Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	4	/	0	1	Signature	

Paper Reference(s)

6664/01

Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Thursday 22 May 2014 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 10 questions in this question paper. The total mark for this paper is 75.
There are 36 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1.

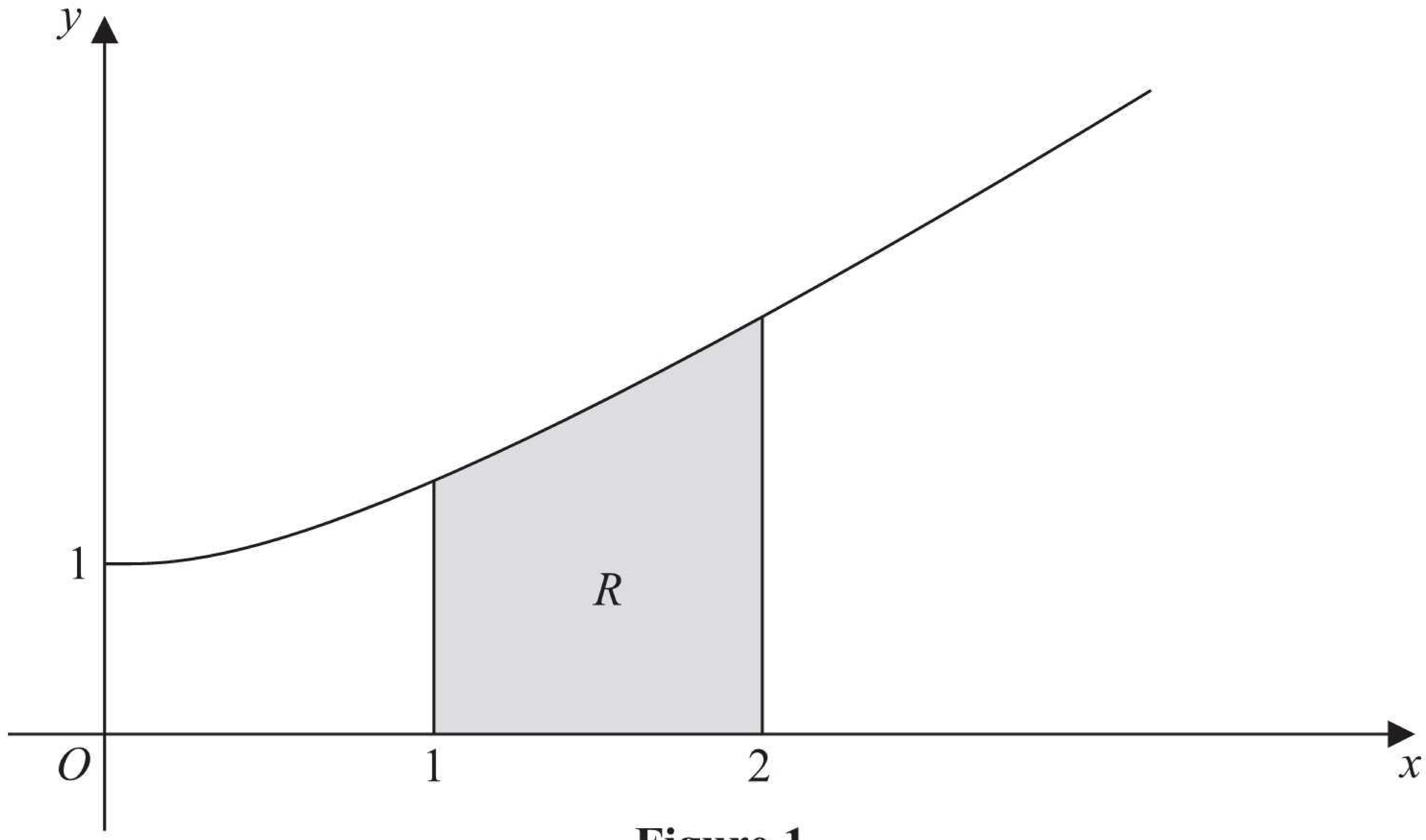

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{(x^2 + 1)}$, $x \geqslant 0$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$

The table below shows corresponding values for x and y for $y = \sqrt{(x^2 + 1)}$.

x	1	1.25	1.5	1.75	2
y	1.414	1.601	1.803	2.016	2.236

(a) Complete the table above, giving the missing value of y to 3 decimal places.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

b) $0.25\left(\frac{1.414}{2} + 1.601 + 1.803 + 2.016 + \frac{2.236}{2}\right)$

$= 1.81$ units^2

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Question 1 continued

Q1

(Total 5 marks)

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2. $$f(x) = 2x^3 - 7x^2 + 4x + 4$$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)

2a) if $(x-2)$ is a factor $f(2) = 0$

$f(2) = 2(2)^3 - 7(2)^2 + 4(2) + 4$

$f(2) = 0$ $\therefore$ $(x-2)$ is a factor.

b)

$$\begin{array}{r} 2x^2 - 3x - 2 \\ x-2 \enclose{longdiv}{2x^3 - 7x^2 + 4x + 4} \\ \underline{2x^3 - 4x^2} \\ -3x^2 + 4x \\ \underline{-3x^2 - 6x} \\ -2x + 4 \\ \underline{-2x + 4} \\ 0 \end{array}$$

$(x-2)(2x^2 - 3x - 2)$

$(x-2)(2x+1)(x-2)$

$(x-2)^2(2x+1)$

Leave blank

Question 2 continued

Q2

(Total 6 marks)

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3. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2-3x)^6$$

giving each term in its simplest form.

