Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 5 / 0 1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Monday 16 June 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
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7	
8	
9	
Total	

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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PEARSON

1. The curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{4x+1}{x-2}, \quad x > 2$$

(a) Show that

$$\mathrm{f}'(x) = \frac{-9}{(x-2)^2}$$

(3)

Given that P is a point on C such that $\mathrm{f}'(x) = -1$,

(b) find the coordinates of P.

(3)

a) $f(x) = \frac{4x+1}{x-2}$

$f'(x) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$u = 4x+1 \qquad v = x-2$

$\frac{du}{dx} = 4 \qquad \frac{dv}{dx} = 1$

$f'(x) = \frac{4(x-2) - 1(4x+1)}{(x-2)^2}$

$= \frac{4x-8-4x-1}{(x-2)^2}$

$= \frac{-9}{(x-2)^2}$

b) P is where $f'(x) = -1$

$\frac{-9}{(x-2)^2} = -1$

$-9 = -1(x-2)^2$

$9 = x^2 - 4x + 4$

$0 = x^2 - 4x - 5$

$0 = (x-5)(x+1)$

$x = 5 \qquad x = -1$

$x > 2 \qquad \therefore x = 5$

Question 1 continued

$$y = \frac{4x+1}{x-2}$$

when $x=5$

$$y = \frac{4(5)+1}{(5)-2}$$

$$= \frac{21}{3}$$

$$= 7$$

P is at $(5,7)$

Q1

(Total 6 marks)

Leave blank

2. Find the exact solutions, in their simplest form, to the equations

(a) $2\ln(2x + 1) - 10 = 0$

(2)

(b) $3^x e^{4x} = e^7$

(4)

2a) $2\ln(2x+1) - 10 = 0$

$2\ln(2x+1) = 10$

$\ln(2x+1) = 5$

$2x+1 = e^5$

$2x = e^5 - 1$

$x = \frac{e^5 - 1}{2}$

2b) $3^x e^{4x} = e^7$

$3^x = \frac{e^7}{e^{4x}}$

$3^x = e^{7-4x}$

$\ln 3^x = \ln e^{7-4x}$

$x \ln 3 = 7 - 4x$

$x\ln 3 + 4x = 7$

$x(\ln 3 + 4) = 7$

$x = \frac{7}{\ln(3) + 4}$

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Question 2 continued

Q2

(Total 6 marks)

P 4 3 1 6 4 A 0 5 3 2

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3. The curve C has equation $x = 8y \tan 2y$

The point P has coordinates $\left(\pi, \frac{\pi}{8}\right)$

(a) Verify that P lies on C.

(1)

(b) Find the equation of the tangent to C at P in the form $ay = x + b$, where the constants a and b are to be found in terms of π.

(7)

3a) $x = 8y \tan 2y$

$\left(\pi, \frac{\pi}{8}\right)$ (x, y) $\quad \pi = 8\left(\frac{\pi}{8}\right) \tan\left(2\left(\frac{\pi}{8}\right)\right)$

$\pi = \pi \tan \frac{\pi}{4}$

$\pi = \pi$

$\left(\pi, \frac{\pi}{8}\right)$ lies on C

3b)

$x = 8y \tan 2y$

$\frac{dx}{dy} = v\frac{du}{dy} + u\frac{dv}{dy}$

$u = 8y \qquad v = \tan 2y$

$\frac{du}{dy} = 8 \qquad \frac{dv}{dy} = 2\sec^2 2y$

$\frac{dx}{dy} = 8\tan 2y + 16y\sec^2 2y$

when $y = \frac{\pi}{8}$

$\frac{dx}{dy} = 8\tan\left(2\frac{\pi}{8}\right) + 16\left(\frac{\pi}{8}\right)\sec^2\left(2\cdot\frac{\pi}{8}\right)$

$= 8\tan\left(\frac{\pi}{4}\right) + \frac{2\pi}{\cos^2\left(\frac{\pi}{4}\right)}$

