

1. On a particular day the height above sea level, x metres, and the mid-day temperature, $y^\circ\text{C}$, were recorded in 8 north European towns. These data are summarised below

$$S_{xx} = 3\,535\,237.5 \quad \sum y = 181 \quad \sum y^2 = 4305 \quad S_{xy} = -23\,726.25$$

- (a) Find S_{yy} (2)
- (b) Calculate, to 3 significant figures, the product moment correlation coefficient for these data. (2)
- (c) Give an interpretation of your coefficient. (1)

A student thought that the calculations would be simpler if the height above sea level, h , was measured in kilometres and used the variable $h = \frac{x}{1000}$ instead of x .

- (d) Write down the value of S_{hh} (1)
- (e) Write down the value of the correlation coefficient between h and y . (1)

$$\begin{aligned} \text{a/} \quad S_{yy} &= \sum y^2 - \frac{(\sum y)^2}{n} \\ &= 4305 - \frac{(181)^2}{8} \\ &= 209.875 \end{aligned}$$

$$\begin{aligned} \text{b/} \quad r &= \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} \\ r &= \frac{-23726.25}{\sqrt{(3535237.5)(209.875)}} \\ &= -0.871 \quad (3\text{sf}) \end{aligned}$$

c/ As the sea level increases the mid-day temperature decreases.



Question 1 continued

d/ 3.5352375

e/ -0.871 (3sf)

Q1

(Total 7 marks)



2. The random variable $X \sim N(\mu, 5^2)$ and $P(X < 23) = 0.9192$

(a) Find the value of μ .

(4)

(b) Write down the value of $P(\mu < X < 23)$.

(1)

$$a) \quad z = \frac{X - \mu}{\sigma}$$

$$p = 0.9192 \quad \therefore z = 1.40 \quad (\text{tables})$$

$$1.40 = \frac{23 - \mu}{5}$$

$$7 = 23 - \mu$$

$$\mu = 16$$

$$b) \quad 0.4192 \quad (\mu \text{ is at } 50\%)$$



3. The discrete random variable Y has probability distribution

y	1	2	3	4
$P(Y=y)$	a	b	0.3	c

$3y+2$ 5 8 11 14
 0.1 0.4 0.2 = 1

where a , b and c are constants.

The cumulative distribution function $F(y)$ of Y is given in the following table

y	1	2	3	4
$F(y)$	0.1	0.5	d	1.0

where d is a constant.

$+0.4$ $+0.3$ $+0.2$
 0.8

(a) Find the value of a , the value of b , the value of c and the value of d .

(5)

(b) Find $P(3Y + 2 \geq 8)$.

(2)

a) $a = 0.1$
 $b = 0.4$
 $c = 0.2$
 $d = 0.8$

b/ $P(2, 3, 4) = \underline{\underline{0.9}}$



4. Past records show that the times, in seconds, taken to run 100 m by children at a school can be modelled by a normal distribution with a mean of 16.12 and a standard deviation of 1.60

A child from the school is selected at random.

- (a) Find the probability that this child runs 100 m in less than 15 s. (3)

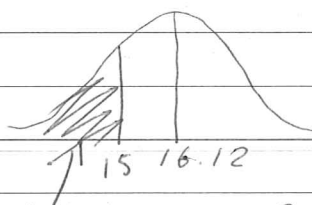
On sports day the school awards certificates to the fastest 30% of the children in the 100 m race.