(4)

(b) Hence, or otherwise, find the first 3 terms, in ascending powers of x, of the expansion of

$$\left(1+\frac{x}{2}\right)(2-3x)^6$$

(3)

3a) $(2-3x)^6$

1 6 15

$1(2)^6 + 6(2)^5(-3x) + 15(2)^4(-3x)^2$

$64 - 576x + 2160x^2$

3b) $\left(1+\frac{x}{2}\right)\left(64-576x+2160x^2\right)$

$64 + 32x - 576x + 2160x^2 - 288x^2$

$64 - 544x + 1872x^2$

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Question 3 continued

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Question 3 continued

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Question 3 continued

(Total 7 marks)

Q3

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4. Use integration to find

$$\int_1^{\sqrt{3}} \left(\frac{x^3}{6} + \frac{1}{3x^2} \right) \mathrm{d}x$$

giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(5)

4) $\int_1^{\sqrt{3}} \frac{x^3}{6} + \frac{1}{3x^2} \, dx$

$\int_1^{\sqrt{3}} \frac{1}{6}x^3 + \frac{1}{3}x^{-2} \, dx$

$\left[\frac{\frac{1}{6}x^4}{4} + \frac{\frac{1}{3}x^{-1}}{-1} + c \right]_1^{\sqrt{3}}$

$\left[\frac{1}{24}x^4 - \frac{1}{3}x^{-1} \right]_1^{\sqrt{3}}$

$\left[\frac{1}{24}(\sqrt{3})^4 - \frac{1}{3}(\sqrt{3})^{-1} \right] - \left[\frac{1}{24}(1)^4 - \frac{1}{3}(1)^{-1} \right]$

$\left(\frac{9}{24} - \frac{1}{3\sqrt{3}} \right) - \left(\frac{1}{24} - \frac{1}{3} \right)$

$\frac{6 - \sqrt{3}}{9}$

$\frac{6}{9} - \frac{1}{9}\sqrt{3}$

$\frac{2}{3} - \frac{1}{9}\sqrt{3}$

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Question 4 continued

(Total 5 marks)

Q4

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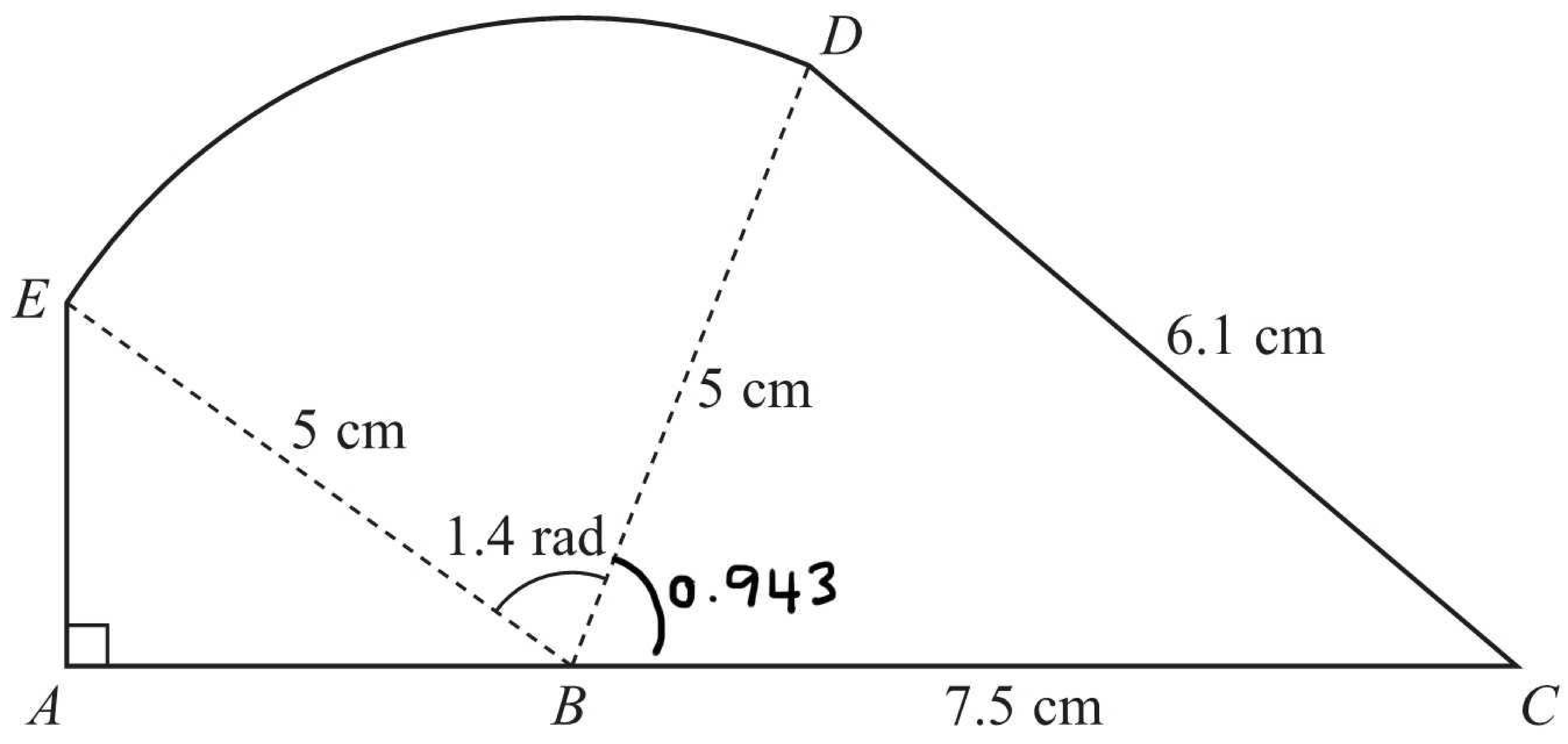