$= 8 + 4\pi$

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Question 3 continued

$$\frac{dx}{dy} = 8 + 4\pi$$

$$\frac{dy}{dx} = \frac{1}{8+4\pi}$$

$\left(\pi, \frac{\pi}{8}\right)$
$x \quad y$

$$y = mx + c$$

$$\frac{\pi}{8} = \frac{1}{8+4\pi}(\pi) + c$$

$$\frac{\pi}{8} = \frac{\pi}{8+4\pi} + c$$

$$c = \frac{\pi}{8} - \frac{\pi}{8+4\pi}$$

$$y = \frac{1}{8+4\pi}x + \frac{\pi}{8} - \frac{\pi}{8+4\pi}$$

$$(8+4\pi)y = x + \frac{(8+4\pi)\pi}{8} - \pi$$

$$= x + \frac{8\pi + 4\pi^2}{8} - \frac{8\pi}{8}$$

$$(8+4\pi)y = x + \frac{4\pi^2}{8}$$

$$\underline{\underline{(8+4\pi)y = x + \frac{\pi^2}{2}}}$$

Q3

(Total 8 marks)

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4.

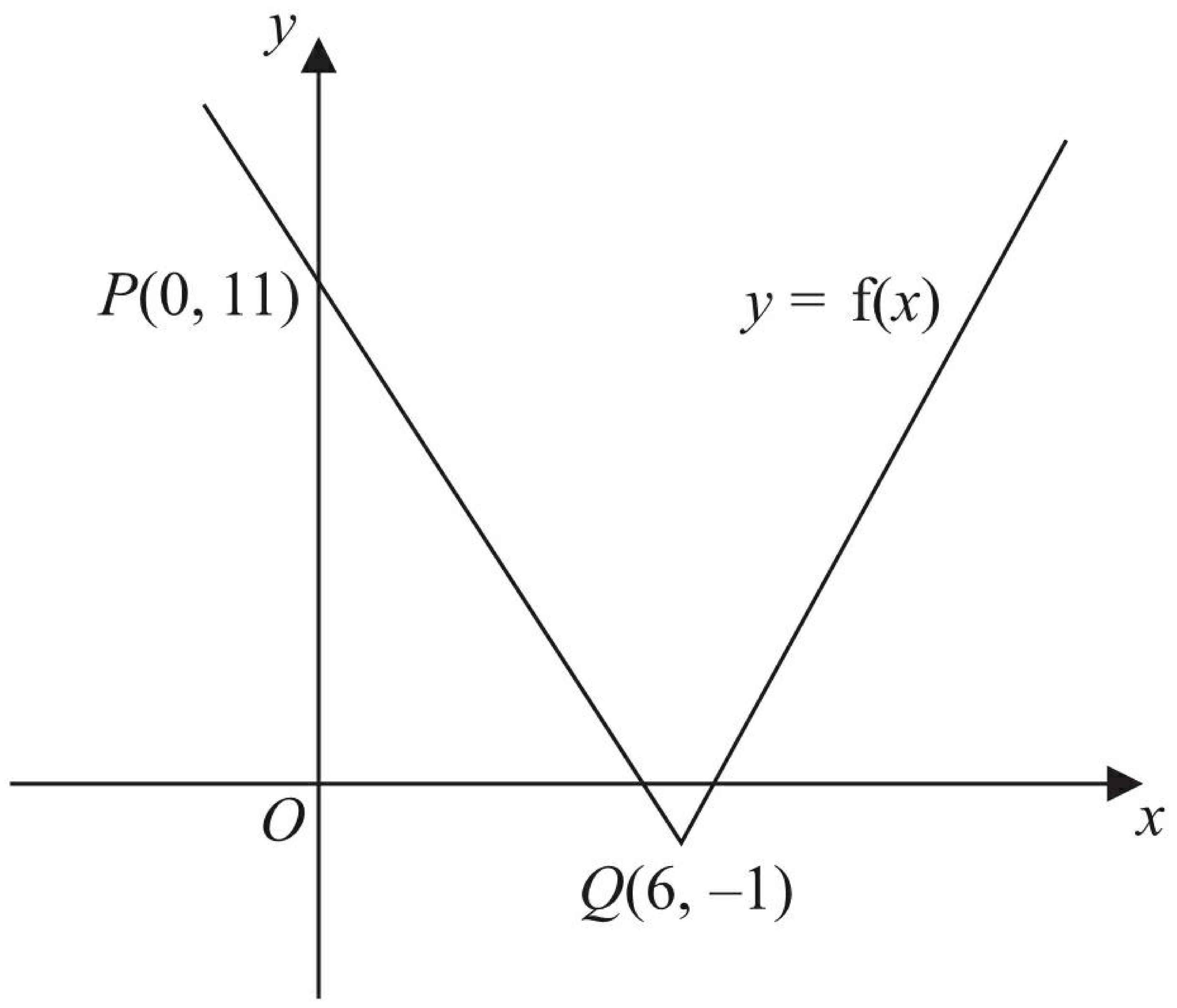


Figure 1

Figure 1 shows part of the graph with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $Q(6, -1)$.