- (b) Estimate, to 2 decimal places, the slowest time taken to run 100 m for which a child will be awarded a certificate. (4)

$$\begin{aligned} a) \quad Z &= \frac{X - \mu}{\sigma} \\ &= \frac{15 - 16.12}{1.6} \end{aligned}$$

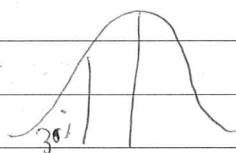
$$= -0.7$$

$$Z = -0.7 \quad \therefore \quad P = 0.7580$$



small bit $\therefore P = 1 - 0.7580$
 $= \underline{\underline{0.242}}$

b/



$$P = 0.7 \quad \therefore \quad Z = -0.52$$

\uparrow
 - (left of graph)

$$-0.52 = \frac{X - 16.12}{1.6}$$

$$X = 15.288 \text{ seconds}$$

$$= 15.29 \text{ seconds (2dp)}$$



5. A class of students had a sudoku competition. The time taken for each student to complete the sudoku was recorded to the nearest minute and the results are summarised in the table below.

Time	Mid-point, x	Frequency, f
2 - 8	5	2
9 - 12	10.5	7
13 - 15	14	5
^{15.5} 16 - 18 ^{18.5}	17	8
19 - 22	20.5	4
23 - 30	26.5	4

(You may use $\sum fx^2 = 8603.75$)

- (a) Write down the mid-point for the 9 - 12 interval. (1)
- (b) Use linear interpolation to estimate the median time taken by the students. (2)
- (c) Estimate the mean and standard deviation of the times taken by the students. (5)

The teacher suggested that a normal distribution could be used to model the times taken by the students to complete the sudoku.

- (d) Give a reason to support the use of a normal distribution in this case. (1)

On another occasion the teacher calculated the quartiles for the times taken by the students to complete a different sudoku and found

$$Q_1 = 8.5 \quad Q_2 = 13.0 \quad Q_3 = 21.0$$

- (e) Describe, giving a reason, the skewness of the times on this occasion. (2)

b/ $15.5 + \frac{1}{8}(3)$
 $= 15.875$

c/ $\text{mean} = \frac{\sum fx}{n} = \frac{477.5}{30} = 15.916$



Question 5 continued

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$
$$= \sqrt{\frac{8603.75}{30} - (15.91\bar{6})^2}$$

$$= 5.783717566$$

$$= 5.784 \text{ (3dp)}$$

d) The mean is almost equal to the median

e) The median is closer to the lower quartile.
This is a positive skew.



6. Jake and Kamil are sometimes late for school.
The events J and K are defined as follows

J = the event that Jake is late for school
 K = the event that Kamil is late for school

$$P(J) = 0.25, P(J \cap K) = 0.15 \text{ and } P(J' \cap K') = 0.7$$

On a randomly selected day, find the probability that

- (a) at least one of Jake or Kamil are late for school, (1)

- (b) Kamil is late for school. (2)

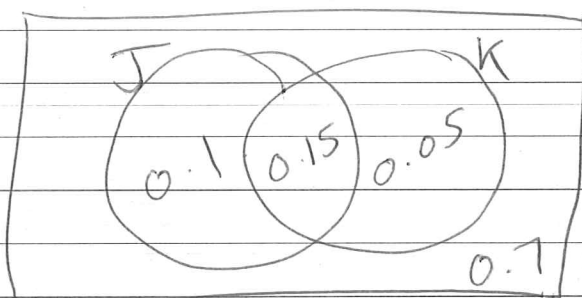
Given that Jake is late for school,

- (c) find the probability that Kamil is late. (3)

The teacher suspects that Jake being late for school and Kamil being late for school are linked in some way.

- (d) Determine whether or not J and K are statistically independent. (2)

- (e) Comment on the teacher's suspicion in the light of your calculation in (d). (1)



a) 0.3

b) 0.2

c) $\frac{0.15}{0.25} = 0.6$



Question 6 continued

d/ if independent: $P(J) \times P(K) = P(J \cap K)$

$$0.25 \times 0.2 = 0.05$$

$$P(J \cap K) \neq 0.05$$

J and K are not statistically independent

e) it appears that there is a link between Jake and Kamil being late for school.



