Write your name here

Surname	Other names

Pearson
Edexcel GCE

Centre Number	Candidate Number

Core Mathematics C1

Advanced Subsidiary

Wednesday 17 May 2017 – Morning
Time: 1 hour 30 minutes

Paper Reference
6663/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Calculators may NOT be used in this examination.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

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1. Find

$$\int \left(2x^5 - \frac{1}{4x^3} - 5 \right) dx$$

giving each term in its simplest form.

(4)

$$\int 2x^5 - \frac{1}{4}x^{-3} - 5 \; dx$$

$$\frac{2x^6}{6} - \frac{\frac{1}{4}x^{-2}}{-2} - 5x + c$$

$$\frac{1}{3}x^6 + \frac{1}{8}x^{-2} - 5x + c$$

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2. Given

$$y = \sqrt{x} + \frac{4}{\sqrt{x}} + 4, \qquad x > 0$$

find the value of $\frac{dy}{dx}$ when $x = 8$, writing your answer in the form $a\sqrt{2}$, where a is a rational number.

(5)

$$y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}} + 4$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-\frac{3}{2}}$$

$$= \frac{1}{2\sqrt{x}} - \frac{2}{(\sqrt{x})^3}$$

when $x = 8$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{8}} - \frac{2}{(\sqrt{8})^3} \qquad \sqrt{8} = 2\sqrt{2}$$

$$= \frac{1}{2(2\sqrt{2})} - \frac{2}{(2\sqrt{2})^3}$$

$$2\sqrt{2} \times 2\sqrt{2} \times 2\sqrt{2}$$

$$\cancel{8} \; 16\sqrt{2}$$

$$= \frac{1}{4\sqrt{2}} - \frac{2}{16\sqrt{2}}$$

$$= \frac{1}{4\sqrt{2}} - \frac{1}{8\sqrt{2}}$$

$$= \frac{2}{8\sqrt{2}} - \frac{1}{8\sqrt{2}}$$

$$= \frac{1}{8\sqrt{2}} \quad \begin{matrix}\times\sqrt{2} \\ \times\sqrt{2}\end{matrix}$$

$$= \frac{\sqrt{2}}{16}$$

$$= \frac{1}{16}\sqrt{2}$$

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3. A sequence $a_1, a_2, a_3, \dots$ is defined by

$$a_1 = 1$$
$$a_{n+1} = \frac{k(a_n + 1)}{a_n}, \qquad n \geqslant 1$$

where k is a positive constant.

(a) Write down expressions for a_2 and a_3 in terms of k, giving your answers in their simplest form.

(3)

Given that $\sum_{r=1}^{3} a_r = 10$

(b) find an exact value for k.

(3)

a/ $a_2 = \frac{k(1+1)}{1}$

$= 2k$

$a_3 = \frac{\cancel{k}(2k+1)}{2\cancel{k}}$

$= \frac{1}{2}(2k+1)$ or $k + \frac{1}{2}$

b/ $1 + 2k + \frac{1}{2}(2k+1) = 10$

$2k + k + \frac{1}{2} = 9$

$3k = \frac{17}{2}$

$k = \frac{17}{6}$

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4. A company, which is making 140 bicycles each week, plans to increase its production. The number of bicycles produced is to be increased by d each week, starting from 140 in week 1, to $140 + d$ in week 2, to $140 + 2d$ in week 3 and so on, until the company is producing 206 in week 12.

(a) Find the value of d.

(2)

After week 12 the company plans to continue making 206 bicycles each week.

(b) Find the total number of bicycles that would be made in the first 52 weeks starting from and including week 1.

(5)

$U_{12} = 206 \quad a = 140 \quad d = d$

$U_n = a + (n-1)d$

$206 = 140 + 11d$

$66 = 11d$

$d = 6$

b/ $S_{12} = \frac{12}{2}(2(140) + 11(6)) \quad S_n = \frac{n}{2}(2a + (n-1)d)$

$= 6(280 + 66)$

$= 6(346)$

$= 2076$

	300	40	6
6	1800	240	36

40 weeks producing 206

$40 \times 206 = 8240$

$2076 + 8240 = 10316$

	200	6
40	8000	240

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5. $$f(x) = x^2 - 8x + 19$$

(a) Express f(x) in the form $(x + a)^2 + b$, where a and b are constants.