Figure 2

The shape *ABCDEA*, as shown in Figure 2, consists of a right-angled triangle *EAB* and a triangle *DBC* joined to a sector *BDE* of a circle with radius 5 cm and centre *B*.

The points *A*, *B* and *C* lie on a straight line with *BC* = 7.5 cm.

Angle *EAB* = $\frac{\pi}{2}$ radians, angle *EBD* = 1.4 radians and *CD* = 6.1 cm.

(a) Find, in cm^2, the area of the sector *BDE*.
(2)

(b) Find the size of the angle *DBC*, giving your answer in radians to 3 decimal places.
(2)

(c) Find, in cm^2, the area of the shape *ABCDEA*, giving your answer to 3 significant figures.
(5)

a) $\frac{\theta}{2\pi} \times \pi r^2$

$\frac{1.4}{2} \times 5^2 = 17.5\text{cm}^2$

b) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $a = 6.1$, $b = 5$, $c = 7.5$

$\cos A = \frac{5^2 + 7.5^2 - 6.1^2}{2(5)(7.5)}$

$\cos A = 0.5872$

$A = 0.943^c$ (3dp)

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Question 5 continued

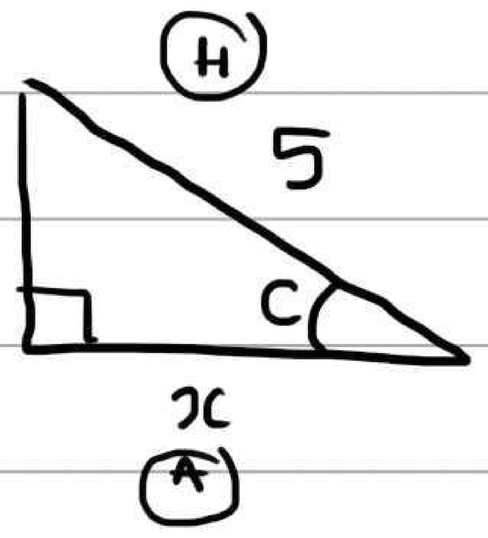
Area of BDC $= \frac{1}{2}ab \sin C$

$= \frac{1}{2}(5)(7.5)\sin(0.943)$

$= 15.177 \text{ cm}^2$ (3 dp) (B)

Angle $A\hat{B}E = \pi - 1.4 - 0.943$

$= 0.798$ 3dp (C)



$\cos(0.798) = \frac{x}{5}$

$5\cos(0.798) = x$

$3.489 = x$ (3dp) (D)

Area of ABE $= \frac{1}{2}(3.489)(5)\sin(0.798)$

$= 6.248 \text{ cm}^2$ (3dp)

Total Area $= 6.248 + 15.177 + 17.5$

$= 38.9 \text{ cm}^2$ (3sf)

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Question 5 continued

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Question 5 continued

Q5

(Total 9 marks)

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6. The first term of a geometric series is 20 and the common ratio is $\frac{7}{8}$

The sum to infinity of the series is S_∞

(a) Find the value of S_∞

(2)

The sum to N terms of the series is S_N

(b) Find, to 1 decimal place, the value of S_{12}

(2)

(c) Find the smallest value of N, for which

$$S_\infty - S_N < 0.5$$

(4)

6a) $S_\infty = \frac{a}{1-r}$ $a=20$ $r=7/8$

$S_\infty = \frac{20}{1-7/8}$

$= 160$

b) $S_{12} = \frac{a(1-r^n)}{1-r}$

$= \frac{20(1-\frac{7}{8}^{12})}{1-7/8}$

$= 127.8$ (1dp)

c) $S_\infty - S_N < 0.5$

$160 - \frac{20(1-7/8^N)}{1-7/8} < 0.5$

$160 - 160(1-7/8^N) < 0.5$

$159.5 < 160(1-7/8^N)$

$\frac{159.5}{160} < 1 - 7/8^N$

$7/8^N < \frac{0.5}{160}$

$S_\infty - S_N = 0.5$ when $N = \log_{7/8}(\frac{1}{320})$

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Question 6 continued

$N = 43.198...$

The gap between S_∞ and S_N is decreasing

$\therefore N = 44$

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Question 6 continued

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Question 6 continued

Q6

(Total 8 marks)

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7. (i) Solve, for $0 \leqslant \theta < 360°$, the equation

$$9\sin(\theta + 60°) = 4$$

giving your answers to 1 decimal place.
You must show each step of your working.

