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Write your name here

Surname	Other names

Pearson
Edexcel GCE

Centre Number	Candidate Number

Core Mathematics C3

Advanced

Tuesday 21 June 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
6665/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1. The functions f and g are defined by

$$f : x \to 7x - 1, \qquad x \in \mathbb{R}$$

$$g : x \to \frac{4}{x-2}, \qquad x \neq 2, x \in \mathbb{R}$$

(a) Solve the equation $fg(x) = x$

(4)

(b) Hence, or otherwise, find the largest value of a such that $g(a) = f^{-1}(a)$

(1)

a) $fg(x) = x$

$7\left(\frac{4}{x-2}\right) - 1 = x$

$\frac{28}{x-2} - 1 = x$

$\frac{28}{x-2} = x + 1$

$28 = (x+1)(x-2)$

$28 = x^2 - 2x + x - 2$

$28 = x^2 - x - 2$

$0 = x^2 - x - 30$

$0 = (x+5)(x-6)$

$x = -5 \quad x = 6$

b) $a = 6$

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2. $$y = \frac{4x}{x^2 + 5}$$

(a) Find $\frac{dy}{dx}$, writing your answer as a single fraction in its simplest form.

(4)

(b) Hence find the set of values of x for which $\frac{dy}{dx} < 0$

(3)

a) $$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$u = 4x \qquad v = x^2 + 5$

$\frac{du}{dx} = 4 \qquad \frac{dv}{dx} = 2x$

$$\frac{dy}{dx} = \frac{(x^2+5)(4) - (4x)(2x)}{(x^2+5)^2}$$

$$= \frac{4x^2 + 20 - 8x^2}{(x^2+5)^2}$$

$$= \frac{20 - 4x^2}{(x^2+5)^2}$$

b/ $$\frac{dy}{dx} < 0$$

$$\frac{20 - 4x^2}{(x^2+5)^2} < 0$$

$20 - 4x^2 < 0$

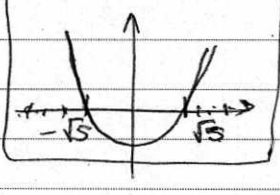
$20 < 4x^2$

$5 < x^2$

$x = \pm\sqrt{5}$

$x < -\sqrt{5}$ or $x > \sqrt{5}$

$x^2 - 5 > 0$



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3. (a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. Give the exact value of R and give the value of α to 2 decimal places.

(3)

(b) Hence solve, for $0 \leqslant \theta < 360°$,

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15$$

Give your answers to one decimal place.

(5)

(c) Use your solutions to parts (a) and (b) to deduce the smallest positive value of θ for which

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15$$

Give your answer to one decimal place.

(2)

a/ $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$R\cos(\theta+\alpha) = R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$

$R\cos(\theta+\alpha) = 2\cos\theta - \cancel{2}\sin\theta$

$R\cos\alpha = 2$

$R\sin\alpha = 1$

$\tan\alpha = \frac{1}{2}$ $\qquad R^2 = 1^2 + 2^2$

$\alpha = 26.57°$ $\qquad R^2 = 5$

$R = \sqrt{5}$

$\sqrt{5}\cos(\theta + 26.57)$

b/ $\dfrac{2}{\sqrt{5}\cos(\theta+26.57)-1} = 15$

$2 = 15\left(\sqrt{5}\cos(\theta+26.57) - 1\right)$

$2 = 15\sqrt{5}\cos(\theta + 25.67) - 15$

$17 = 15\sqrt{5}\cos(\theta+25.67)$

$\dfrac{17}{15\sqrt{5}} = \cos(\theta + 25.67)$

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Question 3 continued

$\theta + 26.57 = 59.54629008,$
300.4537099

$\theta = 33.0^\circ, 273.9^\circ$

c/ $\theta - 26.57 = 59.54629008$
$\theta = 86.1^\circ$

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4.