(2)

The curve C with equation $y = f(x)$ crosses the y-axis at the point P and has a minimum point at the point Q.

(b) Sketch the graph of C showing the coordinates of point P and the coordinates of point Q.

(3)

(c) Find the distance PQ, writing your answer as a simplified surd.

(3)

a) $(x-4)^2 - 16 + 19$

$(x-4)^2 + 3$

b) min point $(4, 3)$ Q

crosses y when $x = 0$ $y = 19$ P

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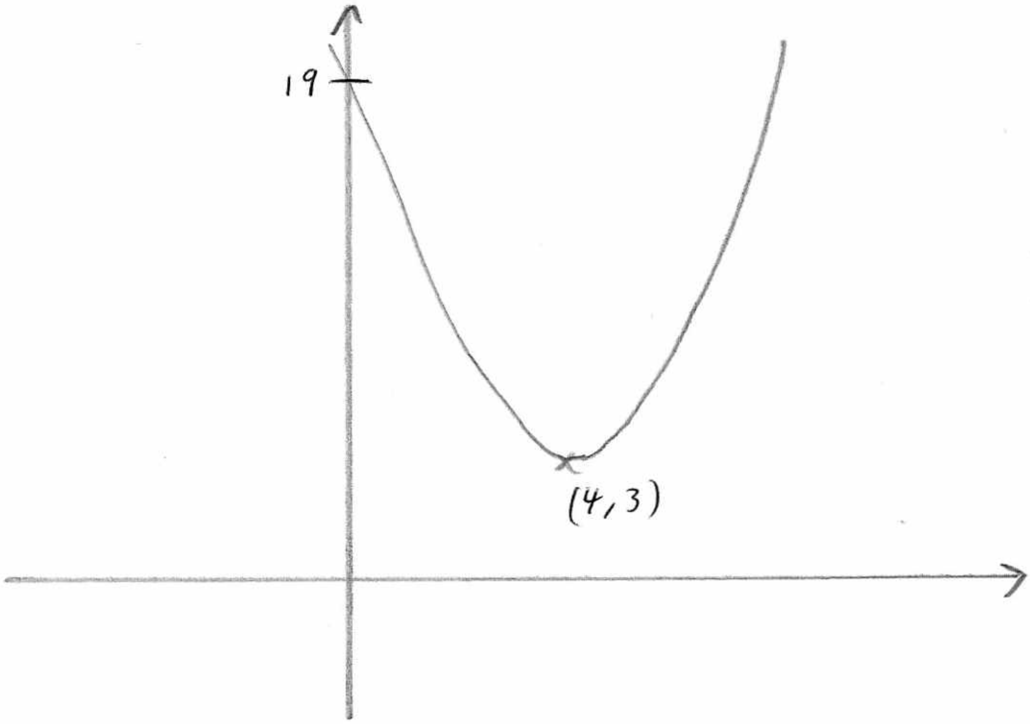
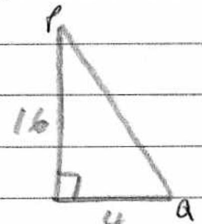
Question 5 continued



c/ Distance between (0, 19) and (4, 3)



$PQ^2 = 4^2 + 16^2$

$PQ^2 = 272$

$PQ = \sqrt{272}$

$= \underline{\underline{4\sqrt{17}}}$

	10	6
10	100	60
6	60	36

$16^2 = \underline{\underline{256}}$

$\sqrt{272}$

2×136

$2 \times 2 \times 68$

$2 \times 2 \times 2 \times 34$

$2 \times 2 \times 2 \times 2 \times 17$

16×17

Q5

(Total 8 marks)

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6. (a) Given $y = 2^x$, show that

$$2^{2x+1} - 17(2^x) + 8 = 0$$

can be written in the form

$$2y^2 - 17y + 8 = 0$$

(2)

