Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	5	/	0	1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Thursday 13 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only		

Team Leader's use only		

Question Number	Leave Blank
1	
2	
3	
4	
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6	
7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. Given that

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \quad x \neq \pm 2$$

find the values of the constants a, b, c, d and e.

(4)

$$\begin{array}{r|l} & 3x^2 - 2x + 7 \\ \hline x^2 + 0x - 4 & 3x^4 - 2x^3 - 5x^2 + 0x - 4 \\ & 3x^4 + 0x^3 - 12x^2 \\ & \quad -2x^3 + 7x^2 + 0x \\ & \quad -2x^3 + 0x^2 + 8x \\ & \qquad 7x^2 - 8x - 4 \\ & \qquad 7x^2 + 0x - 28 \\ & \qquad\quad -8x + 24 \end{array}$$

$$3x^2 - 2x + 7 + \frac{-8x + 24}{x^2 - 4}$$

$a = 3 \quad b = -2 \quad c = 7 \quad d = -8 \quad e = 24$

2. Given that

$$f(x) = \ln x, \quad x > 0$$

sketch on separate axes the graphs of

(i) $y = f(x)$,

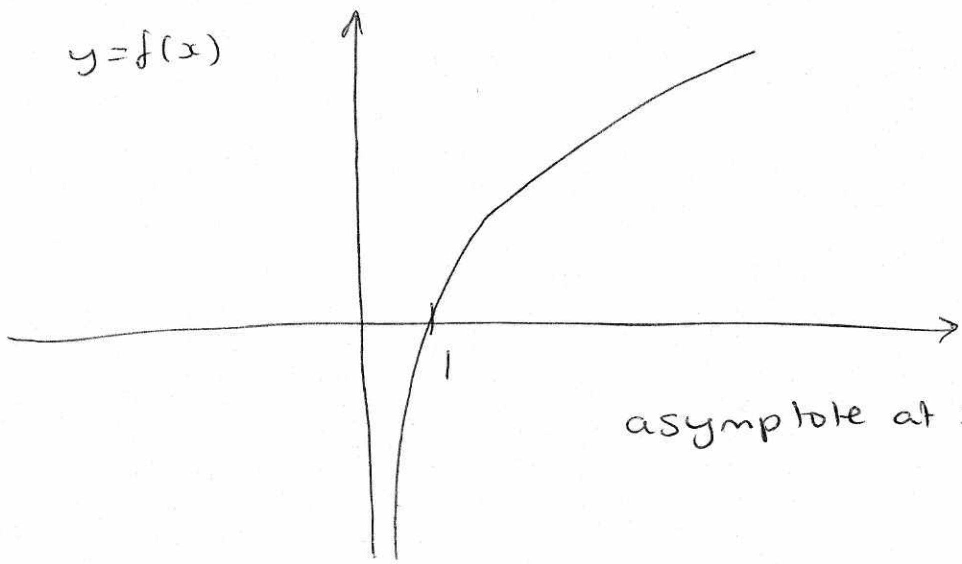
(ii) $y = |f(x)|$,

(iii) $y = -f(x - 4)$.

Show, on each diagram, the point where the graph meets or crosses the x-axis.
In each case, state the equation of the asymptote.

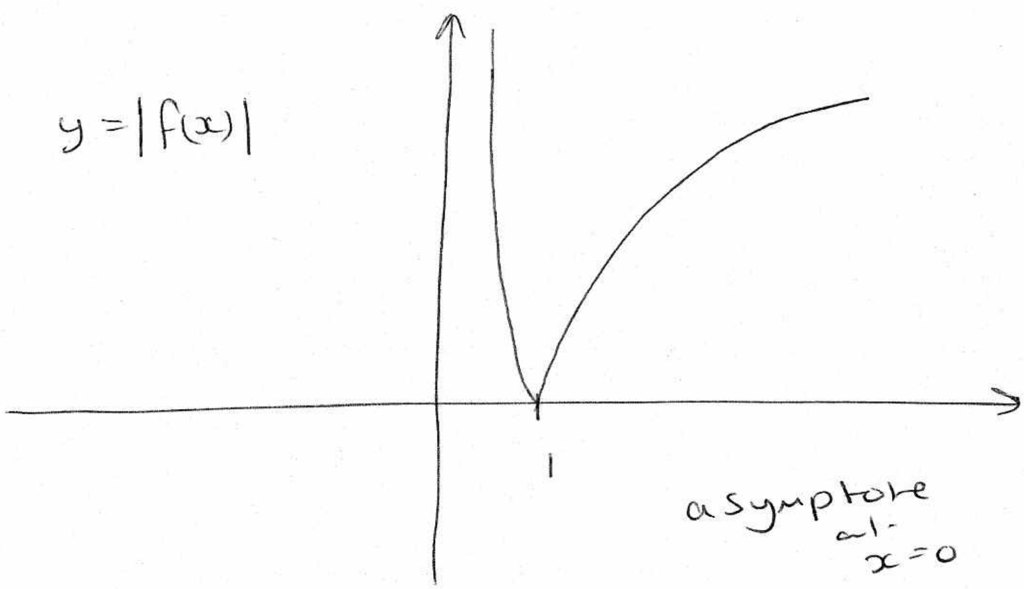
(7)

