Name: _____

IGCSE

Differentiation

Instructions

• Use **black** ink or ball-point pen.
• Answer all questions.
• Answer the questions in the spaces provided
– *there may be more space than you need.*
• Diagrams are **NOT** accurately drawn, unless otherwise indicated.
• You must **show all your working out.**

Information

• The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

• Read each question carefully before you start to answer it.
• Keep an eye on the time.
• Try to answer every question.
• Check your answers if you have time at the end

1 A particle moves along a straight line.
The fixed point O lies on this line.

The displacement of the particle from O at time t seconds, $t \geq 0$, is s metres, where

$$s = t^3 - 5t^2 - 8t + 3$$

Find the value of t for which the particle is instantaneously at rest.

$$v = \frac{ds}{dt} \qquad v = 3t^2 - 10t - 8$$

At rest when $v = 0$

$$3t^2 - 10t - 8 = 0$$
$$(3t + 2)(t - 4) = 0$$
$$3t = \frac{-2}{3} \qquad t = 4$$

t cannot be negative

$t =$ .... 4 ....

(Total for Question 1 is 4 marks)

2 The curve with equation $y = 8x^2 + \frac{2}{x}$ has one stationary point.

Find the co-ordinates of this stationary point.
Show your working clearly.

$$y = 8x^2 + 2x^{-1}$$
$$\frac{dy}{dx} = 16x - 2x^{-2}$$

Stationary point where $\frac{dy}{dx} = 0$

$$16x - 2x^{-2} = 0$$
$$16x = 2x^{-2}$$
$$8x = x^{-2}$$
$$8x = \frac{1}{x^2}$$

$$8x^3 = 1$$
$$x^3 = \frac{1}{8}$$
$$x = \frac{1}{2}$$

$$y = 8\left(\frac{1}{2}\right)^2 + \frac{2}{\left(\frac{1}{2}\right)}$$
$$= 6$$

(.......... $\frac{1}{2}$ .... , ... 6 ....)

(Total for Question 2 is 5 marks)

3 $y = x^3 - 2x^2 - 15x + 5$

(a) Find $\dfrac{dy}{dx}$

$$\frac{dy}{dx} = 3x^2 - 4x - 15$$

$$\frac{dy}{dx} = \underline{3x^2 - 4x - 15}$$

(2)

C is the curve with equation $y = x^3 - 2x^2 - 15x + 5$

(b) Work out the range of values of x for which C has a negative gradient.
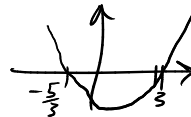
gradient is negative when $\dfrac{dy}{dx} < 0$

$3x^2 - 4x - 15 < 0$

(TYPE INTO CALCULATOR)

ALTERNATIVE

$(3x+5)(x-3) < 0$

$x = -\dfrac{5}{3}$ $x = 3$

$-\dfrac{5}{3} < x < 3$

$$-\frac{5}{3} < x < 3$$

(4)

(Total for Question 3 is 6 marks)

4   The curve C has equation $y = \frac{1}{3}x^3 - 9x + 1$

(a) Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = x^2 - 9$$

$$\frac{dy}{dx} = \underline{ x^2 - 9 }$$

(2)

(b) Find the range of values of x for which C has a negative gradient.

$$x^2 - 9 < 0$$

TYPE INTO CALC.

OR

$$(x + 3)(x - 3) = 0$$

$$-3 < x < 3$$

$$\underline{ -3 < x < 3 }$$

(3)

(Total for Question 4 is 5 marks)

5   A particle P is moving along a straight line that passes through the fixed point O.
The displacement, s metres, of P from O at time t seconds is given by

$$s = t^3 - 6t^2 + 5t - 4$$

Find the value of t for which the acceleration of P is 3 m/s^2

$$v = \frac{ds}{dt} \qquad v = 3t^2 - 12t + 5$$

$$a = \frac{dv}{dt} \qquad \underline{a = 6t - 12}$$

$$6t - 12 = 3$$
$$6t = 15$$
$$t = \frac{15}{6}$$
$$= \frac{5}{2} = 2.5$$

$t = $ ....$2 \cdot 5$........

(Total for Question 5 is 4 marks)