2y$
$BOC = 180 - 2x$

Angles in a triangle add to 180°

$AOC = 360 - (180 - 2y) - (180 - 2x)$

$= 360 - 180 + 2y - 180 + 2x$

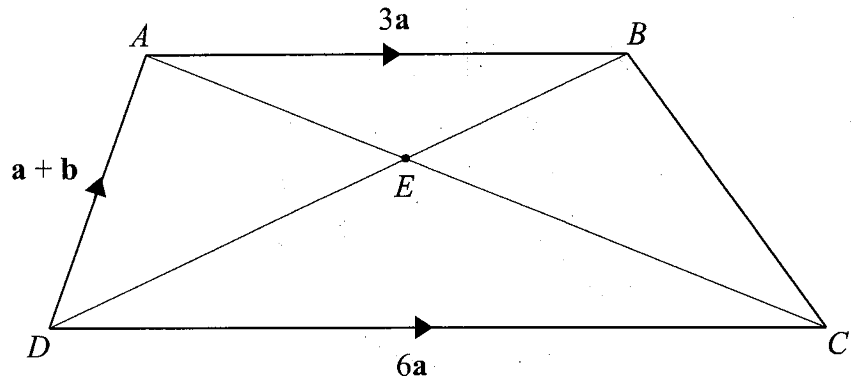
$= \underline{\underline{2x + 2y}}$

Angles around a point add to 360°

$\therefore \; AOC = 2(ABC)$

(Total for Question 6 is 4 marks)

7 The diagram shows a quadrilateral *ABCD*.



$\overrightarrow{AB} = 3a$

$\overrightarrow{DA} = a + b$

$\overrightarrow{DC} = 6a$

E is the point where the line *AC* meets the line *BD*.

Find the ratio of the length of *AE* to the length of *EC*.

$\overrightarrow{AC} = -(a+b) + 6a$

$= 5a - b$

$\overrightarrow{DB} = a + b + 3a$

$= 4a + b$

$\overrightarrow{AE} = x(5a - b)$

$= 5xa - xb$

$\overrightarrow{AE} = -(a+b) + y(4a+b)$

$= -a - b + 4ya + yb$

$= (4y-1)a + (y-1)b$

$5xa - xb = (4y-1)a + (y-1)b$

a// $5x = 4y - 1$

b// $-x = y - 1$

$y = 1 - x$

$5x = 4(1-x) - 1$

$5x = 4 - 4x - 1$

$9x = 3$

$x = \frac{1}{3}$ $\longrightarrow$ $AE = \frac{1}{3} AC$

$AE : EC$

$\frac{1}{3} : \frac{2}{3}$

$1 : 2$

(Total for Question 7 is 5 marks)

8 There are 6 red counters and y blue counters in a bag.

Imogen takes a counter from the bag at random.
Imogen then takes another counter at random from the bag.

The probability that the first counter Imogen takes is red and the second counter Imogen takes is red is $\frac{1}{8}$

Work how many blue counters are in the bag.

$$P(\text{Red, Red}) = \frac{6}{6+y} \times \frac{5}{5+y}$$

$$\frac{6}{6+y} \times \frac{5}{5+y} = \frac{1}{8}$$

$$\frac{30}{(6+y)(5+y)} = \frac{1}{8}$$

$$240 = (6+y)(5+y)$$

$$240 = 30 + 6y + 5y + y^2$$

$$0 = y^2 + 11y - 210$$

$$0 = (y+21)(y-10)$$

$$y = -21 \qquad y = 10$$

y cannot be negative

10

(Total for Question 8 is 5 marks)

9 A circle has the equation $x^2 + y^2 = 13$

(a) Write down the exact length of the radius of the circle.

$\sqrt{13}$

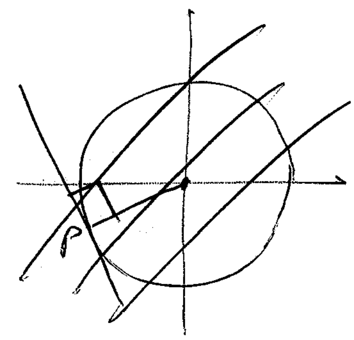
(1)

P is the point (-3,2) on the circle $x^2 + y^2 = 13$

(b) Work out the equation of the tangent to the circle at P.

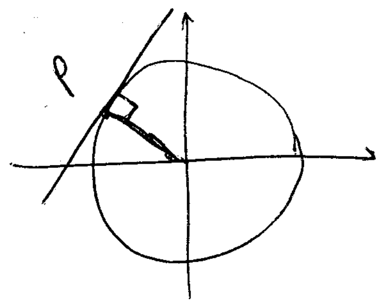
$(0, 0) \quad (-3, 2)$

$x_1 \quad y_1 \quad x_2 \quad y_2$



gradient or radius $= \dfrac{2-0}{-3-0}$

$= -\dfrac{2}{3}$



gradient or tangent $= \dfrac{3}{2}$

$y = \dfrac{3}{2}x + c \qquad (-3, 2)$

$2 = \dfrac{3}{2}(-3) + c$

$2 = \dfrac{-9}{2} + c$

$c = 2 + \dfrac{9}{2}$

$= \dfrac{13}{2}$

$y = \dfrac{3}{2}x + \dfrac{13}{2}$

(4)

(Total for Question 9 is 5 marks)