i/ $y = f(x)$



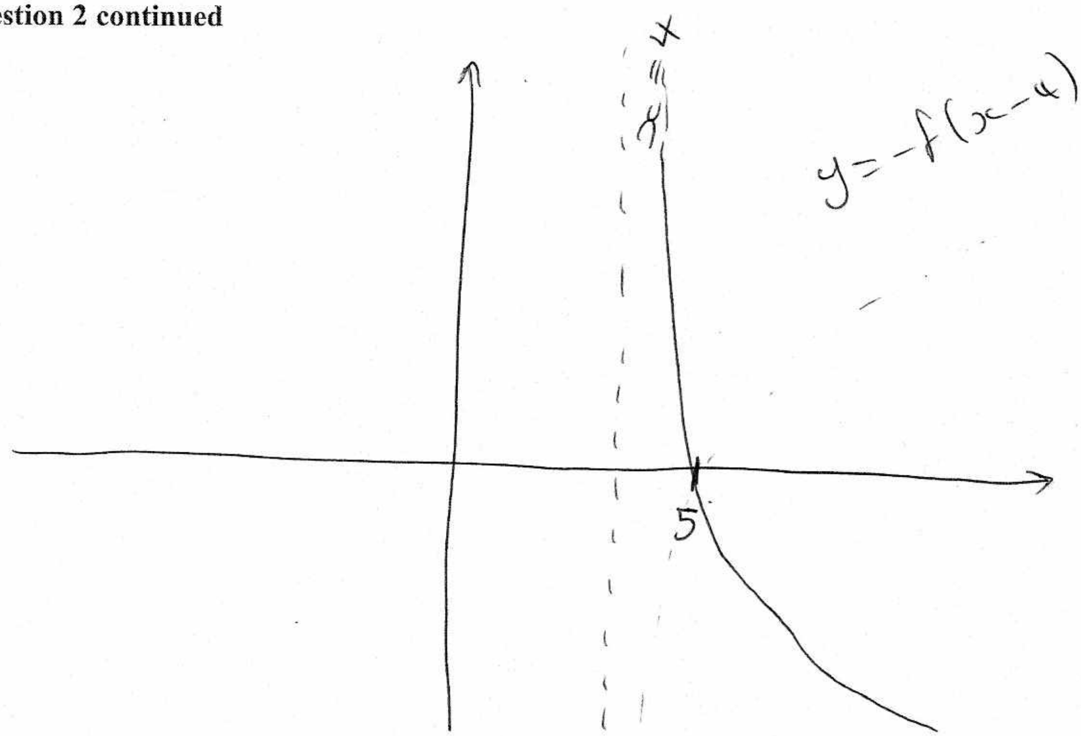
asymptote at $x = 0$

ii/ $y = |f(x)|$



asymptote at $x = 0$

Question 2 continued



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3. Given that

$$2\cos(x + 50)° = \sin(x + 40)°$$

(a) Show, without using a calculator, that

$$\tan x° = \frac{1}{3}\tan 40°$$

(4)

(b) Hence solve, for $0 \leqslant \theta < 360$,

$$2\cos(2\theta + 50)° = \sin(2\theta + 40)°$$

giving your answers to 1 decimal place.

