

Name: \_\_\_\_\_

## GCSE (1 – 9)

# Algebraic Fractions

### Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

### Information

- The marks for each question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1 Simplify fully  $\frac{x^2 + 5x}{x^2 + 7x + 10}$

$$\frac{x(\cancel{x+5})}{(x+2)(\cancel{x+5})}$$

$$\frac{x}{x+2}$$

(Total for question 1 is 2 marks)

2 Simplify fully  $\frac{x^2 - x - 12}{x^2 - 9x + 20}$

$$\frac{(x+3)(\cancel{x-4})}{(x-5)(\cancel{x-4})}$$

$$\frac{x+3}{x-5}$$

(Total for question 2 is 2 marks)

3 Simplify fully  $\frac{3x^2 + 9x}{x^2 - 9}$

$$\frac{3x(\cancel{x+3})}{(x-3)(\cancel{x+3})}$$

$$\frac{3x}{x-3}$$

(Total for question 3 is 2 marks)

4 Simplify fully  $\frac{x+4}{x^2 - 16}$

$$\frac{1(\cancel{x+4})}{(\cancel{x+4})(x-4)}$$

$$\frac{1}{x-4}$$

(Total for question 4 is 2 marks)

- 5 Write  $\frac{3x^2 + 11x - 4}{x^2 + 3x - 4}$  in the form  $\frac{ax + b}{x + c}$  where  $a$ ,  $b$ , and  $c$  are integers.

$$\frac{(3x - 1)(\cancel{x + 4})}{(\cancel{x + 4})(x - 1)}$$

$$\frac{3x - 1}{x - 1}$$

(Total for question 5 is 3 marks)

- 6 Write  $\frac{x^2 + 7x - 18}{2x^2 - x - 6}$  in the form  $\frac{x + a}{bx + c}$  where  $a$ ,  $b$ , and  $c$  are integers.

$$\frac{(x + 9)(\cancel{x - 2})}{(2x + 3)(\cancel{x - 2})}$$

$$\frac{x + 9}{2x + 3}$$

(Total for question 6 is 3 marks)

7 Simplify fully  $\frac{3x+6}{x-4} \div \frac{2x^2+9x+10}{x^2-4x}$

$$\frac{(3x+6)}{(x-4)} \times \frac{x^2-4x}{2x^2+9x+10}$$

$$\frac{(3x+6)(x^2-4x)}{(x-4)(2x^2+9x+10)}$$

$$\frac{3(\cancel{x+2}) \times x(\cancel{x-4})}{(\cancel{x-4})(2x+5)(\cancel{x+2})}$$

$$\frac{3x}{2x+5}$$

$$\frac{3x}{2x+5}$$

(Total for question 7 is 3 marks)

8 Simplify fully  $\frac{2x-2}{x+5} \div \frac{x^2-4x+3}{2x^2+13x+15}$

$$\frac{2(x-1)}{x+5} \times \frac{2x^2+13x+15}{x^2-4x+3}$$

$$\frac{2(x-1)}{(x+5)} \times \frac{(2x+3)(x+5)}{(x-3)(x-1)}$$

$$\frac{2(\cancel{x-1})(2x+3)(\cancel{x+5})}{(\cancel{x+5})(x-3)(\cancel{x-1})}$$

$$\frac{2(2x+3)}{x-3}$$

(Total for question 8 is 3 marks)

9 Solve  $\frac{8}{x+3} + \frac{3}{x+8} = 1$

$$\frac{8(x+8)}{(x+3)(x+8)} + \frac{3(x+3)}{(x+3)(x+8)} = 1$$

$$\frac{8(x+8) + 3(x+3)}{(x+3)(x+8)} = 1$$

$$8(x+8) + 3(x+3) = (x+3)(x+8)$$

$$8x + 64 + 3x + 9 = x^2 + 8x + 3x + 24$$

$$11x + 73 = x^2 + 11x + 24$$

$$73 = x^2 + 24$$

$$0 = x^2 - 49$$

$$(x+7)(x-7) = 0$$

$$x = -7 \text{ or } 7$$

$$x = -7 \text{ or } 7$$

(Total for question 9 is 4 marks)

10 Solve  $\frac{8}{3x-2} + \frac{6}{x+1} = 2$

$$\frac{8(x+1)}{(3x-2)(x+1)} + \frac{6(3x-2)}{(3x-2)(x+1)} = 2$$

$$\frac{8(x+1) + 6(3x-2)}{(3x-2)(x+1)} = 2$$

$$8(x+1) + 6(3x-2) = 2(3x-2)(x+1)$$

$$8x + 8 + 18x - 12 = 2(3x^2 + 3x - 2x - 2)$$

$$26x - 4 = 2(3x^2 + x - 2)$$

$$26x - 4 = 6x^2 + 2x - 4$$

$$0 = 6x^2 - 24x$$

$$0 = 6x(x-4)$$

$$x = 0 \text{ or } x = 4$$

$$x = 0 \text{ or } 4$$

(Total for question 10 is 4 marks)

