Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 5 / 0 1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Thursday 14 June 2012 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
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7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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PEARSON

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1. Express

$$\frac{2(3x+2)}{9x^2-4} - \frac{2}{3x+1}$$

as a single fraction in its simplest form.

(4)

$$\frac{2\cancel{(3x+2)}}{(3x-2)\cancel{(3x+2)}} - \frac{2}{3x+1}$$

$$\frac{2}{3x-2} - \frac{2}{3x+1}$$

$$\frac{2(3x+1)}{(3x-2)(3x+1)} - \frac{2(3x-2)}{(3x-2)(3x+1)}$$

$$\frac{2(3x+1)-2(3x-2)}{(3x-2)(3x+1)}$$

$$\frac{6x+2-6x+4}{(3x-2)(3x+1)}$$

$$\frac{6}{(3x-2)(3x+1)}$$

Leave blank

2. $$f(x) = x^3 + 3x^2 + 4x - 12$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)}, \qquad x \neq -3$$

(3)

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)}, \; n \geqslant 0$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1, x_2 and x_3.

(3)

The root of $f(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

(3)

2a) $x^3 + 3x^2 + 4x - 12 = 0$

$x^2(x+3) + 4x - 12 = 0$

$x^2(x+3) = 12 - 4x$

$x^2 = \dfrac{12-4x}{x+3}$

$x = \sqrt{\dfrac{12-4x}{x+3}}$

$x = \sqrt{\dfrac{4(3-x)}{3+x}}$

b) $x_0 = 1$

$x_1 = 1.41$

$x_2 = 1.20$

$x_3 = 1.31$

c/ $f(1.2715) = -8.21 \times 10^{-3}$

$f(1.2725) = 8.27 \times 10^{-3}$

Change of sign $\therefore \alpha = 1.272$ to 3dp

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3.

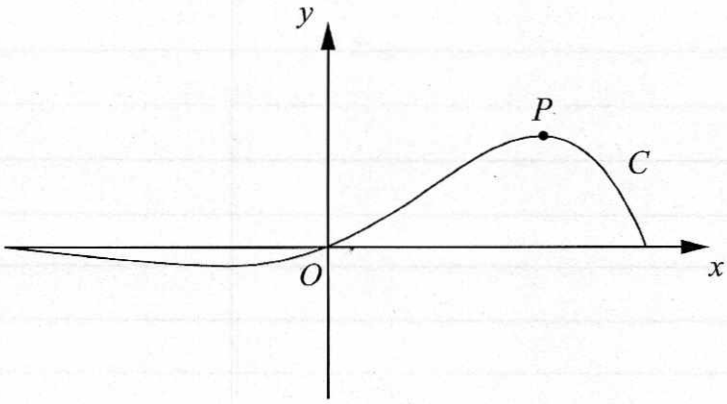

Figure 1

Figure 1 shows a sketch of the curve C which has equation

$$y = e^{x\sqrt{3}} \sin 3x\ , \quad -\frac{\pi}{3} \leqslant x \leqslant \frac{\pi}{3}$$

(a) Find the x coordinate of the turning point P on C, for which $x > 0$
Give your answer as a multiple of π.

(6)

(b) Find an equation of the normal to C at the point where $x = 0$

(3)

$u = e^{\sqrt{3}x}$ $\quad v = \sin 3x$

$\frac{du}{dx} = \sqrt{3}e^{\sqrt{3}x}$ $\quad \frac{dv}{dx} = 3\cos 3x$

$\frac{dy}{dx} = \sqrt{3}e^{\sqrt{3}x} \sin 3x + 3e^{\sqrt{3}x} \cos 3x$

turning point where $\frac{dy}{dx} = 0$

$0 = e^{\sqrt{3}x}(\sqrt{3}\sin 3x + 3\cos 3x)$

$0 = \sqrt{3}\sin 3x + 3\cos 3x$

$0 = \sqrt{3}\tan 3x + 3$

$\tan 3x = -\sqrt{3}$

$3x = \cancel{-60, 120, 300}$

$3x = -\frac{1}{3}\pi, \cancel{-\frac{4}{3}\pi}, \cancel{-\frac{7}{3}\pi}, \frac{2}{3}\pi, \cancel{\frac{5}{3}\pi}$