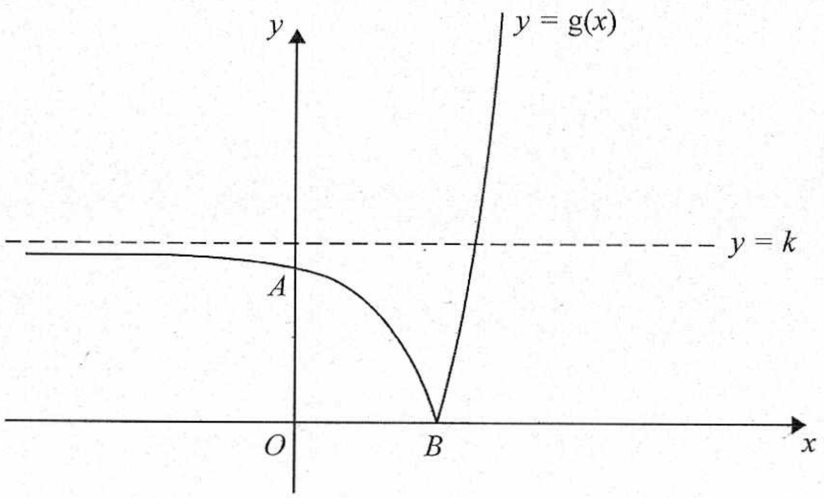

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = |4\mathrm{e}^{2x} - 25|, \qquad x \in \mathbb{R}$$

The curve cuts the y-axis at the point A and meets the x-axis at the point B. The curve has an asymptote $y = k$, where k is a constant, as shown in Figure 1

(a) Find, giving each answer in its simplest form,

(i) the y coordinate of the point A,

(ii) the exact x coordinate of the point B,

(iii) the value of the constant k.

(5)

The equation $\mathrm{g}(x) = 2x + 43$ has a positive root at $x = \alpha$

(b) Show that α is a solution of $x = \frac{1}{2}\ln\left(\frac{1}{2}x + 17\right)$

(2)

The iteration formula

$$x_{n+1} = \frac{1}{2}\ln\left(\frac{1}{2}x_n + 17\right)$$

can be used to find an approximation for α

(c) Taking $x_0 = 1.4$ find the values of x_1 and x_2
Give each answer to 4 decimal places.

(2)

(d) By choosing a suitable interval, show that $\alpha = 1.437$ to 3 decimal places.

(2)

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Question 4 continued

a/ $y = |4e^{2x} - 25|$

crosses ~~x~~ y when $x = 0$

$y = |4e^{2(0)} - 25|$

$= |4 - 25|$

$= \underline{\underline{21}}$

ii/ crosses x when $y = 0$

$0 = |4e^{2x} - 25|$

$0 = 4e^{2x} - 25$

$25 = 4e^{2x}$

$\frac{25}{4} = e^{2x}$

$\frac{5}{2} = e^{x}$

$\ln\frac{5}{2} = x$

iii/ as e^{x} approaches $-\infty$ it gets closer to zero.

$g(x) = |0 - 25|$

$k = \underline{\underline{25}}$

b/ $g(x) = 2x + 43$

$4e^{2x} - 25 = 2x + 43$

$4e^{2x} = 2x + 68$

$e^{2x} = \frac{1}{2}x + 17$

$2x = \ln(\frac{1}{2}x + 17)$

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Question 4 continued

$$x = \frac{1}{2}\ln\left(\frac{1}{2}x + 17\right)$$

c/ $x_1 = \frac{1}{2}\ln\left(\frac{1}{2}(1.4) + 17\right)$

$= 1.4368$

$x_2 = 1.4373$

d/ $4e^{2x} - 2x - 68 = 0$

when $x = 1.4365$ when $x = 1.4375$

$4e^{2(1.4365)} - 2(1.4365) - 68 = -0.1129651982$

$4e^{2(1.4375)} - 2(1.4375) - 68 = 0.02669648585$

change of sign $\therefore$ 1.437 is correct to 3dp.

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5. (i) Find, using calculus, the x coordinate of the turning point of the curve with equation

$$y = e^{3x}\cos 4x, \quad \frac{\pi}{4} \leqslant x < \frac{\pi}{2}$$

Give your answer to 4 decimal places.

(5)

(ii) Given $x = \sin^2 2y$, $0 < y < \frac{\pi}{4}$, find $\frac{dy}{dx}$ as a function of y.

Write your answer in the form

$$\frac{dy}{dx} = p\,\mathrm{cosec}(qy), \qquad 0 < y < \frac{\pi}{4}$$

where p and q are constants to be determined.