(4)

(ii) Solve, for $-\pi \leqslant x < \pi$, the equation

$$2\tan x - 3\sin x = 0$$

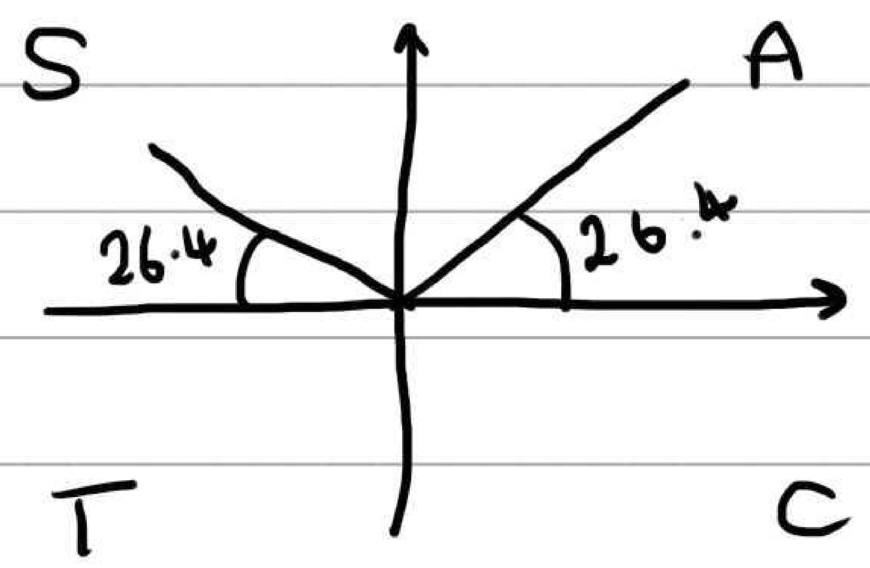
giving your answers to 2 decimal places where appropriate.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(5)

7i) $9\sin(\theta + 60) = 4$

$\sin(\theta + 60) = 4/9$

$\theta + 60 = 26.4$



$\theta + 60 = 26.4,\ 180 - 26.4,\ 360 + 26.4$

$\theta + 60 = 26.4,\ 153.6,\ 386.4$

$0 \leq \theta < 360$ $\quad \theta = 93.6,\ 326.4$

ii) $2\tan x - 3\sin x = 0$

$\frac{2\sin x}{\cos x} - 3\sin x = 0$

$2\sin x - 3\sin x\cos x = 0$

$\sin x(2 - 3\cos x) = 0$

$\sin x = 0 \qquad 2 - 3\cos x = 0$

$x = 0 \qquad 2 = 3\cos x$

$2/3 = \cos x$

$x = 0.84$ (x)

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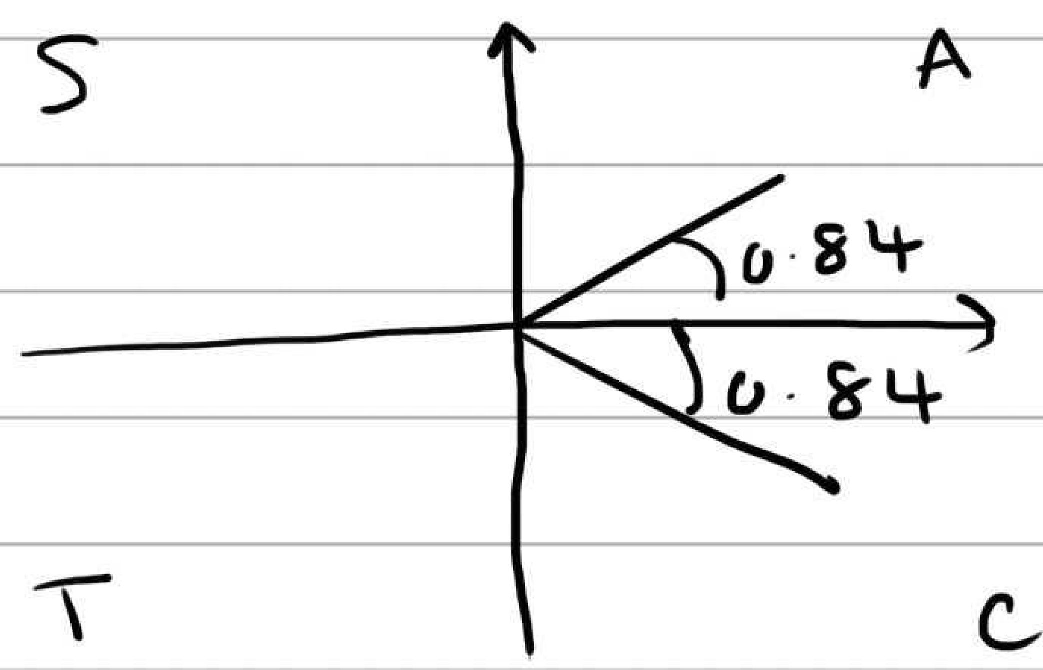
Question 7 continued