(b) Hence solve

$$2^{2x+1} - 17(2^x) + 8 = 0$$

(4)

a) $y = 2^x$

$2^1(2^{2x}) - 17(2^x) + 8 = 0$

$2(2^x)^2 - 17(2^x) + 8 = 0$

$2y^2 - 17y + 8 = 0$

b) $(2y - 1)(y - 8) = 0$

$y = \frac{1}{2} \quad y = 8$

$2^x = \frac{1}{2} \quad 2^x = 8$

$\underline{\underline{x = -1}} \quad \underline{\underline{x = 3}}$

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7. The curve C has equation $y = f(x)$, $x > 0$, where

$$f'(x) = 30 + \frac{6 - 5x^2}{\sqrt{x}}$$

Given that the point $P(4, -8)$ lies on C,

(a) find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

(b) Find $f(x)$, giving each term in its simplest form.

(5)

a/ $(4, -8)$

when $x = 4$

$f'(4) = 30 + \frac{6 - 5(4)^2}{\sqrt{4}}$

$$\boxed{\begin{array}{l} 30 + \frac{6-80}{2} \\ 30 - 37 \end{array}}$$

$= -7$

$m = -7$

$y = -7x + c$ $\quad (4, -8)$ $\quad x \; y$

$-8 = -7(4) + c$

$-8 = -28 + c$

$c = 20$

$y = -7x + 20$

b/ $f'(x) = 30 + \frac{6 - 5x^2}{x^{\frac{1}{2}}}$

$f'(x) = 30 + 6x^{-\frac{1}{2}} - 5x^{3/2}$

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Question 7 continued

$$f(x) = 30x + \frac{6x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{5x^{5/2}}{5/2} + C$$

$$f(x) = 30x + 12x^{\frac{1}{2}} - 2x^{5/2} + C$$

$(4, -8)$

$$-8 = 30(4) + 12(4)^{\frac{1}{2}} - 2(4)^{5/2} + C$$

$$-8 = 120 + 24 - 2(32) + C$$

$$-8 = 120 + 24 - 64 + C$$

$$-8 = 144 - 64 + C$$

$$-8 = 80 + C$$

$$C = -88$$

$$f(x) = 30x + 12x^{\frac{1}{2}} - 2x^{5/2} - 88$$

(Total 9 marks)

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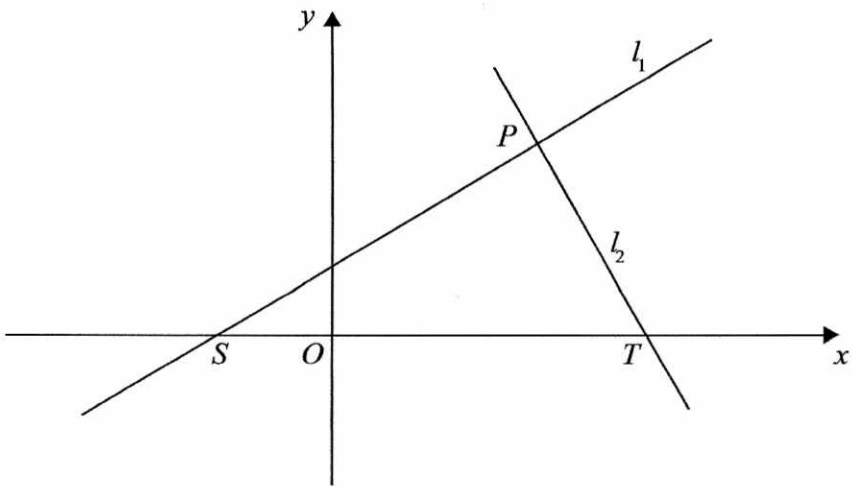
Q7

8.



Not to scale

Figure 1

The straight line l_1, shown in Figure 1, has equation $5y = 4x + 10$

The point P with x coordinate 5 lies on l_1

The straight line l_2 is perpendicular to l_1 and passes through P.

(a) Find an equation for l_2, writing your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(4)

The lines l_1 and l_2 cut the x-axis at the points S and T respectively, as shown in Figure 1.

(b) Calculate the area of triangle SPT.