(4)

$\sin 40 = \cos 50 \qquad \cos 40 = \sin 50$

a) $\cos(A+B) = \cos A\cos B - \sin A\sin B$
$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$2\cos(x+50) = \sin(x+40)$
$2(\cos x\cos 50 - \sin x\sin 50) = \sin x\cos 40 + \cos x\sin 40$
$2\cos x(\cos 50) - 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$
$2\cos x\sin 40 - 2\sin x\cos 40 = \sin x\cos 40 + \cos x\sin 40$
$\cos x\sin 40 = 3\sin x\cos 40$
$\sin 40 = 3\tan x\cos 40$
$\tan 40 = 3\tan x$
$\frac{1}{3}\tan 40 = \tan x$

b) $\tan(2\theta) = \frac{1}{3}\tan 40$
$\tan(2\theta) = 0.279699871$
$2\theta = 15.62629958, 195.6262996,$
$375.6262996, 555.6262996$
$\theta = 7.8°, 97.8°, 187.8°, 277.8°$

4. $$f(x) = 25x^2e^{2x} - 16, \qquad x \in \mathbb{R}$$

(a) Using calculus, find the exact coordinates of the turning points on the curve with equation $y = f(x)$.

(5)

(b) Show that the equation $f(x) = 0$ can be written as $x = \pm\frac{4}{5}e^{-x}$

(1)

The equation $f(x) = 0$ has a root α, where $\alpha = 0.5$ to 1 decimal place.

(c) Starting with $x_0 = 0.5$, use the iteration formula

$$x_{n+1} = \frac{4}{5}e^{-x_n}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(d) Give an accurate estimate for α to 2 decimal places, and justify your answer.

(2)

a/ turning points where $\frac{dy}{dx} = 0$

$u = 25x^2 \qquad v = e^{2x}$

$\frac{du}{dx} = 50x \qquad \frac{dv}{dx} = 2e^{2x}$

$f'(x) = 50xe^{2x} + 50x^2e^{2x}$

$= 50e^{2x}(x + x^2)$

$= (50xe^{2x})(1 + x)$

$x = 0 \qquad x = -1$

$y = 25(0)^2e^{2(0)} - 16 \qquad y = 25(-1)^2e^{2(-1)}$

$= -16 \qquad = 25e^{-2}$

$(0, -16)$ and $(-1, 25e^{-2})$

b) $25x^2e^{2x} - 16 = 0$

$25x^2e^{2x} = 16$

$x^2e^{2x} = \frac{16}{25}$

Leave blank

Question 4 continued

$$(xe^x)^2 = \frac{16}{25}$$

$$xe^x = \pm\sqrt{\frac{16}{25}}$$

$$xe^x = \pm\frac{4}{5}$$

$$x = \pm\frac{4}{5e^x}$$

$$= \pm\frac{4}{5}e^{-x}$$

c/ $x_0 = 0.5$

$$x_1 = \frac{4}{5}e^{-(x_0)}$$

$x_1 = 0.485$
$x_2 = 0.492$
$x_3 = 0.489$

d/ $\alpha = 0.49$ to 2dp

x_2 and x_3 both round to 0.49 to 2dp

$f(0.485) = -0.487$
$f(0.495) = 0.485$

change of sign ∴ solution is 0.49 to 2dp

Leave blank

5. Given that

$$x = \sec^2 3y, \qquad 0 < y < \frac{\pi}{6}$$

(a) find $\frac{dx}{dy}$ in terms of y.

(2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

(4)

(c) Find an expression for $\frac{d^2y}{dx^2}$ in terms of x. Give your answer in its simplest form.

(4)

a/ $x = (\sec 3y)^2$

$\frac{dx}{dy} = 2\sec(3y) \times 3\sec(3y)\tan(3y)$

$= 6\sec^2(3y)\tan(3y)$

b/ $\frac{dy}{dx} = \frac{1}{6\sec^2(3y)\tan(3y)}$ $\qquad x = \sec^2(3y)$

$\frac{dy}{dx} = \frac{1}{6x\tan(3y)}$ $\qquad \boxed{1 + \tan^2 x = \sec^2 x}$

$= \frac{1}{6x\sqrt{x-1}}$ $\qquad \tan^2 3y = \sec^2 3y - 1$

$\tan 3y = \sqrt{\sec^2 3y - 1}$

$= \frac{1}{6x(x-1)^{\frac{1}{2}}}$ $\qquad = \sqrt{x-1}$

c/ $\frac{dy}{dx} = \frac{1}{6}x^{-1}(x-1)^{-\frac{1}{2}}$

$u = \frac{1}{6}x^{-1} \qquad v = (x-1)^{-\frac{1}{2}}$

$\frac{du}{dx} = -\frac{1}{6}x^{-2} \qquad \frac{dv}{dx} = -\frac{1}{2}(x-1)^{-\frac{3}{2}}$