(5)

i/ $y = e^{3x}\cos 4x$

$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

$u = e^{3x}$ $\quad v = \cos 4x$

$\frac{du}{dx} = 3e^{3x}$ $\quad \frac{dv}{dx} = -4\sin 4x$

$\frac{dy}{dx} = 3e^{3x}\cos 4x \cancel{+} - 4e^{3x}\sin 4x$

Turning Point is where $\frac{dy}{dx} = 0$

$0 = 3e^{3x}\cos 4x - 4e^{3x}\sin 4x$

$0 = e^{3x}(3\cos 4x - 4\sin 4x)$

$e^{3x} \neq 0$

$0 = 3\cos 4x - 4\sin 4x$

$4\sin 4x = 3\cos 4x$

$4\tan 4x = 3$

$\tan 4x = 3/4$

$4x = 0.6435011088$

$x = 0.1609$ (4dp)

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Question 5 continued $\cancel{y=}\ x = (\sin 2y)^2$

ii/ $\frac{dx}{dy} = 2(\sin 2y)(2\cos 2y)$

$\frac{dx}{dy} = 4\sin 2y \cos 2y$

$\boxed{\sin 2A = 2\sin A \cos A}$

$\frac{dx}{dy} = 2\sin 4y$

$\frac{dy}{dx} = \frac{1}{2\sin 4y}$

$= \frac{1}{2}\operatorname{cosec} 4y$

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6. $$f(x) = \frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6}, \qquad x > 2, x \in \mathbb{R}$$

(a) Given that

$$\frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6} \equiv x^2 + A + \frac{B}{x - 2}$$

find the values of the constants A and B.

(4)

(b) Hence or otherwise, using calculus, find an equation of the normal to the curve with equation $y = f(x)$ at the point where $x = 3$

(5)

$$\begin{array}{r|l} & x^2 \qquad + 3 \\ x^2 + x - 6 & x^4 + x^3 - 3x^2 + 7x - 6 \\ & x^4 + x^3 - 6x^2 \\ & \qquad\quad 3x^2 + 7x - 6 \\ & \qquad\quad 3x^2 + 3x - 18 \\ & \qquad\qquad\quad 4x + 12 \end{array}$$

$$x^2 + 3 + \frac{4x + 12}{x^2 + x - 6}$$

$$x^2 + 3 + \frac{4\cancel{x}\cancel{(x+3)}}{\cancel{(x+3)}(x-2)}$$

$$x^2 + 3 + \frac{4}{x - 2} \qquad A = 3 \quad B = 4.$$

b/ ~~dy~~ $y = x^2 + 3 + 4(x-2)^{-1}$

$$\frac{dy}{dx} = 2x - 4(x-2)^{-2}$$

when $x=3$ $\quad \frac{dy}{dx} = 2(3) - 4(3-2)^{-2}$

$$= 2$$

$\therefore$ gradient of normal $= -\frac{1}{2}$

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Question 6 continued

when $x=3$

$$y=(3)^2+3+4(3-2)^{-1}$$
$$=16$$

$$y=mx+c$$
$$y=-\tfrac{1}{2}x+c \qquad (3,16)$$
$$16=-\tfrac{1}{2}(3)+c$$

$$16=-\frac{3}{2}+c$$
$$c=\frac{35}{2}$$

$$\underline{y=-\tfrac{1}{2}x+\frac{35}{2}}$$

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7. (a) For $-\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2}$, sketch the graph of $y = g(x)$ where

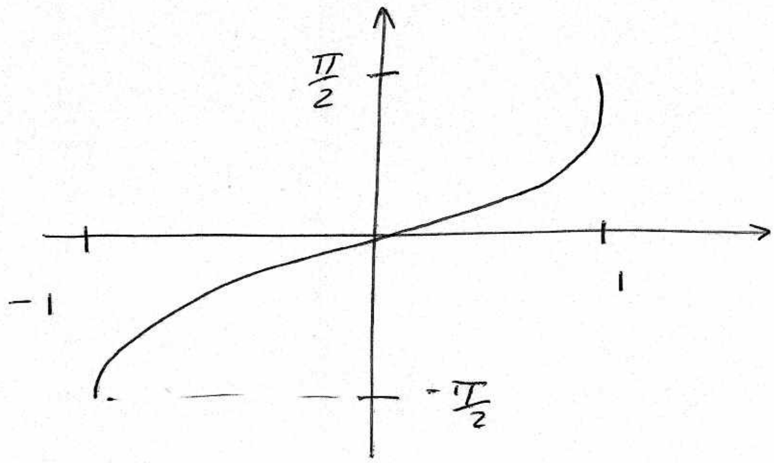
$$g(x) = \arcsin x \qquad -1 \leqslant x \leqslant 1$$