$x = -\frac{1}{9}\pi, \frac{2}{9}\pi$

$x > 0 \quad \therefore \quad x = \frac{2}{9}\pi$

Question 3 continued

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b/ $(0,0)$

when $x=0$

$$\frac{dy}{dx} = \sqrt{3}e^{\sqrt{3}(0)} \sin(3(0)) + 3e^{\sqrt{3}(0)} \cos(3(0))$$

$$= 3$$

$\therefore m = -1/3$

$$\underline{\underline{y = -1/3\, x}}$$

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4.

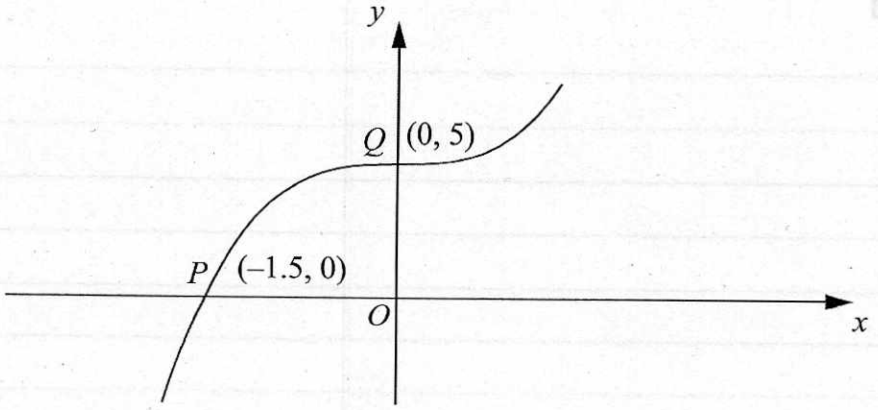


Figure 2

Figure 2 shows part of the curve with equation $y = f(x)$
The curve passes through the points $P(-1.5,\ 0)$ and $Q(0,\ 5)$ as shown.

On separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$

(2)

(b) $y = f(|x|)$

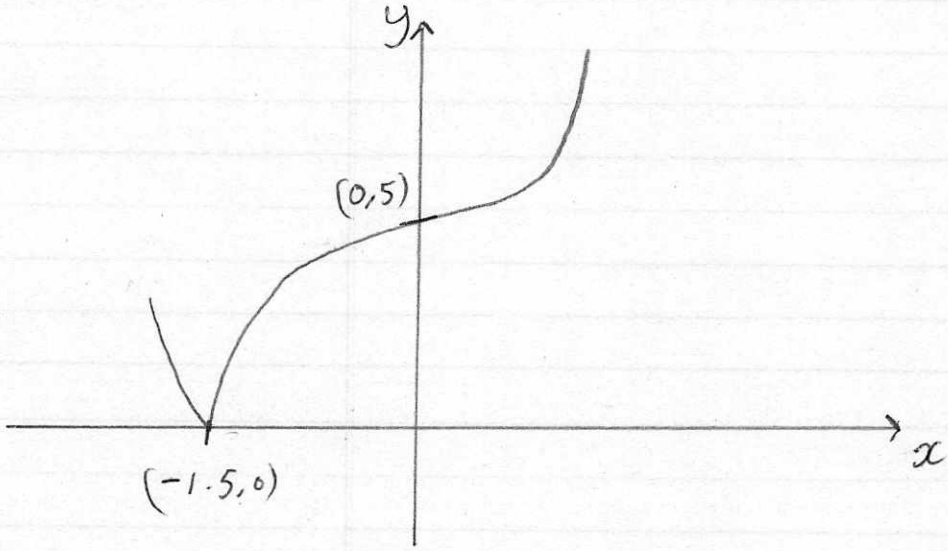
(2)

(c) $y = 2f(3x)$

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

a)