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Question 5 continued

$$\frac{d^2y}{dx^2} = -\frac{1}{6}x^{-2}(x-1)^{-1/2} + \frac{1}{6}x^{-1}\left(-\frac{1}{2}(x-1)^{-3/2}\right)$$

$$= -\frac{1}{6}x^{-2}(x-1)^{-1/2} - \frac{1}{12}x^{-1}(x-1)^{-3/2}$$

$$= \frac{1}{12}x^{-2}(x-1)^{-3/2}\left(-2(x-1) - x\right)$$

$$= \frac{1}{12}x^{-2}(x-1)^{-3/2}\left(-2x+2-x\right)$$

$$= \frac{1}{12x^2(x-1)^{3/2}}(2-3x)$$

$$= \frac{2-3x}{12x^2(x-1)^{3/2}}$$

Leave blank

6. Find algebraically the exact solutions to the equations

(a) $\ln(4-2x)+\ln(9-3x)=2\ln(x+1), \qquad -1<x<2$

(5)

(b) $2^x e^{3x+1}=10$

Give your answer to (b) in the form $\dfrac{a+\ln b}{c+\ln d}$ where a, b, c and d are integers.

(5)

a) $\ln(4-2x)+\ln(9-3x)=2\ln(x+1)$

$\ln((4-2x)(9-3x))=\ln(x+1)^2$

$(4-2x)(9-3x)=(x+1)^2$

$(4-2x)(9-3x)=(x+1)(x+1)$

$36-12x-18x+6x^2=x^2+x+x+1$

$6x^2-30x+36=x^2+2x+1$

$5x^2-32x+35=0$

$(5x-7)(x-5)=0$

$x=\frac{7}{5} \qquad x=5$

$-1<x<2$

$\therefore \quad x=7/5$

b) $2^x e^{3x+1}=10$

$\ln(2^x e^{3x+1})=\ln 10$

$\ln 2^x+\ln e^{3x+1}=\ln 10$

$x\ln 2+3x+1=\ln 10$

~~$3x=\ln 10$~~

$x(\ln 2+3)=\ln(10)-1$

$x=\dfrac{\ln(10)-1}{\ln(2)+3}=\dfrac{-1+\ln(10)}{3+\ln(2)}$

Leave blank

7. The function f has domain $-2 \leqslant x \leqslant 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = f(x)$ is shown in Figure 1.

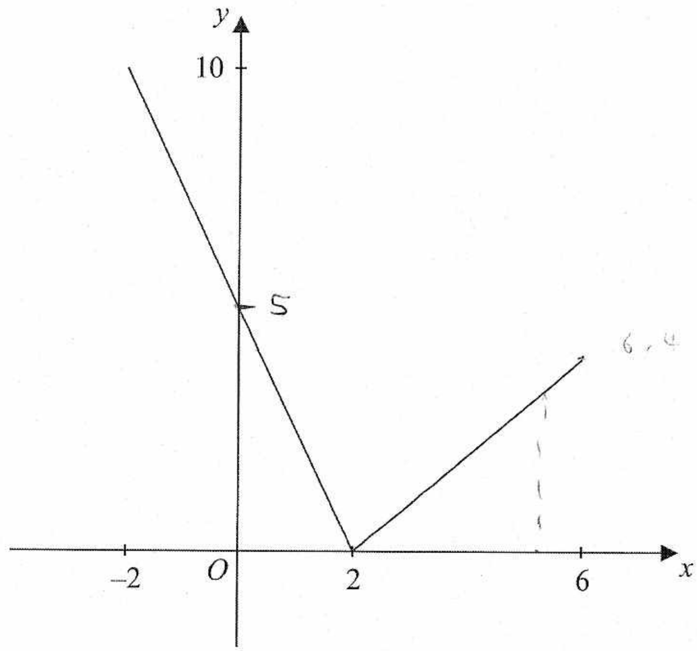

Figure 1

(a) Write down the range of f.
(1)