The graph crosses the y-axis at the point $P(0, 11)$.

Sketch, on separate diagrams, the graphs of

(a) $y = |\mathrm{f}(x)|$ **(2)**

(b) $y = 2\mathrm{f}(-x) + 3$ **(3)**

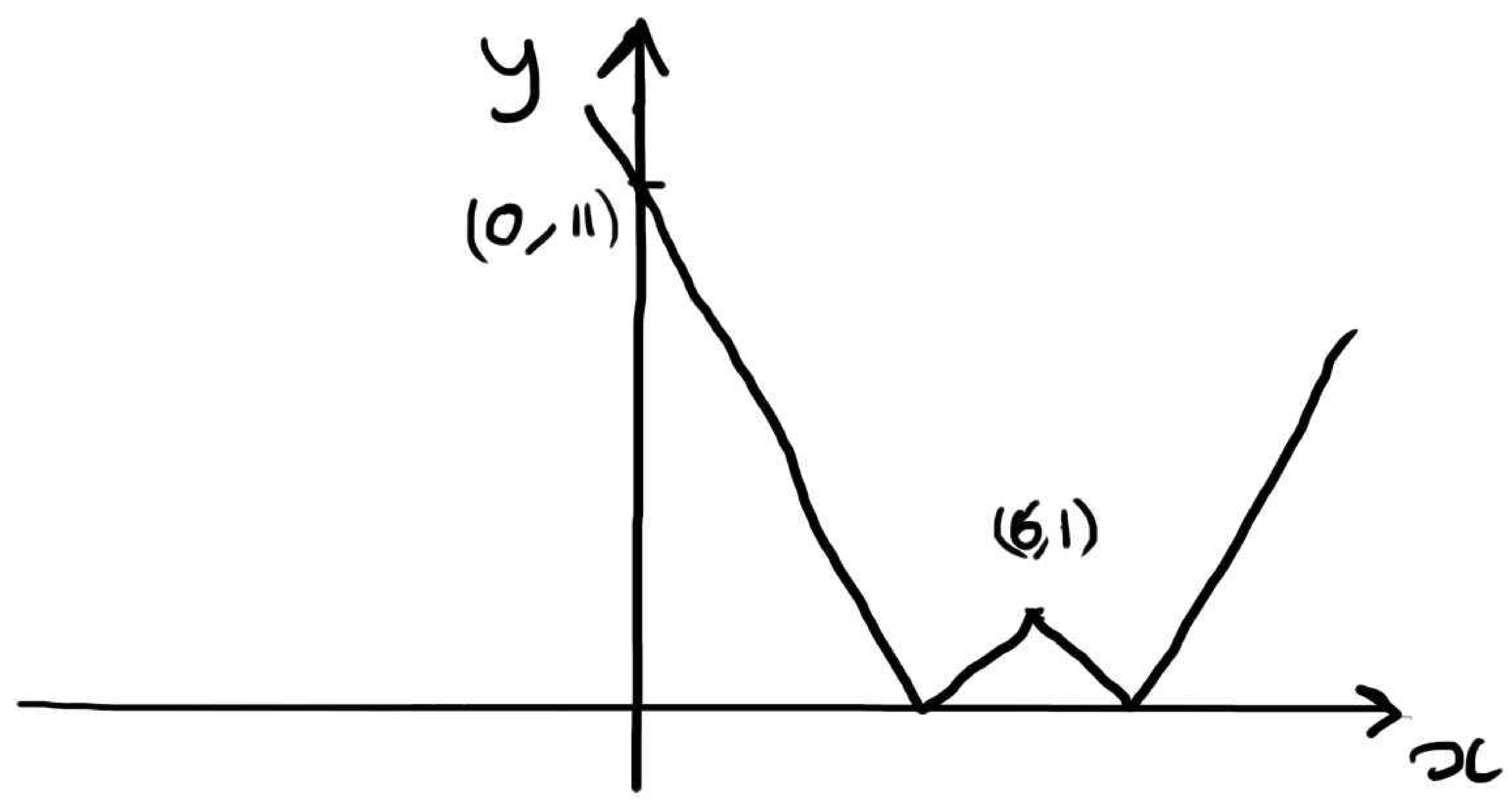
On each diagram, show the coordinates of the points corresponding to P and Q.

