# Mathematics <br> 2022 Paper 1 (Non-Calculator) Higher Tier 

## Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

## Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Answer the questions in the spaces provided
- there may be more space than you need.
- Calculators may not be used.

- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must show all your working out.


## Information

- The total mark for this paper is 80
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.


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## Higher Tier Formulae Sheet

## Perimeter, area and volume

Where $a$ and $b$ are the lengths of the parallel sides and $h$ is their perpendicular separation:

$$
\text { Area of a trapezium }=\frac{1}{2}(a+b) h
$$

Volume of a prism $=$ area of cross section $\times$ length Where $r$ is the radius and $d$ is the diameter:

Circumference of a circle $=2 \pi \mathrm{r}=\pi d$
Area of a circle $=\pi r^{2}$

## Pythagoras' Theorem and Trigonometry


b


In any right-angled triangle where $a, \mathrm{~b}$ and $c$ are the length of the sides and c is the hypotenuse:

$$
a^{2}+b^{2}=c^{2}
$$

In any right-angled triangle $A B C$ where $a, b$ and $c$ are the length of the sides and $c$ is the hypotenuse:

$$
\sin A=\frac{a}{c} \quad \cos A=\frac{b}{c} \quad \tan A=\frac{a}{b}
$$

In any triangle ABC where $\mathrm{a}, \mathrm{b}$ and c are the length of the sides:
sine rule: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule: $a^{2}=b^{2}+c^{2}-2 b c \cos A$
Area of triangle $=\frac{1}{2} a b \sin C$

## Probability

Where $\mathrm{P}(A)$ is the probability of outcome $A$ and $\mathrm{P}(B)$ is the probability of outcome $B$ :

$$
\begin{aligned}
& \mathrm{P}(A \text { or } B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A \text { and } B) \\
& \mathrm{P}(A \text { and } B)=\mathrm{P}(A \text { given } B) \mathrm{P}(B)
\end{aligned}
$$

1 Write 240 as a product of its prime factors.

2 (a) Work out $\frac{3}{4}-\frac{7}{10}$
(b) Work out $2 \frac{1}{3} \times \frac{3}{5}$

Give your answer as a mixed number in its simplest form.

3 In a bag there are only red counters, blue counters, green counters and yellow counters.
A counter is taken at random from the bag.
The table shows the probabilities that the counter will be green or will be yellow.

| Colour | Red | Blue | Green | Yellow |
| :--- | :---: | :---: | :---: | :---: |
| Probability |  |  | 0.35 | 0.20 |

The probability that the counter will be red is twice the probability that the counter will be blue.
There are 21 green counters in the bag.
Work out the number of red counters in the bag.

4 Here is the graph of $y=2 x+6-x^{2}$

(a) Write down the turning point of the graph $y=2 x+6-x^{2}$
$\qquad$
(1)
(b) Use the graph to find the roots of the equation $x^{2}=2 x+6$
$5 \quad 5<2 y<12$ where $y$ is an integer.
(a) Write down all the possible values of $y$.
(b) Solve $4>19-3 x$

6 Dermot has 240 counters.
The counters are either red, or blue, or yellow or green.
$15 \%$ of the counters are red.
$\frac{2}{5}$ of the counters are blue
The ratio of yellow counters to green counters is $3: 1$
Work out the number of yellow counters Dermot has.
$7 \quad$ Here is a cube.


Work out the volume of this cube.
Give your answer in standard form
$\qquad$ . $\mathrm{cm}^{3}$

8

$A B C D E F$ is a hexagon.
Angle $C D E=2 \times$ Angle $B C D$
Work out the size of angle $C D E$.
$9 \quad$ Liquid $\mathbf{A}$ has a density of $1.2 \mathrm{~g} / \mathrm{cm}^{3}$
$150 \mathrm{~cm}^{3}$ of Liquid $\mathbf{A}$ is mixed with some of Liquid $\mathbf{B}$ to make Liquid $\mathbf{C}$.
Liquid $\mathbf{C}$ has a mass of 220 g and a density of $1.1 \mathrm{~g} / \mathrm{cm}^{3}$
Find the density of Liquid B.
$\mathrm{g} / \mathrm{cm}^{3}$

1010 students have a mean height of 150 cm .
4 of the students have a mean height of 165 cm .
Work out the mean height of the other 6 students.

