

Name: _____

GCSE (1 – 9)

Quadratic Simultaneous Equations

Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information

- The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1 Solve the simultaneous equations

$$x^2 + y^2 = 13$$

$$x = y - 5$$

$$(y - 5)^2 + y^2 = 13$$

$$(y - 5)(y - 5) + y^2 = 13$$

$$y^2 - 5y - 5y + 25 + y^2 = 13$$

$$2y^2 - 10y + 25 = 13$$

$$2y^2 - 10y + 12 = 0$$

$$y^2 - 5y + 6 = 0$$

$$(y - 3)(y - 2) = 0$$

$$\underline{y = 3} \quad \underline{y = 2}$$

$$x = (3) - 5 \quad x = (2) - 5$$

$$\underline{\underline{-2}} \quad \underline{\underline{-3}}$$

$$x = \underline{\underline{-2}} \text{ or } \underline{\underline{-3}}$$

$$y = \underline{\underline{3}} \text{ or } \underline{\underline{2}}$$

(Total for question 1 is 5 marks)

2 Solve the simultaneous equations

$$x^2 + y^2 = 17$$

$$y = x - 3$$

$$x^2 + (x - 3)^2 = 17$$

$$x^2 + (x - 3)(x - 3) = 17$$

$$x^2 + x^2 - 3x - 3x + 9 = 17$$

$$2x^2 - 6x + 9 = 17$$

$$2x^2 - 6x - 8 = 0$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$\underline{\underline{x = 4}} \quad \underline{\underline{x = -1}}$$

$$y = (4) - 3$$

$$\underline{\underline{= 1}}$$

$$y = (-1) - 3$$

$$\underline{\underline{= -4}}$$

$$x = \underline{\underline{4 \text{ or } -1}}$$

$$y = \underline{\underline{1 \text{ or } -4}}$$

(Total for question 2 is 5 marks)

3 Solve the simultaneous equations

$$x^2 + y^2 = 34$$

$$x - y = 2$$

$$x = 2 + y$$

$$(2 + y)^2 + y^2 = 34$$

$$(2 + y)(2 + y) + y^2 = 34$$

$$4 + 2y + 2y + y^2 + y^2 = 34$$

$$2y^2 + 4y + 4 = 34$$

$$2y^2 + 4y - 30 = 0$$

$$y^2 + 2y - 15 = 0$$

$$(y + 5)(y - 3) = 0$$

$$\underline{y = -5} \quad \underline{y = 3}$$

$$x = 2 + (-5) \quad x = 2 + (3)$$
$$= \underline{-3} \quad = \underline{5}$$

$$x = \underline{-3 \text{ or } 5}$$

$$y = \underline{-5 \text{ or } 3}$$

(Total for question 3 is 5 marks)

4 Solve the simultaneous equations

$$x^2 + y^2 = 20$$

$$3x = 2 - y$$

$$3x + y = 2$$

$$y = 2 - 3x$$

$$x^2 + (2 - 3x)^2 = 20$$

$$x^2 + (2 - 3x)(2 - 3x) = 20$$

$$x^2 + 4 - 6x - 6x + 9x^2 = 20$$

$$10x^2 - 12x + 4 = 20$$

$$10x^2 - 12x - 16 = 0$$

$$5x^2 - 6x - 8 = 0$$

$$(5x + 4)(x - 2) = 0$$

$$\underline{\underline{x = -\frac{4}{5}}}$$

$$\underline{\underline{x = 2}}$$

$$y = 2 - 3\left(-\frac{4}{5}\right) \quad y = 2 - 3(2)$$

$$= 2 + \frac{12}{5}$$

$$= -4$$

$$= \frac{22}{5}$$

$$x = \dots \frac{-4}{5} \text{ or } 2 \dots$$

$$y = \dots \frac{22}{5} \text{ or } -4 \dots$$

(Total for question 4 is 5 marks)

5 Solve the simultaneous equations

$$x^2 + y^2 = 41$$

$$y = 2x - 3$$

$$x^2 + (2x - 3)^2 = 41$$

$$x^2 + (2x - 3)(2x - 3) = 41$$

$$x^2 + 4x^2 - 6x - 6x + 9 = 41$$

$$5x^2 - 12x + 9 = 41$$

$$5x^2 - 12x - 32 = 0$$

$$(5x + 8)(x - 4) = 0$$

$$x = \underline{\underline{-\frac{8}{5}}} \quad \underline{\underline{x = 4}}$$

$$y = 2\left(-\frac{8}{5}\right) - 3$$

$$= \underline{\underline{-\frac{16}{5} - 3}}$$

$$= \underline{\underline{-\frac{31}{5}}}$$

$$y = 2(4) - 3$$

$$= \underline{\underline{5}}$$

$$x = \underline{\underline{-\frac{8}{5} \text{ or } 4}}$$

$$y = \underline{\underline{-\frac{31}{5} \text{ or } 5}}$$

(Total for question 5 is 5 marks)