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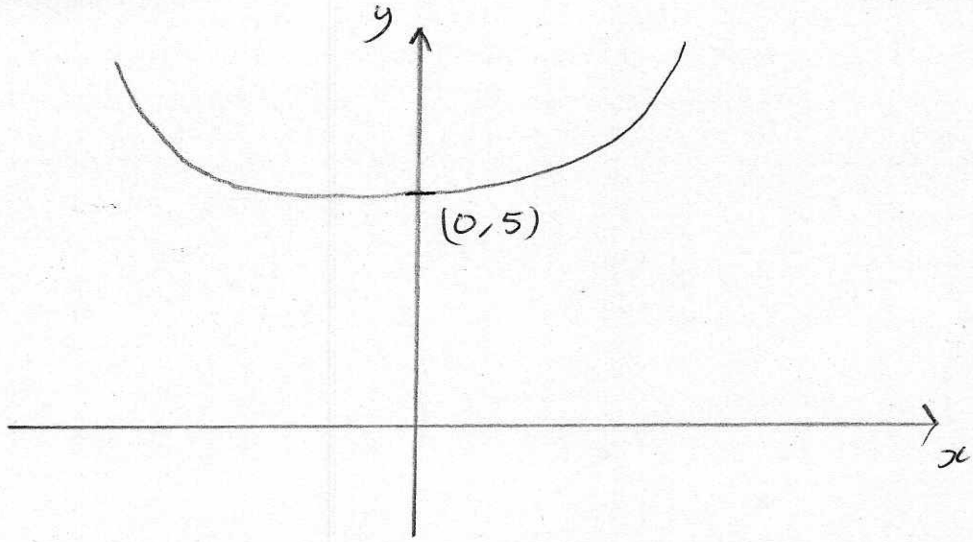
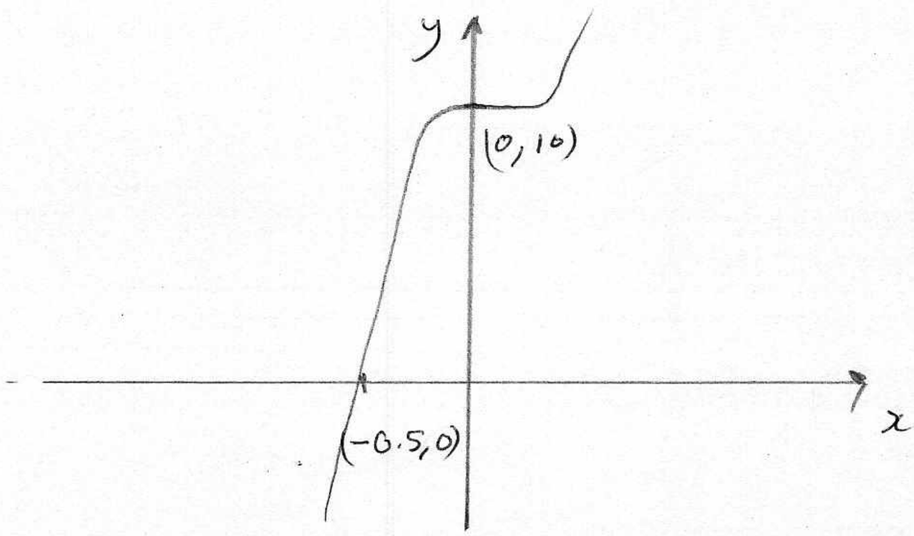
Question 4 continued

b)



c)



Question 5 continued

a) $\dfrac{4}{\sin^2 2\theta} - \dfrac{1}{\sin^2\theta}$

$\dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta}$

$\dfrac{\cancel{4}\,1}{\cancel{4}\sin^2\theta\cos^2\theta} - \dfrac{1}{\sin^2\theta}$

$\dfrac{1}{\sin^2\theta\cos^2\theta} - \dfrac{\cos^2\theta}{\sin^2\theta\cos^2\theta}$