Given that $\mathrm{f}(x) = a|x - b| - 1$, where a and b are constants,

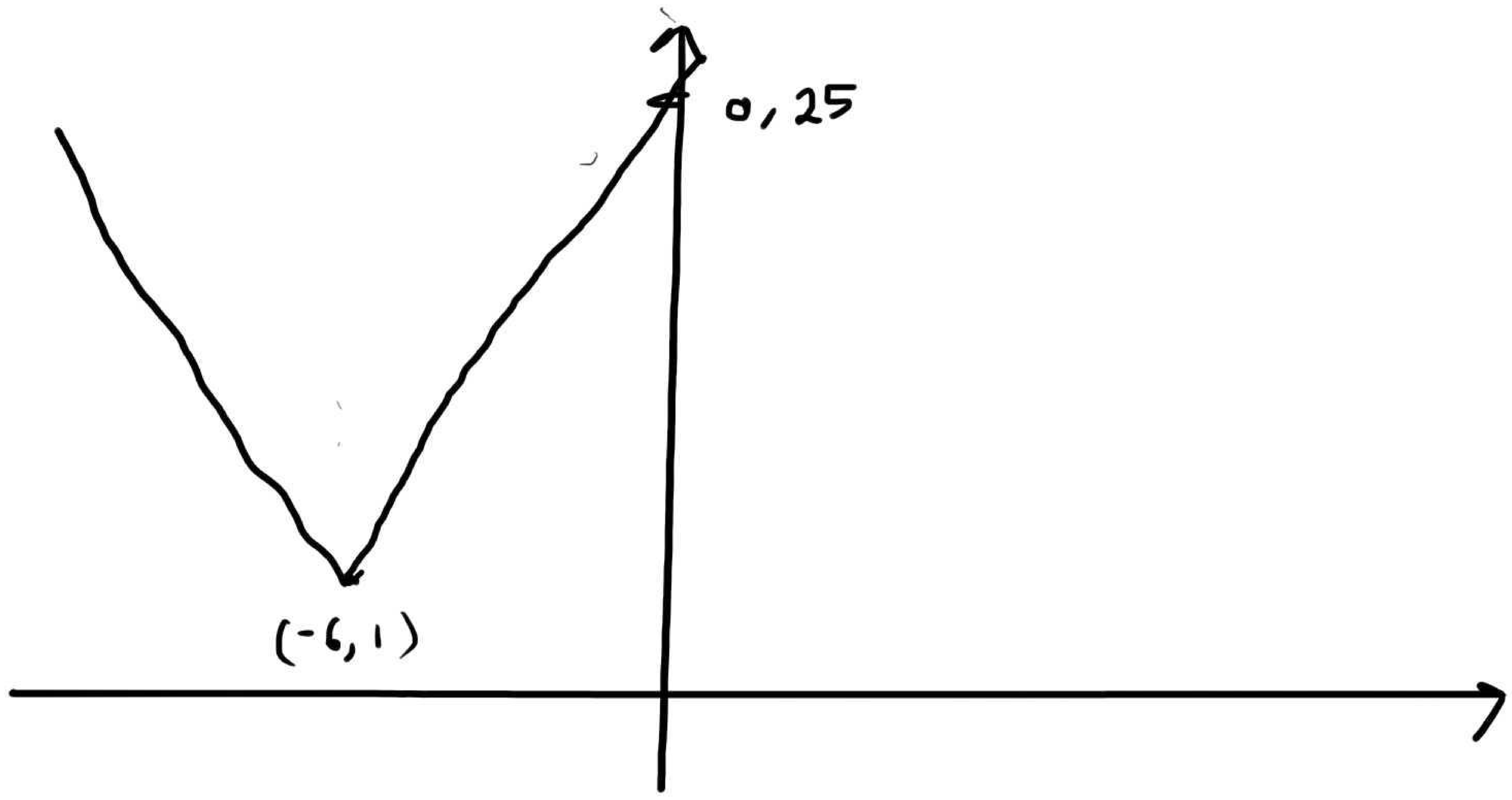
(c) state the value of a and the value of b. **(2)**