$\sin x = 0$

$x = -\pi, 0, \cancel{\pi}$

$\cos x = 2/3$

$x = 0.84, -0.84$

$\underline{x = -\pi, -0.84, 0, 0.84}$

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Question 7 continued

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Question 7 continued

(Total 9 marks)

Q7

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8. (a) Sketch the graph of

$$y = 3^x, \quad x \in \mathbb{R}$$

showing the coordinates of any points at which the graph crosses the axes.

(2)

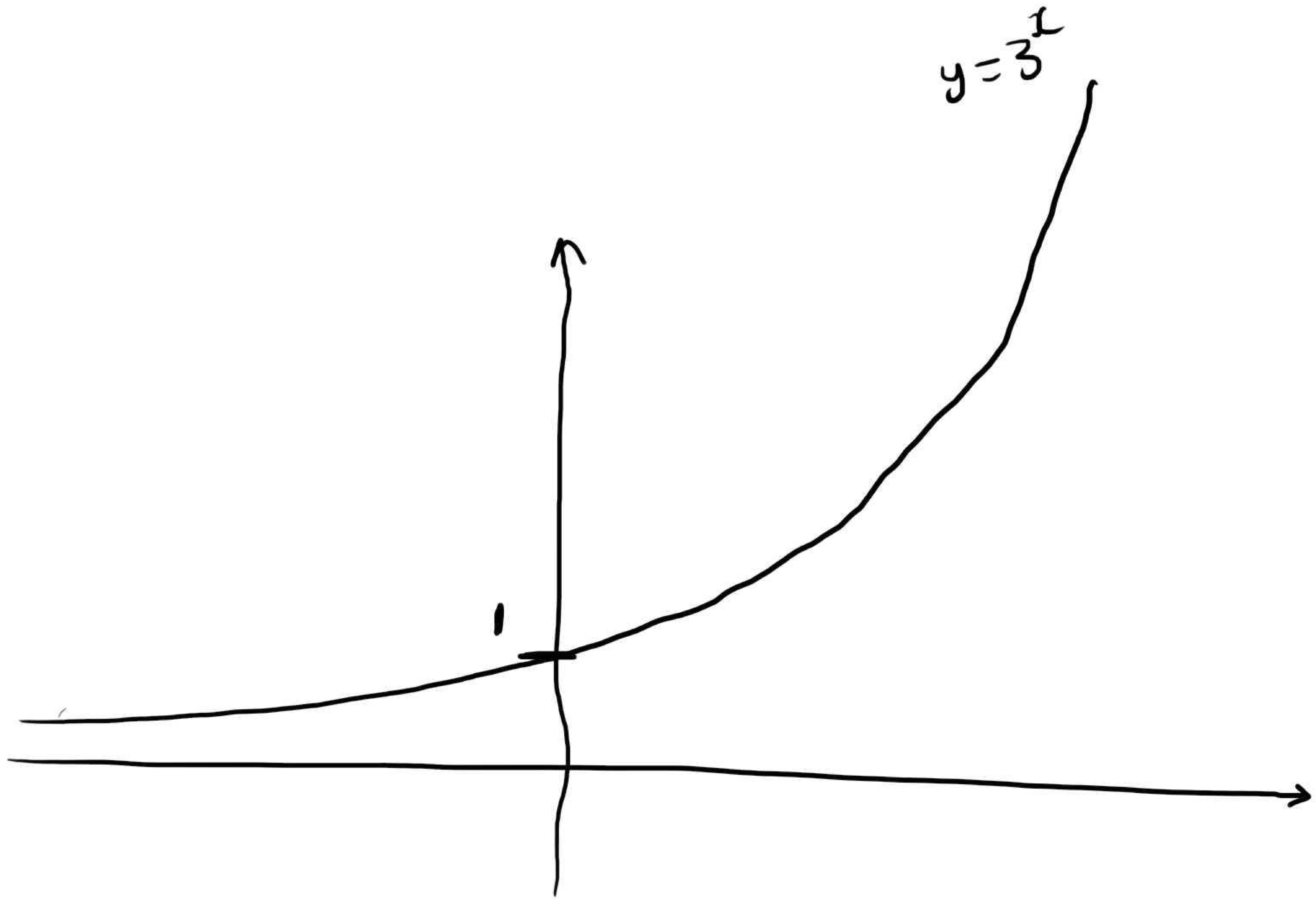
(b) Use algebra to solve the equation

$$3^{2x} - 9(3^x) + 18 = 0$$

giving your answers to 2 decimal places where appropriate.