$\dfrac{1-\cos^2\theta}{\sin^2\theta\cos^2\theta}$

$\dfrac{\sin^2\theta}{\sin^2\theta\cos^2\theta}$

$\dfrac{\sin^2\theta}{\cos^2\theta}$

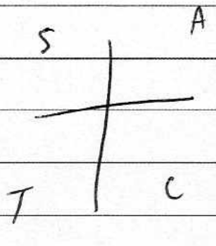
$\sec^2\theta$

c) $\sec^2\theta = 4$

$\sec\theta = \pm\sqrt{4}$

$= \pm 2$

$\cos\theta = \pm\frac{1}{2}$



$\theta = \frac{1}{3}\pi$ $\theta = \frac{2}{3}\pi$

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6. The functions f and g are defined by

$$f : x \mapsto e^x + 2, \qquad x \in \mathbb{R}$$

$$g : x \mapsto \ln x, \qquad x > 0$$

(a) State the range of f.
(1)

(b) Find $fg(x)$, giving your answer in its simplest form.
(2)

(c) Find the exact value of x for which $f(2x+3) = 6$
(4)

(d) Find f^{-1}, the inverse function of f, stating its domain.
(3)

(e) On the same axes sketch the curves with equation $y = f(x)$ and $y = f^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes.
(4)

6a) $f(x) > 2$

b/ $fg(x) = e^{\ln x} + 2$

$= x + 2$

c/ $e^{2x+3} + 2 = 6$

$e^{2x+3} = 4$

$2x + 3 = \ln 4$

$x = \frac{\ln(4) - 3}{2}$

d/ $y = e^x + 2$

$\cancel{y}\ x = e^y + 2$

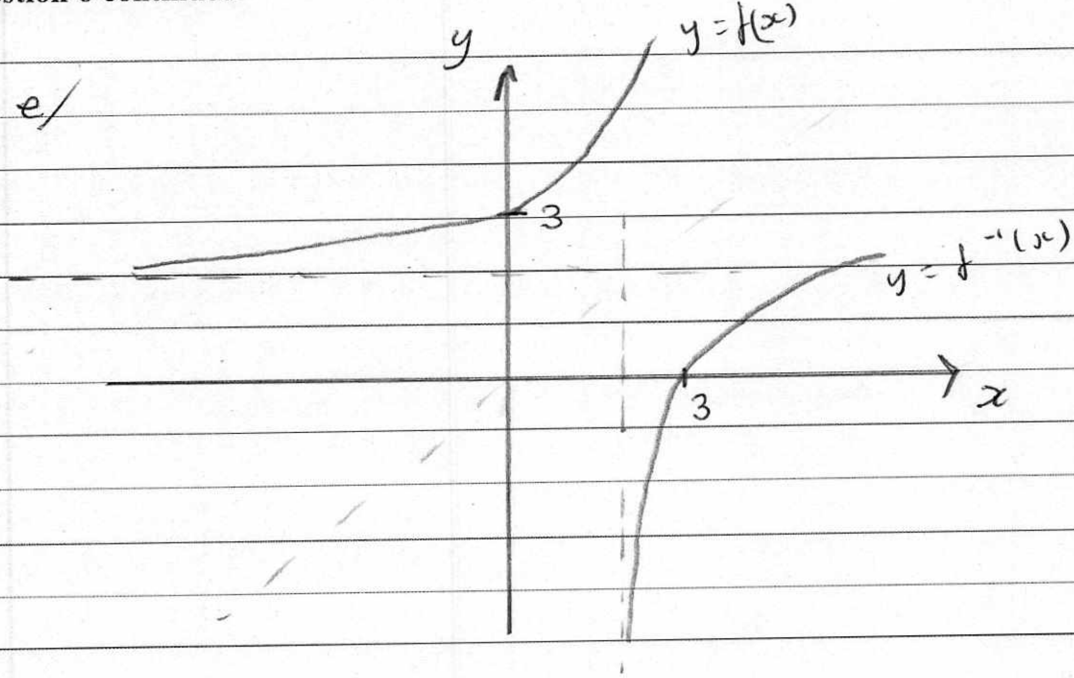
$x - 2 = e^y$

$\ln(x-2) = y$

$f^{-1}(x) = \ln(x-2) \quad \therefore \quad x > 2$

Question 6 continued

e/



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7. (a) Differentiate with respect to x,

(i) $x^{\frac{1}{2}} \ln(3x)$

(ii) $\dfrac{1-10x}{(2x-1)^5}$, giving your answer in its simplest form.