Question 4 continued

a)



b)



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Question 4 continued

Question 4 continued

c) $f(x) = a|x-b| - 1$

$(0, 11)$ $(6, -1)$

$y = a(x-b) - 1$ and $y = -a(x-b) - 1$

both points lie on line with negative gradient

$(0,11)$ $11 = -a(-b) - 1$
$12 = ab$

$(6,-1)$ $-1 = -a(6-b) - 1$
$0 = -a(6-b)$

$a \neq 0$ $\therefore$ $b = 6$

$a = \frac{12}{6}$

$a = 2$

(Total 7 marks)

Q4

P 4 3 1 6 4 A 0 1 1 3 2

Turn over

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5.

$$g(x) = \frac{x}{x+3} + \frac{3(2x+1)}{x^2+x-6}, \quad x > 3$$

(a) Show that $g(x) = \dfrac{x+1}{x-2}, \quad x > 3$ **(4)**

(b) Find the range of g. **(2)**

(c) Find the exact value of a for which $g(a) = g^{-1}(a)$. **(4)**

5a) $\dfrac{x}{x+3} + \dfrac{3(2x+1)}{x^2+x-6}$

$\dfrac{x}{x+3} + \dfrac{3(2x+1)}{(x+3)(x-2)}$

$\dfrac{x(x-2)}{(x+3)(x-2)} + \dfrac{3(2x+1)}{(x+3)(x-2)}$

$\dfrac{x^2-2x+6x+3}{(x+3)(x-2)}$

$\dfrac{x^2+4x+3}{(x+3)(x-2)}$

$\dfrac{\cancel{(x+3)}(x+1)}{\cancel{(x+3)}(x-2)}$

$\dfrac{x+1}{x-2}$

b) $x > 3$, when $x = 3$ $y = 4$

as x increases $y \to 1$

$\therefore$ $1 < g(x) < 4$

P 4 3 1 6 4 A 0 1 2 3 2

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Question 5 continued

c) $g(x) = \dfrac{x+1}{x-2}$

$y = \dfrac{x+1}{x-2}$

$y(x-2) = x+1$

$xy - 2y = x+1$

$xy - x = 1+2y$

$x(y-1) = 1+2y$

$x = \dfrac{1+2y}{y-1}$

$g^{-1}(x) = \dfrac{1+2x}{x-1}$

$g^{-1}(x) = g(x)$

$\dfrac{1+2x}{x-1} = \dfrac{x+1}{x-2}$

$(1+2x)(x-2) = (x+1)(x-1)$

$2x^2 + x - 4x - 2 = x^2 + x - x - 1$

$2x^2 - 3x - 2 = x^2 - 1$

$x^2 - 3x - 1 = 0$

$a=1 \quad b=-3 \quad c=-1$

$x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$

$= \dfrac{3 \pm \sqrt{9+4}}{2}$

$= \dfrac{3 \pm \sqrt{13}}{2}$

$x > 3 \quad \therefore \quad x = \dfrac{3+\sqrt{13}}{2}$

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Question 5 continued

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Question 5 continued

Q5

(Total 10 marks)

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6.