(5)

8a)



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Question 8 continued

b) $3^{2x} - 9(3^{x}) + 18 = 0$

$(3^{x} - 3)(3^{x} - 6) = 0$

$3^{x} = 3$ $\quad$ $3^{x} = 6$

$x = 1$ $\quad$ $x = \log_3 6$

$x = 1.63$ 2dp

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Question 8 continued

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Question 8 continued

Q8

(Total 7 marks)

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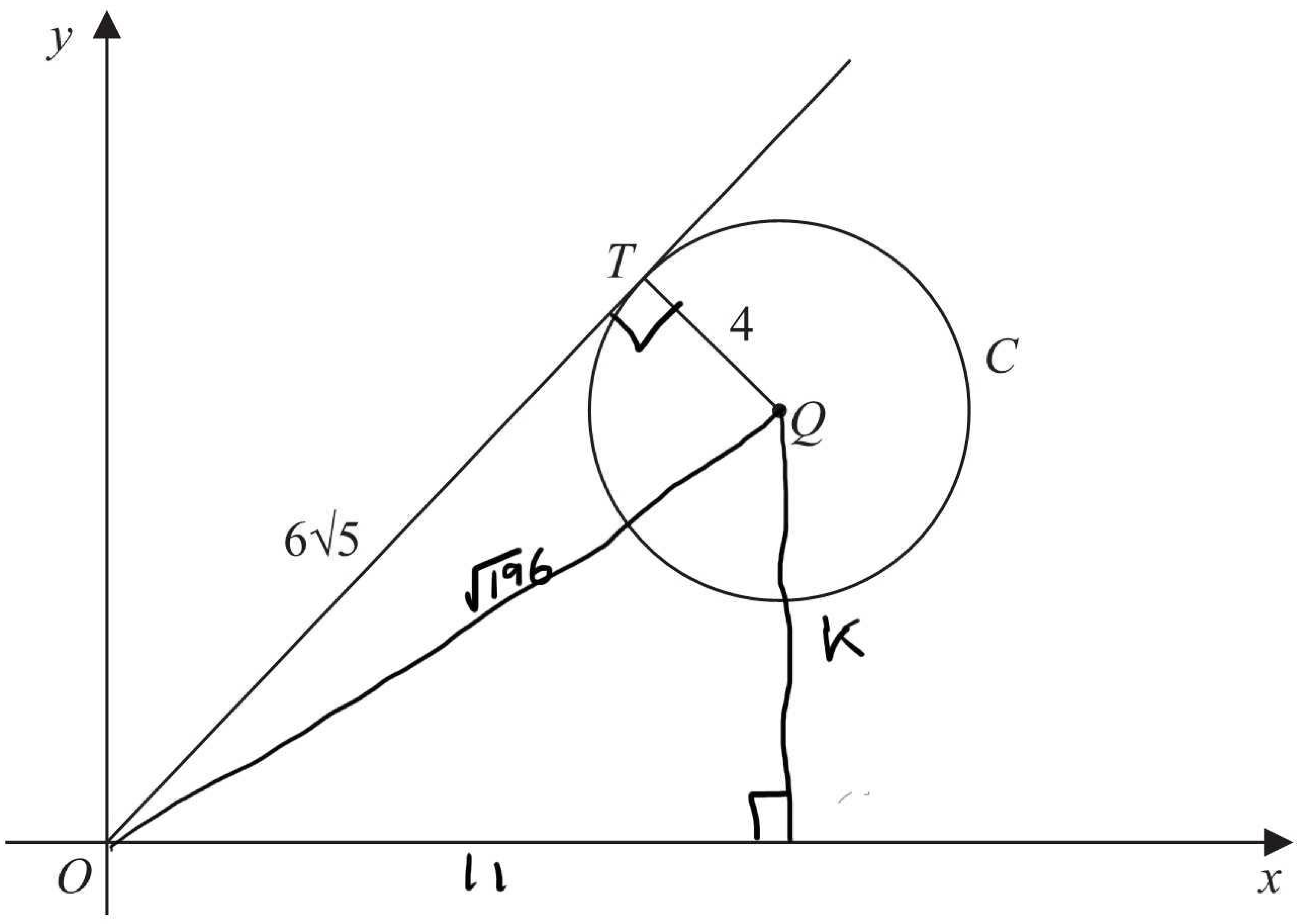

Figure 3

Figure 3 shows a circle C with centre Q and radius 4 and the point T which lies on C.

The tangent to C at the point T passes through the origin O and $OT = 6\sqrt{5}$

Given that the coordinates of Q are $(11, k)$, where k is a positive constant,

(a) find the exact value of k,

(3)

(b) find an equation for C.