(2)

(b) Find the exact value of x for which

$$3g(x+1) + \pi = 0$$

(3)



Question 7 continued

$$3 \arcsin(x+1) + \pi = 0$$
$$\arcsin(x+1) = \frac{-\pi}{3}$$
$$x+1 = \sin\left(\frac{-\pi}{3}\right)$$
$$x+1 = \frac{-\sqrt{3}}{2}$$
$$x = \frac{-\sqrt{3}}{2} - 1$$

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Q7

(Total 5 marks)

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8. (a) Prove that

$$2\cot 2x + \tan x \equiv \cot x \qquad x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise, solve, for $-\pi \leqslant x < \pi$,

$$6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2$$

Give your answers to 3 decimal places.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

a) $2\cot 2x + \tan x \equiv \cot x$

$$\boxed{\tan 2x = \frac{2\tan x}{1-\tan^2 x}}$$

$$2\left(\frac{1-\tan^2 x}{2\tan x}\right) + \tan x$$

$$\frac{2 - 2\tan^2 x}{2\tan x} + \tan x$$

$$\frac{1-\tan^2 x}{\tan x} + \tan x$$

$$\frac{1-\tan^2 x}{\tan x} + \frac{\tan^2 x}{\tan x}$$

$$\frac{1}{\tan x}$$

$$\underline{\underline{\cot x}}$$

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Question 8 continued

b/ $3\cot x = \operatorname{cosec}^2 x - 2$

$$\boxed{1 + \cot^2 x = \operatorname{cosec}^2 x}$$

$$3\cot x = 1 + \cot^2 x - 2$$
$$3\cot x = \cot^2 x - 1$$
$$0 = \cot^2 x - 3\cot x - 1$$

$a=1 \quad b=-3 \quad c=-1$

$$\cot x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{13}}{2}$$

$\cot x = 3.302775638$ or -0.3027756377

$\tan x = \frac{1}{3.302775638}$ or $\frac{1}{-0.3027756377}$

$x = 0.294, -2.848, -1.277, 1.865$

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9. The amount of an antibiotic in the bloodstream, from a given dose, is modelled by the formula

$$x = De^{-0.2t}$$

where x is the amount of the antibiotic in the bloodstream in milligrams, D is the dose given in milligrams and t is the time in hours after the antibiotic has been given.

A first dose of 15 mg of the antibiotic is given.

(a) Use the model to find the amount of the antibiotic in the bloodstream 4 hours after the dose is given. Give your answer in mg to 3 decimal places.

(2)

A second dose of 15 mg is given 5 hours after the first dose has been given. Using the same model for the second dose,

(b) show that the **total** amount of the antibiotic in the bloodstream 2 hours after the second dose is given is 13.754 mg to 3 decimal places.

(2)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the total amount of the antibiotic in the bloodstream is 7.5 mg.

(c) Show that $T = a\ln\left(b + \frac{b}{e}\right)$, where a and b are integers to be determined.

(4)

a/ $x = De^{-0.2t}$

$x = 15e^{-0.2(4)}$

$= 6.740$ mg (3dp)

b/ total $= 15e^{-0.2(7)} + 15e^{-0.2(2)}$

$= 13.754$ (3dp) mg.

c/ $7.5 = 15e^{-0.2(5+T)} + 15e^{-0.2T}$

$\frac{1}{2} = e^{-0.2(5+T)} + e^{-0.2T}$

$\frac{1}{2} = e^{-1-0.2T} + e^{-0.2T}$

$\frac{1}{2} = (e^{-1})(e^{-0.2T}) + e^{-0.2T}$

$\frac{1}{2} = e^{-0.2T}(e^{-1} + 1)$

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Question 9 continued

$$\frac{1}{2(e^{-1}+1)} = e^{-0.2T}$$

$$\ln\left(\frac{1}{2e^{-1}+2}\right) = -0.2T$$

$$\ln(2e^{-1}+2)^{-1} = -0.2T$$

$$-\ln(2e^{-1}+2) = -0.2T$$

$$\ln(2e^{-1}+2) = 0.2T$$

$$5\ln(2e^{-1}+2) = T$$

$$5\ln\left(\frac{2}{e}+2\right) = T$$

$\underline{\underline{a=5}}$ $\underline{\underline{b=2}}$