11 The points A, B, C and D lie in order on a straight line.

$$
A B: B D=2: 5 \text { and } A C: C D=4: 7
$$

Find $A B: B C: C D$

12 Jon plays a game where he can win, draw or lose.
The probability Jon wins any game 0.5 .
The probability Jon draws any game is 0.3
Jon plays two games.
Work out the probability Jon wins exactly one game.

13 The cumulative frequency table shows the height, in cm , of some tomato plants.

| Height | Cumulative Frequency |
| :---: | :---: |
| $140<\mathrm{h} \leqslant 150$ | 7 |
| $140<\mathrm{h} \leqslant 160$ | 17 |
| $140<\mathrm{h} \leqslant 170$ | 32 |
| $140<\mathrm{h} \leqslant 180$ | 51 |
| $140<\mathrm{h} \leqslant 190$ | 57 |
| $140<\mathrm{h} \leqslant 200$ | 60 |

(a) On the grid, plot a cumulative frequency graph for this information.

(b) Find an estimate for the inter quartile range.

14 Find the value of $\left(\frac{64}{125}\right)^{-\frac{2}{3}}$

15 Write 0.16 as a fraction in its simplest form.
$16 \quad A B C$ is an isosceles triangle.
Angle $A C B=$ Angle $A B C$
The area of the triangle is $60 \mathrm{~cm}^{2}$
Calculate the perimeter of the triangle.

..cm

17 Here are the equations of 5 straight lines.

$$
\begin{aligned}
& P: y=2 x+5 \\
& Q: y=-2 x+5 \\
& R: y=x+5 \\
& S: y=-\frac{1}{2} x+6 \\
& T: y=\frac{1}{2} x+1
\end{aligned}
$$

(a) Write down the letter of the line that is parallel to $y=x+6$
(b) Write down the letter of the line that is perpendicular to $y=2 x-1$

18 The diagram shows a cuboid.
All the measurements are in centimetres.
The total surface area of the cuboid is $172 \mathrm{~cm}^{2}$
Find the value of $x$.


19 Write $(3-\sqrt{8})^{2}$ in the form $\mathrm{a}+\mathrm{b} \sqrt{2}$, where $a$ and $b$ are integers.

20 Given that

$$
2 x+1: x+2=x+8: 3 x-4
$$

Find the possible values of $x$.

21 Here is a speed-time graph.


Work out an estimate for the acceleration when $t=2$.
$\mathrm{ms}^{-2}$
(Total for Question 21 is $\mathbf{2}$ marks)

22 The table shows pairs of values for $x$ and $y$

| $x$ | 2 | 3 |
| :---: | :---: | :---: |
| $y$ | 32 | 72 |

(i) Tick the correct statement below.

$$
\begin{aligned}
& y \propto x \\
& y \propto x^{2} \\
& y \propto x^{3}
\end{aligned}
$$

(ii) Write a formula for $y$ in terms of $x$
$23 B A C$ is a sector of a circle, centre $A$.
$A C$ is 10 cm .


Find the area of the shaded segment.
Give your answer in terms of $\pi$.
$\mathrm{cm}^{2}$

24 A circle has the equation $x^{2}+y^{2}=7$
$P$ is the point $(-2, \sqrt{3})$ on the circle $x^{2}+y^{2}=7$
Work out the equation of the tangent to the circle at $P$.

25

$\overrightarrow{O A}=2 a$
$\overrightarrow{O B}=2 b$
$N$ is the point on $O B$ such that $O N: N B=2: 1$
$M$ is the midpoint of $A B$
$P$ is the point on $O M$ such that $A P N$ is a straight line.
Find $O P: P M$