6 Solve the simultaneous equations

$$x^2 + y^2 = 20$$

$$2x + y = 3$$

$$y = 3 - 2x$$

$$x^2 + (3 - 2x)^2 = 20$$

$$x^2 + (3 - 2x)(3 - 2x) = 20$$

$$x^2 + 9 - 6x - 6x + 4x^2 = 20$$

$$5x^2 - 12x + 9 = 20$$

$$5x^2 - 12x - 11 = 0$$

This will not factorise.

$$a = 5 \quad b = -12 \quad c = -11$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(5)(-11)}}{2(5)}$$

$$x = \underline{\underline{3.11}} \text{ (3sf)} \quad \text{or} \quad \underline{\underline{-0.708}} \text{ (3sf)}$$

$$y = 3 - 2("3.11") \quad y = 3 - 2("-0.708")$$

$$= \underline{\underline{-3.22}} \text{ (3sf)} \quad = \underline{\underline{4.42}} \text{ 3sf}$$

$$x = \underline{\underline{3.11}} \text{ or } \underline{\underline{-0.708}}$$

$$y = \underline{\underline{-3.22}} \text{ or } \underline{\underline{4.42}}$$

(Total for question 6 is 5 marks)

7 Solve the simultaneous equations

$$x^2 + y^2 = 27$$

$$2x - y = 3$$

$$2x = 3 + y$$

$$2x - 3 = y$$

$$x^2 + (2x - 3)^2 = 27$$

$$x^2 + (2x - 3)(2x - 3) = 27$$

$$x^2 + 4x^2 - 6x - 6x + 9 = 27$$

$$5x^2 - 12x + 9 = 27$$

$$5x^2 - 12x - 18 = 0$$

$$a = 5 \quad b = -12 \quad c = -18$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(5)(-18)}}{2(5)}$$

$$= \underline{\underline{3.44}} \text{ (3sf)} \quad \text{or} \quad \underline{\underline{-1.04}} \text{ (3sf)}$$

$$y = 2(\text{Ans}) - 3$$

$$= \underline{\underline{3.89}} \text{ (3sf)}$$

$$y = 2(\text{Ans}) - 3$$

$$= -5.09 \text{ (3sf)}$$

$$x = \underline{\underline{3.44}} \text{ or } \underline{\underline{-1.04}}$$

$$y = \underline{\underline{3.89}} \text{ or } \underline{\underline{-5.09}}$$

(Total for question 7 is 5 marks)

8 Solve algebraically the simultaneous equations

$$x^2 - 3y^2 = 13$$

$$2x + 3y = 4$$

$$2x + 3y = 4$$

$$2x = 4 - 3y$$

$$x = \frac{4 - 3y}{2}$$

$$\left(\frac{4 - 3y}{2}\right)^2 - 3y^2 = 13$$

$$\left(\frac{4 - 3y}{2}\right)\left(\frac{4 - 3y}{2}\right) - 3y^2 = 13$$

$$\frac{16 - 12y - 12y + 9y^2}{4} - 3y^2 = 13$$

$$\frac{16 - 24y + 9y^2}{4} - 3y^2 = 13$$

$$16 - 24y + 9y^2 - 12y^2 = 52$$

$$-3y^2 - 24y + 16 = 52$$

$$-3y^2 - 24y - 36 = 0$$

$$3y^2 + 24y + 36 = 0$$

$$y^2 + 8y + 12 = 0$$

$$(y + 2)(y + 6) = 0$$

$$y = -2 \quad y = -6$$

$$x = \frac{4 - 3(-2)}{2}$$

$$= 5$$

$$x = \frac{4 - 3(-6)}{2}$$

$$= 11$$

$$\underline{x = 5, y = -2 \quad \text{or} \quad x = 11, y = -6}$$

(Total for question is 5 marks)

9 Solve algebraically the simultaneous equations

$$2x^2 - y^2 = 14$$

$$3x + 2y = 3$$

$$3x + 2y = 3$$

$$3x = 3 - 2y$$

$$x = \frac{3 - 2y}{3}$$

$$2\left(\frac{3-2y}{3}\right)^2 - y^2 = 14$$

$$2\left(\frac{3-2y}{3}\right)\left(\frac{3-2y}{3}\right) - y^2 = 14$$

$$2\left(\frac{9-6y-6y+4y^2}{9}\right) - y^2 = 14$$

$$2\left(\frac{9-12y+4y^2}{9}\right) - y^2 = 14$$

$$\frac{18-24y+8y^2}{9} - y^2 = 14$$

$$18-24y+8y^2-9y^2 = 126$$

$$-y^2 - 24y + 18 = 126$$

$$-y^2 - 24y - 108 = 0$$

$$y^2 + 24y + 108 = 0$$

$$(y+6)(y+18) = 0$$

$$y = -6 \quad y = -18$$

$$x = \frac{3 - 2(-6)}{3}$$

$$= 5$$

$$x = \frac{3 - 2(-18)}{3}$$

$$= 13$$

$$\underline{x = 5 \text{ and } y = -6 \text{ or } x = 13 \text{ and } y = -18}$$

(Total for question is 5 marks)