(b) Find ff(0).
(2)

The function g is defined by

$$g : x \rightarrow \frac{4 + 3x}{5 - x}, \qquad x \in \mathbb{R}, \qquad x \neq 5$$

(c) Find $g^{-1}(x)$
(3)

(d) Solve the equation $gf(x) = 16$
(5)

a/ $0 \leqslant f(x) \leqslant 10$

b/ $f(0) = 5$
$f(5) = \cancel{1.3} = \underline{\underline{3}}$

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Question 7 continued

c) $y = \dfrac{4+3x}{5-x}$

$x = \dfrac{4+3y}{5-y}$

$x(5-y) = 4+3y$

$5x - xy = 4+3y$

$5x-4 = 3y+xy$

$5x-4 = y(3+x)$

$\dfrac{5x-4}{3+x} = y$

$g^{-1}(x) = \dfrac{5x-4}{3+x}$

d) $g^{-1}(16) = \dfrac{5(16)-4}{3+(16)}$

$= 4$

$\therefore \quad f(x) = 4$

$x = 6 \quad$ or $\quad x = 2/5$

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8.

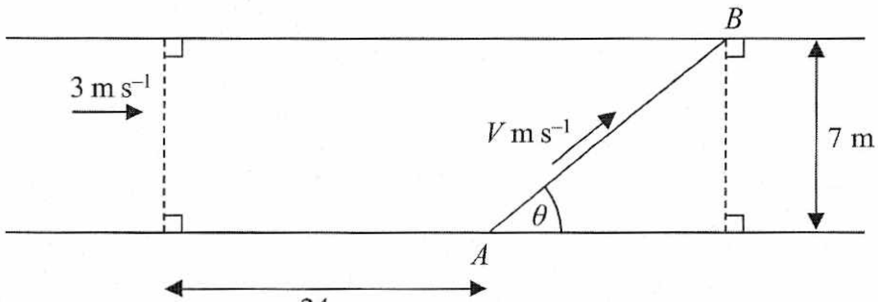

Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1}.
Kate is 24 m ahead of John when she starts to cross the road from the fixed point A.
John passes her as she reaches the other side of the road at a variable point B, as shown in Figure 2.
Kate's speed is V m s^{-1} and she moves in a straight line, which makes an angle θ, $0 < \theta < 150°$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24\sin\theta + 7\cos\theta}, \qquad 0 < \theta < 150°$$

(a) Express $24\sin\theta + 7\cos\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places. **(3)**

Given that θ varies,

(b) find the minimum value of V. **(2)**

Given that Kate's speed has the value found in part (b),

(c) find the distance AB. **(3)**

Given instead that Kate's speed is 1.68 m s^{-1},

(d) find the two possible values of the angle θ, given that $0 < \theta < 150°$. **(6)**

a) $24\sin\theta + 7\cos\theta$

$R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$R\sin\alpha = 24 \qquad R\cos\alpha = 7$

Question 8 continued

$\tan\alpha = \frac{24}{7}$ $\qquad$ $R^2 = 24^2 + 7^2$

$\qquad\qquad\qquad\qquad R = \sqrt{24^2 + 7^2}$

$\underline{\underline{\alpha = 73.74^\circ}}$ $\qquad\qquad\qquad = \underline{\underline{25}}$

$25\cos(\theta - 73.74)$

b/

$V = \frac{21}{25\cos(\theta - 73.74)}$

Max value of $25\cos(\theta - 73.74)$ is 25

$\therefore V_{max} = \frac{21}{25} = 0.84\ ms^{-1}$

c/ $\cos(\theta - 73.74) = 1$

$\theta - 73.74 = 0$

$\theta = 73.74$

$\sin\theta = \frac{O}{H}$

$\sin(73.74) = \frac{7}{AB}$

$AB = \frac{7}{\sin(73.74)}$

$= 7.29\ m\ (3sf)$

d/ $1.68 = \frac{21}{25\cos(\theta - 73.74)}$

$25\cos(\theta - 73.74) = 12.5$

$\cos(\theta - 73.74) = 1/2$

$\theta - 73.74 = 60, -60$

$\theta = 13.74, 133.74$