(2)

a) $OQ^2 = 4^2 + (6\sqrt{5})^2$

$OQ^2 = 196$

$k^2 = OQ^2 - 11^2$

$k^2 = 196 - 121$

$k^2 = 75$

$k = \sqrt{75}$

$= \underline{5\sqrt{3}}$

b) $(x-a)^2 + (y-b)^2 = r^2$

$\underline{(x-11)^2 + (y-5\sqrt{3})^2 = 16}$

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Question 9 continued

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Question 9 continued

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Question 9 continued

Q9

(Total 5 marks)

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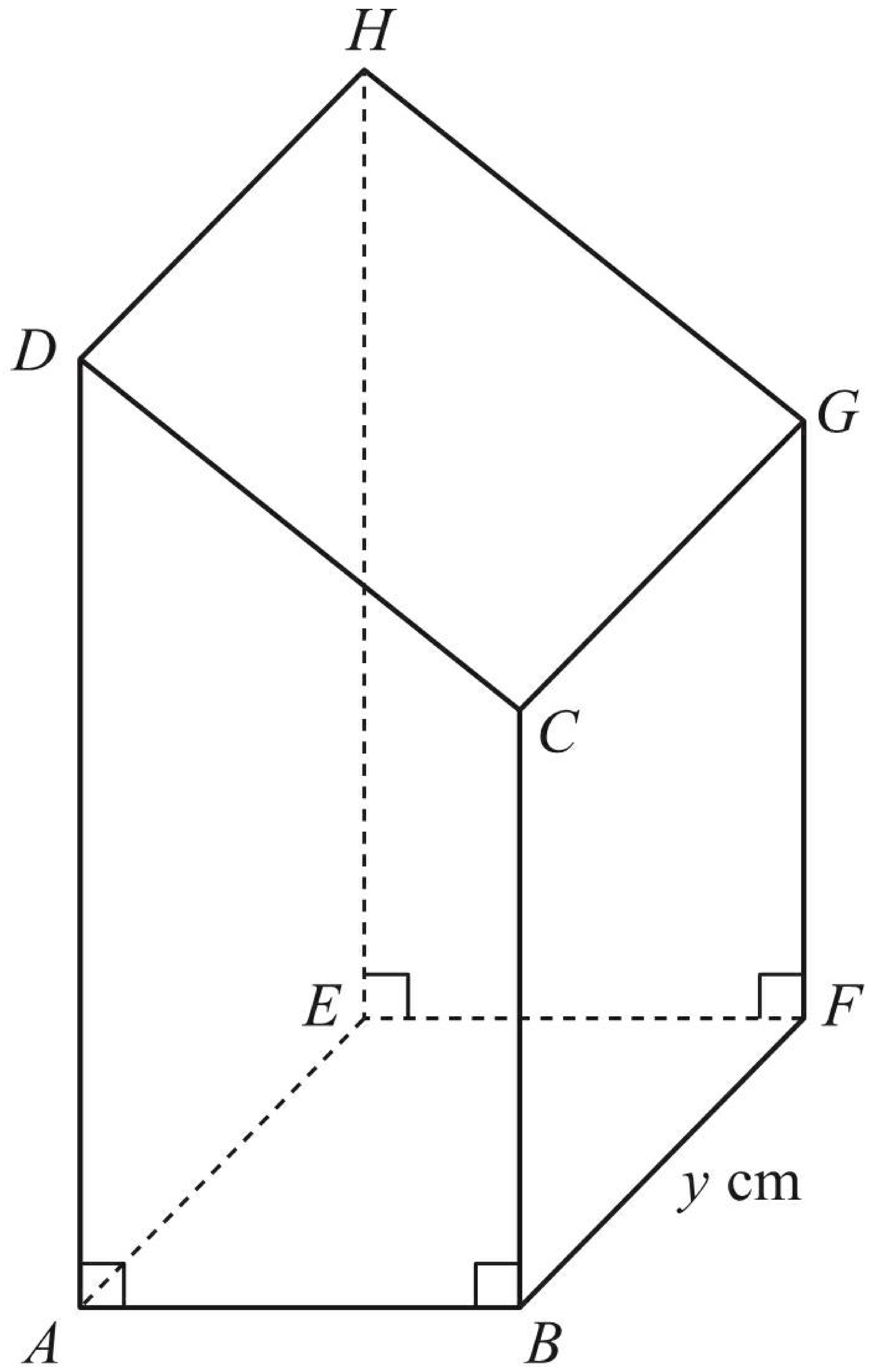