(6)

(b) Given that $x = 3\tan 2y$ find $\dfrac{dy}{dx}$ in terms of x.

(5)

7a) $u = x^{1/2}$ $\quad v = \ln 3x$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$ $\quad \frac{dv}{dx} = \frac{1}{x}$

$\frac{1}{2}x^{-1/2} \ln 3x + x^{-1/2}$

ii) $u = 1-10x$ $\quad v = (2x-1)^5$

$\frac{du}{dx} = -10$ $\quad \frac{dv}{dx} = \cancel{5}\ 10(2x-1)^4$

$\dfrac{-10(2x-1)^5 - 10(1-10x)(2x-1)^4}{(2x-1)^{10}}$

$\dfrac{-10(2x-1) - 10(1-10x)}{(2x-1)^6}$

$\dfrac{-20x + 10 - 10 + 100x}{(2x-1)^6}$

$\dfrac{80x}{(2x-1)^6}$

b) $\frac{dx}{dy} = 6\sec^2 2y$ $\qquad \sin^2 2y + \cos^2 2y = 1$

$\tan^2 2y + 1 = \sec^2 2y$

$\cancel{\frac{dy}{dx} = \frac{1}{6}\cos^2 2y}$

$\frac{dx}{dy} = 6(\tan^2 2y + 1)$

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Question 7 continued

$$\frac{dx}{dy}\,\frac{dy}{dx} = 6\tan^2 2y + 6$$

$$\frac{3\tan 2y}{3} = \frac{x}{3}$$

$$\tan^2 2y = \frac{x^2}{9}$$

$$\frac{dx}{dy}\,\frac{dy}{dx} = 6\left(\frac{x^2}{9}\right) + 6$$

$$= \frac{2}{3}x^2 + 6$$

$$\frac{dy}{dx} = \frac{1}{\frac{2}{3}x^2 + 6}$$

$$= \frac{3}{2x^2 + 18}$$

Turn over

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8. $$f(x) = 7\cos 2x - 24\sin 2x$$

Given that $f(x) = R\cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(a) find the value of R and the value of α. **(3)**

(b) Hence solve the equation

$$7\cos 2x - 24\sin 2x = 12.5$$

for $0 \leqslant x < 180°$, giving your answers to 1 decimal place. **(5)**

(c) Express $14\cos^2 x - 48\sin x\cos x$ in the form $a\cos 2x + b\sin 2x + c$, where a, b, and c are constants to be found. **(2)**

(d) Hence, using your answers to parts (a) and (c), deduce the maximum value of

$$14\cos^2 x - 48\sin x\cos x$$

(2)

8 a)

$R\cos(2x+\alpha) = R\cos 2x\cos\alpha - R\sin 2x\sin\alpha$

$R\cos\alpha = 7 \qquad R\sin\alpha = 24$

$\tan\alpha = \frac{24}{7}$

$\alpha = 73.7 \quad (3sf)$

$R = \sqrt{7^2+24^2}$
$= 25$

$25\cos(2x + 73.7)$

b) $25\cos(2x+73.7) = 12.5$

$\cos(2x + 73.7) = \frac{1}{2}$

$2x + 73.7 = 60, 300, 420$

$x = 113.1, 173.1$

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Question 8 continued

c/ $\cos 2x = 2\cos^2 x - 1$
$\sin 2x = 2\sin x \cos x$

$14\cos^2 x - 48\sin x\cos x$
$7(2\cos^2 x) - 24(2\sin x\cos x)$
$7(\cos 2x + 1) - 24\sin 2x$
$7\cos 2x + 7 - 24\sin 2x$
$7\cos 2x - 24\sin 2x + 7$

$a = 7 \quad b = -24 \quad c = 7$

d/ $25\cos(2x + 73.7) + 7$

max value = $\underline{\underline{32}}$