11 Solve  $\frac{2}{5-x} + \frac{3}{x+7} = 1$

$$\frac{2(x+7)}{(5-x)(x+7)} + \frac{3(5-x)}{(5-x)(x+7)} = 1$$

$$\frac{2(x+7) + 3(5-x)}{(5-x)(x+7)} = 1$$

$$2(x+7) + 3(5-x) = (5-x)(x+7)$$

$$2x + 14 + 15 - 3x = 5x + 35 - x^2 - 7x$$

$$29 - x = 35 - 2x - x^2$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$x = -3 \quad x = 2$$

$$x = -3 \text{ or } 2$$

(Total for question 11 is 4 marks)

12 Solve  $\frac{7}{x+1} - \frac{4}{3x-2} = 1$

$$\frac{7(3x-2)}{(x+1)(3x-2)} - \frac{4(x+1)}{(x+1)(3x-2)} = 1$$

$$\frac{7(3x-2) - 4(x+1)}{(x+1)(3x-2)} = 1$$

$$7(3x-2) - 4(x+1) = (x+1)(3x-2)$$

$$21x - 14 - 4x - 4 = 3x^2 - 2x + 3x - 2$$

$$17x - 18 = 3x^2 + x - 2$$

$$0 = 3x^2 - 16x + 16$$

$$(3x-4)(x-4) = 0$$

$$x = \frac{4}{3} \quad x = 4$$

$$x = \frac{4}{3} \text{ or } 4$$

(Total for question 12 is 4 marks)

13 Given that

$$2x+1 : x+2 = x+8 : 3x-4$$

Find the possible values of  $x$ .

$$\frac{2x+1}{x+2} = \frac{x+8}{3x-4}$$

$$(2x+1)(3x-4) = (x+8)(x+2)$$

$$6x^2 - 8x + 3x - 4 = x^2 + 2x + 8x + 16$$

$$6x^2 - 5x - 4 = x^2 + 10x + 16$$

$$5x^2 - 15x - 20 = 0$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$x=4 \quad x=-1$$

$$x=4 \text{ or } x=-1$$

(Total for question 13 is 4 marks)

14 Given that

$$x-1 : 2x-3 = x+2 : 3x-2$$

Find the possible values of  $x$ .

$$\frac{x-1}{2x-3} = \frac{x+2}{3x-2}$$

$$(x-1)(3x-2) = (x+2)(2x-3)$$

$$3x^2 - 2x - 3x + 2 = 2x^2 - 3x + 4x - 6$$

$$3x^2 - 5x + 2 = 2x^2 + x - 6$$

$$x^2 - 6x + 8 = 0$$

$$(x-2)(x-4) = 0$$

$$x=2 \quad x=4$$

$$x=2 \text{ or } x=4$$

(Total for question 14 is 4 marks)

15 Given that

$$x+9 : 5x-1 = x+7 : 2x-3$$

Find the possible values of  $x$ .

$$\frac{x+9}{5x-1} = \frac{x+7}{2x-3}$$

$$(x+9)(2x-3) = (x+7)(5x-1)$$

$$2x^2 - 3x + 18x - 27 = 5x^2 - x + 35x - 7$$

$$2x^2 + 15x - 27 = 5x^2 + 34x - 7$$

$$0 = 3x^2 + 19x + 20$$

$$0 = (3x+4)(x+5)$$

$$x = -\frac{4}{3} \quad x = -5$$

$$x = -\frac{4}{3} \text{ or } x = -5$$

(Total for question 15 is 4 marks)

16 Given that

$$5-3x : 9-x = 3x+7 : 4-x$$

Find the possible values of  $x$ .

$$\frac{5-3x}{9-x} = \frac{3x+7}{4-x}$$

$$(5-3x)(4-x) = (3x+7)(9-x)$$

$$20 - 5x - 12x + 3x^2 = 27x - 3x^2 + 63 - 7x$$

$$3x^2 - 17x + 20 = 20x - 3x^2 + 63$$

$$6x^2 - 37x - 43 = 0$$

$$(6x-43)(x+1) = 0$$

$$x = \frac{43}{6} \quad x = -1$$

$$x = \frac{43}{6} \text{ or } x = -1$$

(Total for question 16 is 4 marks)