Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	5	/	0	1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Tuesday 15 June 2010 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. (a) Show that

$$\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$$

(2)

(b) Hence find, for $-180° \leqslant \theta < 180°$, all the solutions of

$$\frac{2\sin 2\theta}{1+\cos 2\theta} = 1$$

Give your answers to 1 decimal place.

(3)

a) $\frac{\sin 2\theta}{1+\cos 2\theta}$

$\frac{2\sin\theta\cos\theta}{1+\cos^2\theta-\sin^2\theta}$

$\frac{2\sin\theta\cos\theta}{(1-\sin^2\theta)+\cos^2\theta}$

$\frac{2\sin\theta\cos\theta}{\cos^2\theta+\cos^2\theta}$

$\frac{2\sin\theta\cos\theta}{2\cos^2\theta}$

$\frac{\sin\theta}{\cos\theta}$

$\tan\theta$

$2\tan\theta = 1$

b) $\tan\theta = \frac{1}{2}$

$\theta = ~~45, 225, -135~~$

26.6, -153.4

~~$\theta = -135°, 45°$~~

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Question 1 continued

Q1

(Total 5 marks)

Turn over

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2. A curve C has equation

$$y = \frac{3}{(5-3x)^2}, \quad x \neq \frac{5}{3}$$

The point P on C has x-coordinate 2. Find an equation of the normal to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

$y = 3(5-3x)^{-2}$ (CHAIN RULE)

$\frac{dy}{dx} = 18(5-3x)^{-3}$

when $x = 2$.

$\frac{dy}{dx} = 18(5-6)^{-3}$

$= -18$

$\therefore \quad m = 1/18$

$y = \frac{3}{(5-3(2))^2} = 3$

$y = mx + c$

$3 = 1/18(2) + c$

$3 = 1/9 + c$

$27/9 = 1/9 + c$

$26/9 = c$

$y = 1/18 \, x + 26/9$

$18y = x + 52$

$x - 18y + 52 = 0$

Leave blank

Question 2 continued

Q2

(Total 7 marks)

Turn over

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3. $$f(x) = 4\operatorname{cosec} x - 4x + 1, \quad \text{where } x \text{ is in radians.}$$

(a) Show that there is a root α of $f(x) = 0$ in the interval $[1.2, 1.3]$. **(2)**

(b) Show that the equation $f(x) = 0$ can be written in the form

$$x = \frac{1}{\sin x} + \frac{1}{4}$$ **(2)**

(c) Use the iterative formula

$$x_{n+1} = \frac{1}{\sin x_n} + \frac{1}{4}, \quad x_0 = 1.25,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places. **(3)**

(d) By considering the change of sign of $f(x)$ in a suitable interval, verify that $\alpha = 1.291$ correct to 3 decimal places. **(2)**

3a) $4\operatorname{cosec} x - 4x + 1 = 0$

$f(1.2) = 4\operatorname{cosec}(1.2) - 4(1.2) + 1 = 0.4916655$

$f(1.3) = 4\operatorname{cosec}(1.3) - 4(1.3) + 1 = -0.0487198$

change of sign ∴ root is interval $[1.2, 1.3]$

b) $0 = \frac{4}{\sin x} - 4x + 1$

$4x = \frac{4}{\sin x} + 1$

$x = \frac{1}{\sin x} + \frac{1}{4}$

c) $x_0 = 1.25$

$x_1 = 1.3038$

$x_2 = 1.2867$

$x_3 = 1.2917$

Leave blank

Question 3 continued

d/ upper bound = 1.2915
Lower bound = 1.2905

$f(1.2915) = \frac{4}{\sin(1.2915)} - 4(1.2915) + 1 = -4.8\times10^{-3}$

$f(1.2905) = \frac{4}{\sin(1.2905)} - 4(1.2905) + 1 = 4.5\times10^{-4}$

Change of sign $\therefore \alpha = 1.291$ to (3dp)

Turn over

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Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 9 marks)

Leave blank

4. The function f is defined by

$$f : x \mapsto |2x - 5|, \quad x \in \mathbb{R}$$

(a) Sketch the graph with equation $y = f(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(2)

(b) Solve $f(x) = 15 + x$.