7. A teacher took a random sample of 8 children from a class. For each child the teacher recorded the length of their left foot, f cm, and their height, h cm. The results are given in the table below.

f	23	26	23	22	27	24	20	21
h	135	144	134	136	140	134	130	132

(You may use $\sum f = 186$ $\sum h = 1085$ $S_{ff} = 39.5$ $S_{hh} = 139.875$ $\sum fh = 25291$)

(a) Calculate S_{fh} (2)

(b) Find the equation of the regression line of h on f in the form $h = a + bf$.
Give the value of a and the value of b correct to 3 significant figures. (5)

(c) Use your equation to estimate the height of a child with a left foot length of 25 cm. (2)

(d) Comment on the reliability of your estimate in (c), giving a reason for your answer. (2)

The left foot length of the teacher is 25 cm.

(e) Give a reason why the equation in (b) should not be used to estimate the teacher's height. (1)

$$S_{fh} = \sum fh - \frac{(\sum f)(\sum h)}{n}$$

$$= 25291 - \frac{(186)(1085)}{8}$$

$$= 64.75$$

$$b/ \quad b = \frac{S_{fh}}{S_{ff}} \quad a = \bar{h} - b\bar{f}$$

$$= \frac{64.75}{39.5}$$

$$= 1.639240506$$



Question 7 continued

$$\bar{h} = \frac{1085}{8}$$

$$= 135.625$$

$$\bar{A} = \frac{186}{8}$$

$$= 23.25$$

$$a = 135.625 - 1.639240506(23.25)$$

$$a = 97.51265823$$

$$h = 97.5 + 1.64f$$

$$c) A = 25$$

$$h = 97.5 + 1.64(25)$$

$$= \underline{\underline{138.5 \text{ cm}}}$$

d) ~~3~~ 25 is within the range provided so it is a reliable estimate

e) It should not because the data provided was for children



8. A spinner is designed so that the score S is given by the following probability distribution.

s	0	1	2	4	5
$P(S = s)$	p	0.25	0.25	0.20	0.20

- (a) Find the value of p . (2)
- (b) Find $E(S)$. (2)
- (c) Show that $E(S^2) = 9.45$ (2)
- (d) Find $\text{Var}(S)$. (2)

Tom and Jess play a game with this spinner. The spinner is spun repeatedly and S counters are awarded on the outcome of each spin. If S is even then Tom receives the counters and if S is odd then Jess receives them. The first player to collect 10 or more counters is the winner.

- (e) Find the probability that Jess wins after 2 spins. (2)
- (f) Find the probability that Tom wins after exactly 3 spins. (4)
- (g) Find the probability that Jess wins after exactly 3 spins. (3)

a) $p = 0.1$

b) $0 \times 0.1 + 1 \times 0.25 + 2 \times 0.25 + 4 \times 0.2 + 5 \times 0.2$
 $= 2.55$

c) $E(S^2) = 0^2 \times 0.1 + 1^2 \times 0.25 + 2^2 \times 0.25 + 4^2 \times 0.2 + 5^2 \times 0.2$
 $= 9.45$

d) $\text{Var}(S) = 9.45 - 2.55^2$
 $= 2.9475$



Question 8 continued

$$\begin{aligned} e/ \quad P(5,5) &= 0.2 \times 0.2 \\ &= \underline{\underline{0.04}} \end{aligned}$$

$$f/ \quad P(4,4,2) = 0.2 \times 0.2 \times 0.25 = \frac{1}{100}$$

$$P(4,2,4) = \frac{1}{100}$$

$$P(2,4,4) = \frac{1}{100}$$

$$\begin{aligned} P(4,4,4) &= 0.2 \times 0.2 \times 0.2 = 0.008 \\ &= \underline{\underline{0.03}} + 0.008 = \underline{\underline{0.038}} \end{aligned}$$

$$g/ \quad P(5,5,5') = 0.2 \times 0.2 \times 0.8 = \frac{4}{125}$$

$$P(5,5',5) = \frac{4}{125}$$

$$P(5',5,5) = \frac{4}{125}$$

$$= \underline{\underline{\frac{12}{125}}}$$