Figure 4

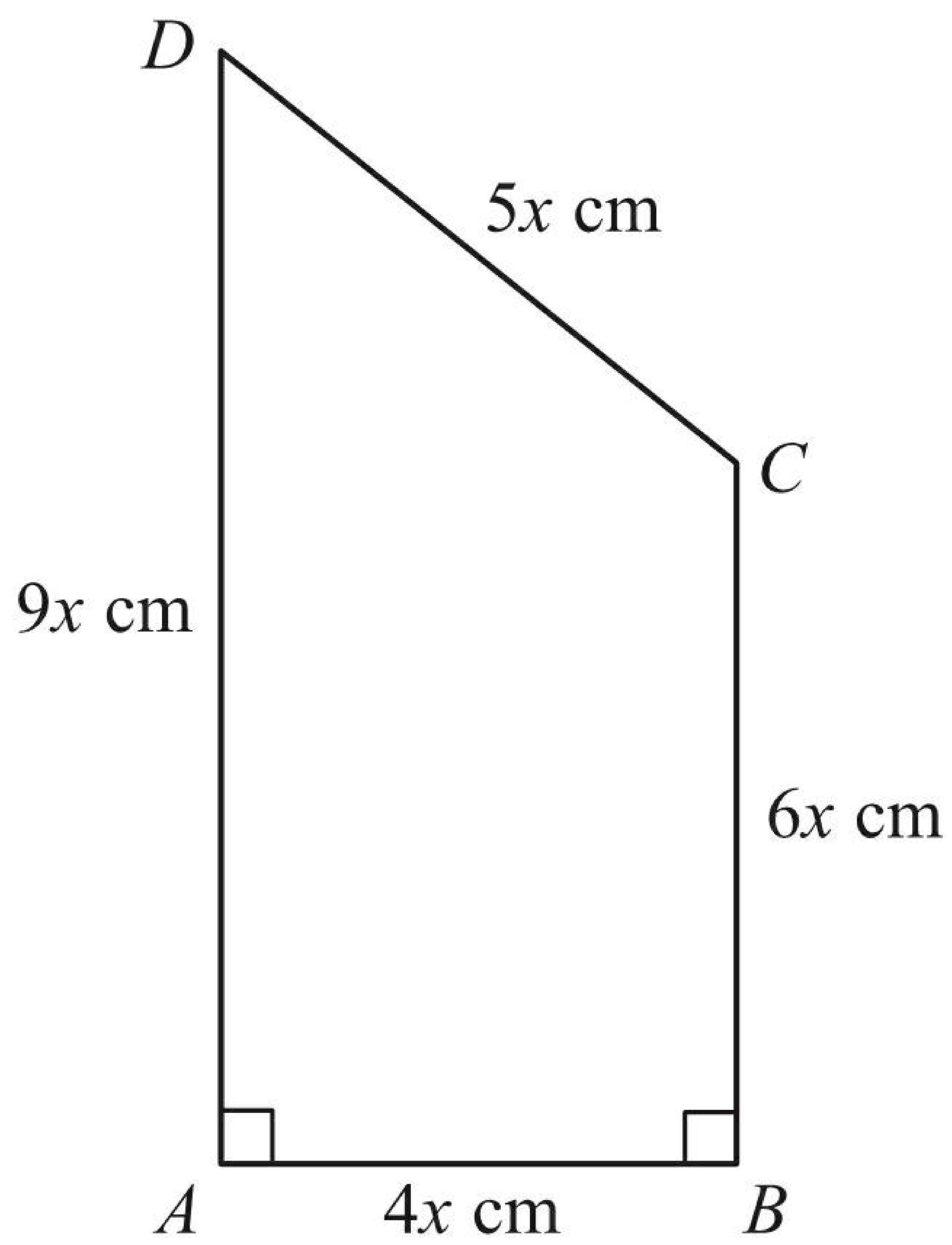

Figure 5

Figure 4 shows a closed letter box *ABFEHGCD,* which is made to be attached to a wall of a house.

The letter box is a right prism of length y cm as shown in Figure 4. The base *ABFE* of the prism is a rectangle. The total surface area of the six faces of the prism is S cm^2.

The cross section *ABCD* of the letter box is a trapezium with edges of lengths $DA = 9x$ cm, $AB = 4x$ cm, $BC = 6x$ cm and $CD = 5x$ cm as shown in Figure 5.
The angle $DAB = 90°$ and the angle $ABC = 90°$.

The volume of the letter box is 9600 cm^3.

(a) Show that

$$y = \frac{320}{x^2}$$

(2)

(b) Hence show that the surface area of the letter box, S cm^2, is given by

$$S = 60x^2 + \frac{7680}{x}$$

(4)

(c) Use calculus to find the minimum value of S.

(6)

(d) Justify, by further differentiation, that the value of S you have found is a minimum.

(2)

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Question 10 continued

10a) volume = area of cross section × length

$$9600 = \left(\frac{9x+6x}{\cancel{x}}\right)\frac{2}{\cancel{4}}x \times y$$

$$9600 = 15x(2x) \times y$$

$$9600 = 30x^2 \times y$$

$$\frac{9600}{30x^2} = y$$

$$y = \frac{320}{x^2}$$

b) surface area : front : $30x^2$

back : $30x^2$

base : $4xy$

side : $6xy$

side 2 : $9xy$

top : $5xy$

∴ surface area $= 60x^2 + 24xy$

$y = \frac{320}{x^2}$

$$= 60x^2 + 24x\left(\frac{320}{x^2}\right)$$

$$= 60x^2 + \frac{7680}{x}$$

c) $S = 60x^2 + \frac{7680}{x}$

Min value when $\frac{dS}{dx} = 0$

$$\frac{dS}{dx} = 120x - 7680x^{-2}$$

$$0 = 120x - 7680x^{-2}$$

$$7680x^{-2} = 120x$$

$$7680 = 120x^3$$

$$64 = x^3$$

$x = 4$

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Question 10 continued

when $x = 4$

$$S = 60(4)^2 + \frac{7680}{4}$$

$$= \underline{\underline{2880}}$$

d) $\frac{dS}{dx} = 120x - 7680x^{-2}$

$$\frac{d^2S}{dx^2} = 120 + 15360x^{-3}$$

when $x = 4$

$$\frac{d^2S}{dx^2} = 120 + \frac{15360}{(4)^3}$$

+ve $\therefore$ it is a minimum

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Question 10 continued

Turn over

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Question 10 continued

Q10

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END