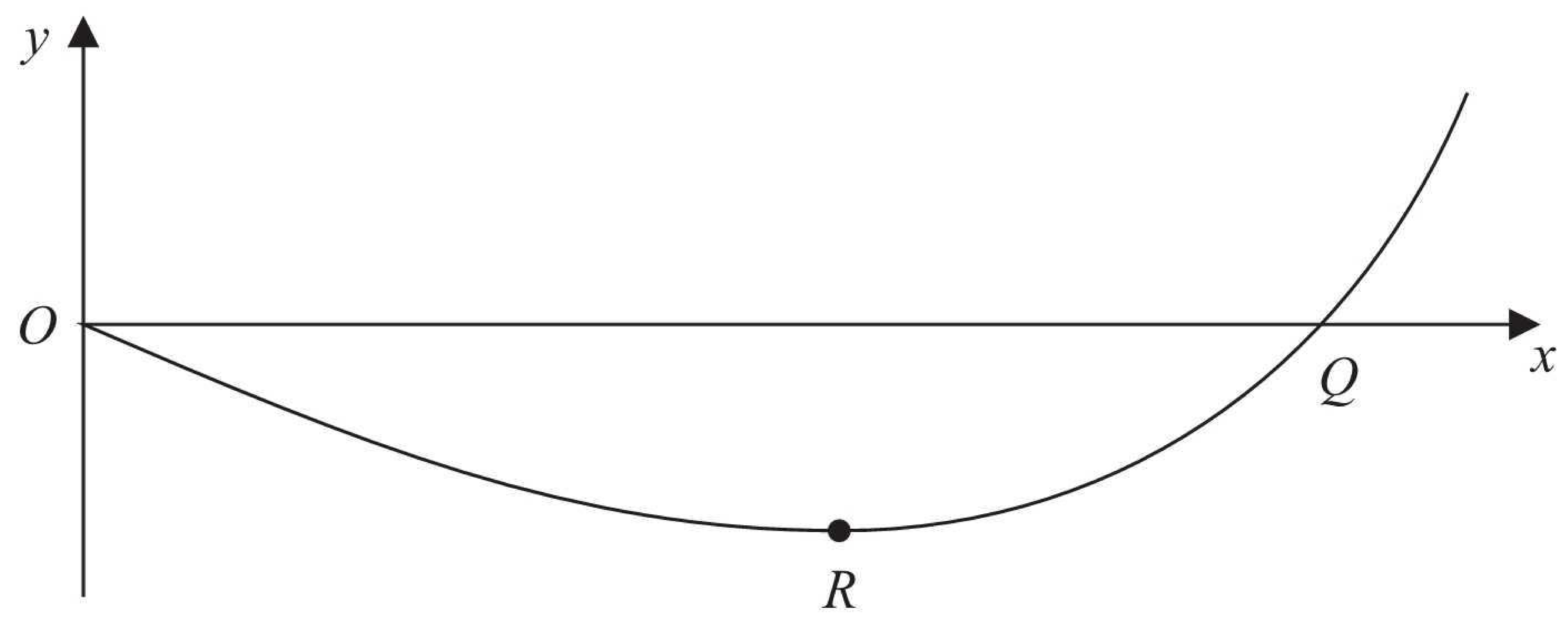

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 2\cos\left(\frac{1}{2}x^2\right) + x^3 - 3x - 2$$

The curve crosses the x-axis at the point Q and has a minimum turning point at R.

(a) Show that the x coordinate of Q lies between 2.1 and 2.2

(2)

(b) Show that the x coordinate of R is a solution of the equation

$$x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$$

(4)

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{2}{3}x_n\sin\left(\frac{1}{2}x_n^{\ 2}\right)}, \quad x_0 = 1.3$$

(c) find the values of x_1 and x_2 to 3 decimal places.

(2)

$y = 2\cos(\frac{1}{2}x^2) + x^3 - 3x - 2$

6a) x coordinate of Q is where $y=0$

$y = 2\cos(\frac{1}{2}x^2) + x^3 - 3x - 2$

when $x = 2.1$ $\quad y = 2\cos(\frac{1}{2}(2.1)^2) + (2.1)^3 - 3(2.1) - 2$

$y = -0.22$ (2dp)

when $x = 2.2$ $\quad y = 2\cos(\frac{1}{2}(2.2)^2) + (2.2)^3 - 3(2.2) - 2$

$y = 0.55$ (2dp)

change of sign $\therefore$ coordinate lies between 2.1 and 2.2

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Question 6 continued

b) x coordinate of R is where $\frac{dy}{dx} = 0$

$$y = 2\cos\left(\tfrac{1}{2}x^2\right) + x^3 - 3x - 2$$

$$\frac{dy}{dx} = -2x\sin\left(\tfrac{1}{2}x^2\right) + 3x^2 - 3$$

$$0 = -2x\sin\left(\tfrac{1}{2}x^2\right) + 3x^2 - 3$$

$$3x^2 = 2x\sin\left(\tfrac{1}{2}x^2\right) + 3$$

$$x^2 = \tfrac{2}{3}x\sin\left(\tfrac{1}{2}x^2\right) + 1$$

$$x = \sqrt{\tfrac{2}{3}x\sin\left(\tfrac{1}{2}x^2\right) + 1}$$

$$x = \sqrt{1 + \tfrac{2}{3}x\sin\left(\tfrac{1}{2}x^2\right)}$$

c) $$x_{n+1} = \sqrt{1 + \tfrac{2}{3}(x_n)\sin\left(\tfrac{1}{2}(x_n)^2\right)}$$