(3)

The function g is defined by

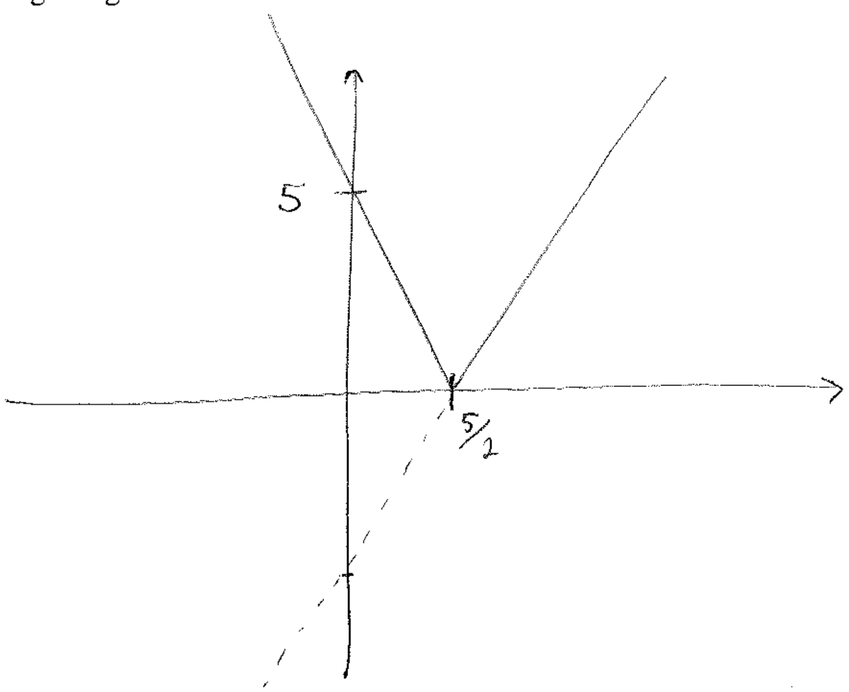
$$g : x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leqslant x \leqslant 5$$

(c) Find $fg(2)$.

(2)

(d) Find the range of g.

(3)



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Question 4 continued

b/ $|2x-5| = 15 + x$

$2x - 5 = 15 + x$ $\quad$ $-2x + 5 = 15 + x$

$x = 20$ $\quad$ $-10 = 3x$

$\frac{-10}{3} = x$

c/ $g(x) = x^2 - 4x + 1$

$g(2) = (2)^2 - 4(2) + 1$

$= 4 - 8 + 1$

$= -3$

$f(-3) = |2(-3) - 5|$

$= 11$

d/ $x^2 - 4x + 1$ $\quad$ $(5)^2 - 4(5) + 1 = 6$

$(x-2)^2 - 3$

$g(x) \geq -3$

$-3 \leq g(x) \leq 6$

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Question 4 continued

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Question 4 continued

Q4

(Total 10 marks)

5.

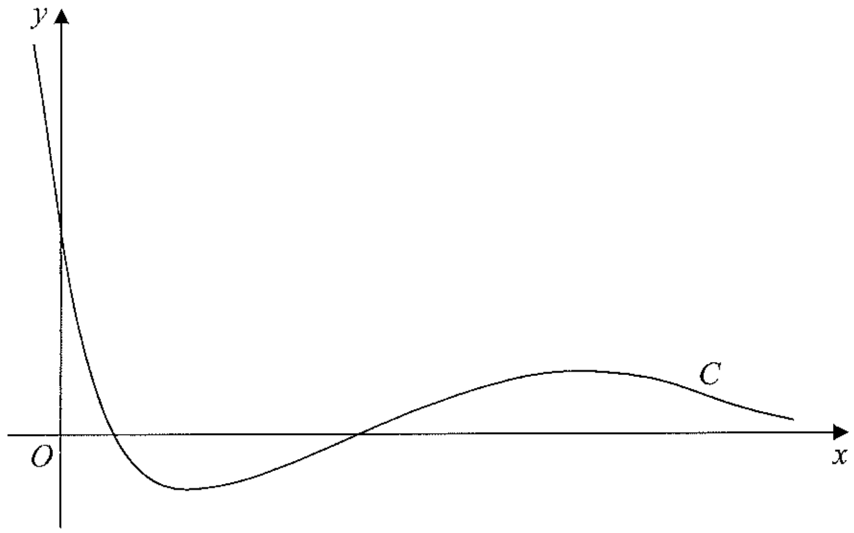

Figure 1

Figure 1 shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

(a) Find the coordinates of the point where C crosses the y-axis. **(1)**

(b) Show that C crosses the x-axis at $x = 2$ and find the x-coordinate of the other point where C crosses the x-axis. **(3)**