$x_0 = 1.3$

$x_1 = 1.284$

$x_2 = 1.276$

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Question 6 continued

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Question 6 continued

Q6

(Total 8 marks)

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7. (a) Show that

$$\operatorname{cosec} 2x + \cot 2x = \cot x, \qquad x \neq 90n°, \qquad n \in \mathbb{Z}$$

(5)

(b) Hence, or otherwise, solve, for $0 \leqslant \theta < 180°$,

$$\operatorname{cosec}(4\theta + 10°) + \cot(4\theta + 10°) = \sqrt{3}$$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

7a) $\dfrac{1}{\sin 2x} + \dfrac{\cos 2x}{\sin 2x} = \cot x$

$\dfrac{1 + \cos 2x}{\sin 2x} = \cot x$

$\dfrac{1 + \cos 2x}{2\sin x\cos x} = \cot x$

$\dfrac{1 + \cos^2 x - \sin^2 x}{2\sin x \cos x} = \cot x$

$\dfrac{1 + \cos^2 x - (1 - \cos^2 x)}{2\sin x \cos x} = \cot x$

$\dfrac{1 + \cos^2 x - 1 + \cos^2 x}{2\sin x \cos x} = \cot x$

$\dfrac{2\cos^2 x}{2\sin x \cos x} = \cot x$

$\dfrac{2\cos x}{2\sin x} = \cot x$

$\cot x = \cot x$

shown.

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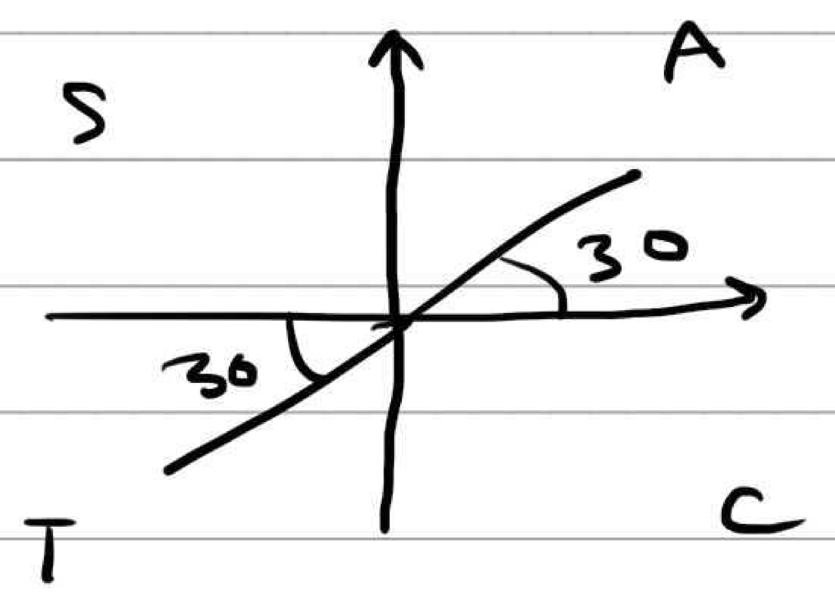
Question 7 continued

b)

$$\cot(2\theta + 5) = \sqrt{3}$$

$$\tan(2\theta + 5) = \frac{1}{\sqrt{3}}$$

$$2\theta + 5 = 30^\circ, 210^\circ$$



$$\theta = 12.5^\circ, 102.5^\circ$$

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Question 7 continued

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Question 7 continued

(Total 10 marks)

Q7

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8. A rare species of primrose is being studied. The population, P, of primroses at time t years after the study started is modelled by the equation

$$P = \frac{800e^{0.1t}}{1 + 3e^{0.1t}}, \qquad t \geqslant 0, \quad t \in \mathbb{R}$$

(a) Calculate the number of primroses at the start of the study.
(2)

(b) Find the exact value of t when $P = 250$, giving your answer in the form $a \ln(b)$ where a and b are integers.
(4)

(c) Find the exact value of $\frac{dP}{dt}$ when $t = 10$. Give your answer in its simplest form.
(4)

(d) Explain why the population of primroses can never be 270
(1)

8a) At start of study $t=0$

$$P = \frac{800e^0}{1+3e^0}$$

$$= \frac{800}{4}$$

$$= \underline{\underline{200}}$$

b) $P=250$

$$250 = \frac{800e^{0.1t}}{1+3e^{0.1t}}$$

$$250(1+3e^{0.1t}) = 800e^{0.1t}$$

$$250 + 750e^{0.1t} = 800e^{0.1t}$$

$$250 = 50e^{0.1t}$$

$$5 = e^{0.1t}$$

$$\ln 5 = 0.1t$$

$$\underline{\underline{10 \ln 5}} = t$$

c) $$\frac{dP}{dt} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Question 8 continued

$u = 800e^{0.1t}$ $\quad\quad v = 1 + 3e^{0.1t}$

$\frac{du}{dt} = 80e^{0.1t}$ $\quad\quad \frac{dv}{dt} = 0.3e^{0.1t}$

$$\frac{dP}{dt} = \frac{(1+3e^{0.1t})\,80e^{0.1t} - (800e^{0.1t})(0.3e^{0.1t})}{(1+3e^{0.1t})^2}$$

$$= \frac{80e^{0.1t} + 240e^{0.2t} - 240e^{0.2t}}{1 + 6e^{0.1t} + 9e^{0.2t}}$$

$$= \frac{80e^{0.1t}}{1+6e^{0.1t}+9e^{0.2t}}$$

when $t = 10$

$$\frac{dP}{dt} = \frac{80e}{1+6e+9e^2}$$

d) $\quad P = \frac{800e^{0.1t}}{1+3e^{0.1t}}$

as the value of t increases it will get closer to $\frac{800}{3}$

$$\frac{800}{3} < 270$$

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Question 8 continued

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Question 8 continued

Q8

(Total 11 marks)

9. (a) Express $2\sin\theta - 4\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the value of α to 3 decimal places.

(3)

$$\mathrm{H}(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$$

Find

(b) (i) the maximum value of $\mathrm{H}(\theta)$,

(ii) the smallest value of θ, for $0 \leqslant \theta < \pi$, at which this maximum value occurs.

(3)

Find

(c) (i) the minimum value of $\mathrm{H}(\theta)$,

(ii) the largest value of θ, for $0 \leqslant \theta < \pi$, at which this minimum value occurs.

(3)

9a) $R\sin(\theta-\alpha) = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$

$= 2\sin\theta - 4\cos\theta$

$R\cos\alpha = 2$

$R\sin\alpha = 4$

$\tan\alpha = 2$ $\quad R^2 = 2^2 + 4^2$

$\alpha = \underline{1.107}$ $\quad R^2 = 20$

$R = \underline{\sqrt{20}} = 2\sqrt{5}$

$2\sin\theta - 4\cos\theta = \underline{2\sqrt{5}\cdot\sin(\theta - 1.107)}$

b)

i) $H(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$

$= 4 + 5(2\sqrt{5}\sin(3\theta - 1.107))^2$

max value of $\sin(3\theta - 1.107) = 1$

$\therefore$ max $H(\theta) = 4 + 5(2\sqrt{5})^2$

$= \underline{104}$

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Question 9 continued

ii) Max value of $\sin(x)$ is when $x = \pi/2$

$$3\theta - 1.107 = \pi/2$$

$$3\theta = 2.677...$$

$$\theta = 0.893 \text{ (3sf)}$$

c) $H(\theta) = 4 + 5(2\sqrt{5}\sin(3\theta - 1.107))^2$

min value $H(\theta) = 4 + 5(0)$

$$= 4$$

ii) $\sin(x) = 0$ at $0, \pi, 2\pi ...$

$$3\theta - 1.107 = 0$$

$$3\theta = 1.107$$

$$\theta = 0.369$$

$$3\theta - 1.107 = 2\pi$$

$$\theta = \frac{2\pi + 1.107}{3}$$

$$\theta = 2.46 \text{ (3sf)}$$

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Question 9 continued

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Question 9 continued

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Question 9 continued

Q9

(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

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