(c) Find $\frac{dy}{dx}$. **(3)**

(d) Hence find the exact coordinates of the turning points of C. **(5)**

a/ when $x = 0$ $y = 2$. $(0, 2)$

b/ $0 = (2x^2 - 5x + 2)e^{-x}$

$0 = (2x - 1)(x - 2)e^{-x}$

$x = \frac{1}{2}$ $x = 2$

c/ $u = 2x^2 - 5x + 2$ $v = e^{-x}$

$\frac{du}{dx} = 4x - 5$ $\frac{dv}{dx} = -e^{-x}$

$\frac{dy}{dx} = -e^{-x}(2x^2 - 5x + 2) + e^{-x}(4x - 5)$

$= e^{-x}(-2x^2 + 5x - 2 + 4x - 5)$

$= e^{-x}(-2x^2 + 9x - 7)$

Leave blank

Question 5 continued

d) $-e^{-x}(2x^2-9x+7) = 0$

$-e^{-x}(2x-7)(x-1) = 0$

$x = 7/2 \quad x = 1$

$y = 9e^{-7/2} \quad y = -e^{-1}$

$(7/2, 9e^{-7/2}) \quad (1, -e^{-1})$

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Question 5 continued

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Question 5 continued

Q5

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6.

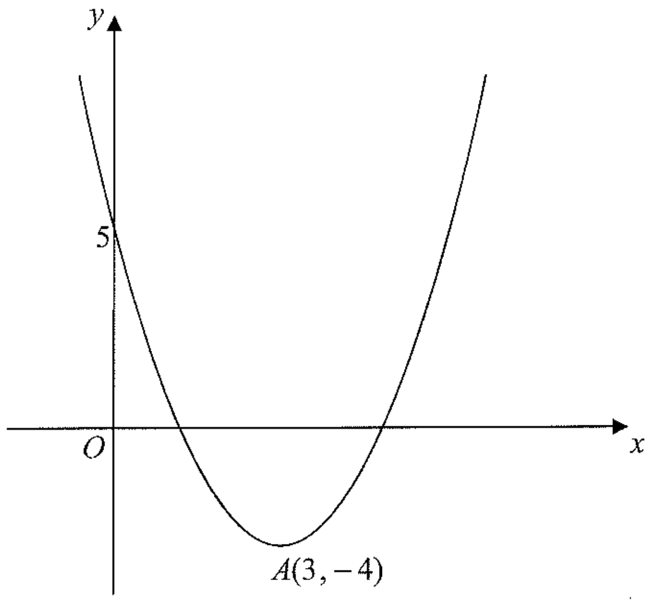

Figure 2

Figure 2 shows a sketch of the curve with the equation $y = \mathrm{f}(x),\ x \in \mathbb{R}$.
The curve has a turning point at $A(3, -4)$ and also passes through the point $(0, 5)$.

(a) Write down the coordinates of the point to which A is transformed on the curve with equation

(i) $y = |\mathrm{f}(x)|$,

(ii) $y = 2\mathrm{f}(\frac{1}{2}x)$.

(4)

(b) Sketch the curve with equation

$$y = \mathrm{f}(|x|)$$

On your sketch show the coordinates of all turning points and the coordinates of the point at which the curve cuts the y-axis.

(3)

The curve with equation $y = \mathrm{f}(x)$ is a translation of the curve with equation $y = x^2$.

(c) Find $\mathrm{f}(x)$.

(2)

(d) Explain why the function f does not have an inverse.

(1)

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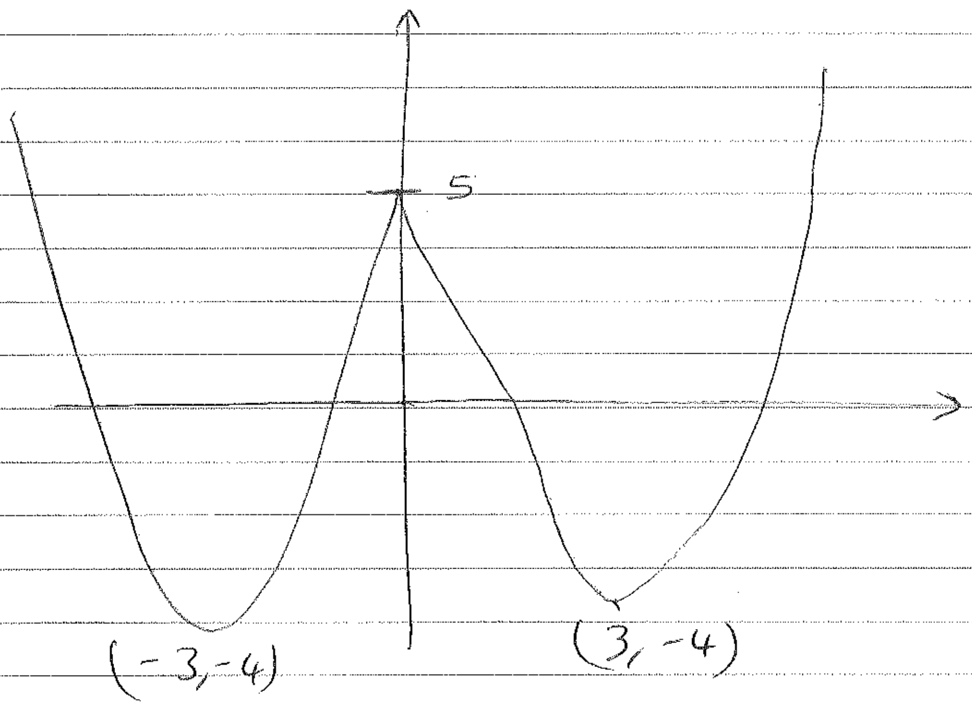
Question 6 continued

6 a i/ (3, 4)

ii/ (6, −8)

b/



c/ $y = (x-3)^2 - 4$

d/ it is a many to one function

Turn over

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Question 6 continued

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Question 6 continued

Q6

(Total 10 marks)

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7. (a) Express $2\sin\theta - 1.5\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 4 decimal places.

(3)

(b) (i) Find the maximum value of $2\sin\theta - 1.5\cos\theta$.

(ii) Find the value of θ, for $0 \leqslant \theta < \pi$, at which this maximum occurs.

(3)

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right), \quad 0 \leqslant t < 12,$$

where t hours is the number of hours after midday.

(c) Calculate the maximum value of H predicted by this model and the value of t, to 2 decimal places, when this maximum occurs.

(3)

(d) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

(6)

a/ $R\sin(\theta - \alpha) = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$

$R\cos\alpha = 2$

$R\sin\alpha = 1.5$

$\tan\alpha = \frac{1.5}{2}$ $\quad R^2 = 1.5^2 + 2^2$

$R = 2.5$

$\alpha = 0.6435011088$

$\alpha = 0.6435$

$2.5\sin(\theta - 0.6435)$

b i/ 2.5

ii/ $\sin(\theta - 0.6435) = 1$

$\theta - 0.6435 = \pi/2$

$\theta = 2.21$ (2dp)

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Question 7 continued

c/ $H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right)$

$= 6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right)$

$\text{Max}_H = 8.5$

$\frac{4\pi t}{25} = 2.21$ (FROM PART B)

$t = 4.41$

d/ $7 = 6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right)$

$1 = 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right)$

$\frac{1}{2.5} = \sin\left(\frac{4\pi t}{25} - 0.6435\right)$

$0.4115\ldots = \frac{4\pi t}{25} - 0.6435$
$2.730\ldots$

$t =$ 2hrs 6mins , 6hrs 43mins

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Question 7 continued

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8. (a) Simplify fully

$$\frac{2x^2+9x-5}{x^2+2x-15}$$

(3)

Given that

$$\ln(2x^2+9x-5)=1+\ln(x^2+2x-15), \quad x \neq -5,$$

(b) find x in terms of e.

(4)

8a) $\frac{(2x-1)(x+5)}{(x-3)(x+5)}$

$\frac{2x-1}{x-3}$

b/ $\ln(2x^2+9x-5) = 1 + \ln(x^2+2x-15)$

$\ln(2x^2+9x-5) - \ln(x^2+2x-15) = 1$

$\ln\left(\frac{2x^2+9x-5}{x^2+2x-15}\right) = 1$

$\ln\left(\frac{2x-1}{x-3}\right) = 1$

$\frac{2x-1}{x-3} = e^1$

$2x-1 = e^1(x-3)$

$2x-1 = xe-3e$

$2x-xe = 1-3e$

$x(2-e) = 1-3e$

$x = \frac{1-3e^1}{2-e^1}$

Leave blank

Question 7 continued

Q7

(Total 15 marks)

Turn over

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Question 8 continued

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Question 8 continued

Q8

(Total 7 marks)

TOTAL FOR PAPER: 75 MARKS

END