(4)

a/ $5y = 4x + 10$

$y = \frac{4}{5}x + 2$

when $x = 5$ $\quad y = \frac{4}{5}(5) + 2$

$= 6$

perpendicular gradient $= -\frac{5}{4}$

$y = -\frac{5}{4}x + c$

$6 = -\frac{5}{4}(5) + c$

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Question 8 continued

$$6 = -\frac{25}{4} + c$$

$$\cancel{\frac{24}{c}}$$

$$\frac{24}{4} = -\frac{25}{4} + c$$

$$\frac{49}{4} = c$$

$$y = -\frac{5}{4}x + \frac{49}{4}$$

$$4y = -5x + 49$$

$$5x + 4y - 49 = 0$$

b/ Cross x when $y = 0$

l_2 $5x + 4(0) - 49 = 0$

$5x - 49 = 0$

$5x = 49$

$x = \frac{49}{5}$

l_1: $5(0) = 4x + 10$

$-10 = 4x$

$x = -\frac{10}{4}$

$= -\frac{5}{2}$

$$\text{Distance } ST = \frac{49}{5} + \frac{5}{2}$$

$$= \frac{98}{10} + \frac{25}{10} = \frac{123}{10}$$

$$\text{Area of } SPT = \frac{1}{2}\left(\frac{123}{10}\right)(6)$$

$$= 3\left(\frac{123}{10}\right)$$

$$= \frac{369}{10} \text{ units}^2$$

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9. (a) On separate axes sketch the graphs of

(i) $y = -3x + c$, where c is a positive constant,

(ii) $y = \dfrac{1}{x} + 5$

On each sketch show the coordinates of any point at which the graph crosses the y-axis and the equation of any horizontal asymptote.

(4)

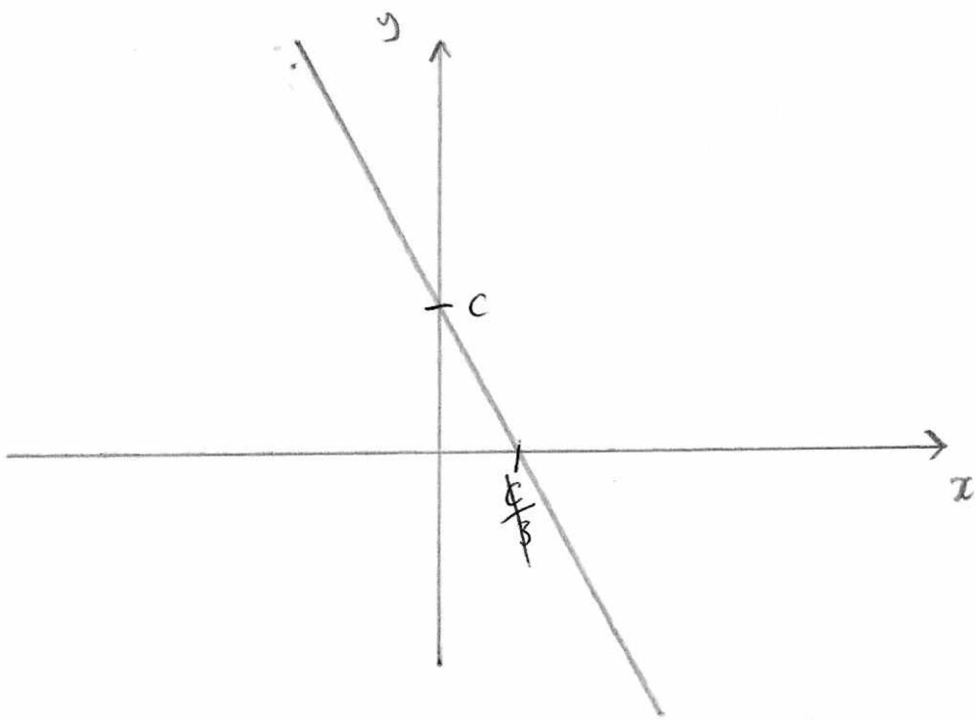
Given that $y = -3x + c$, where c is a positive constant, meets the curve $y = \dfrac{1}{x} + 5$ at two distinct points,

(b) show that $(5 - c)^2 > 12$

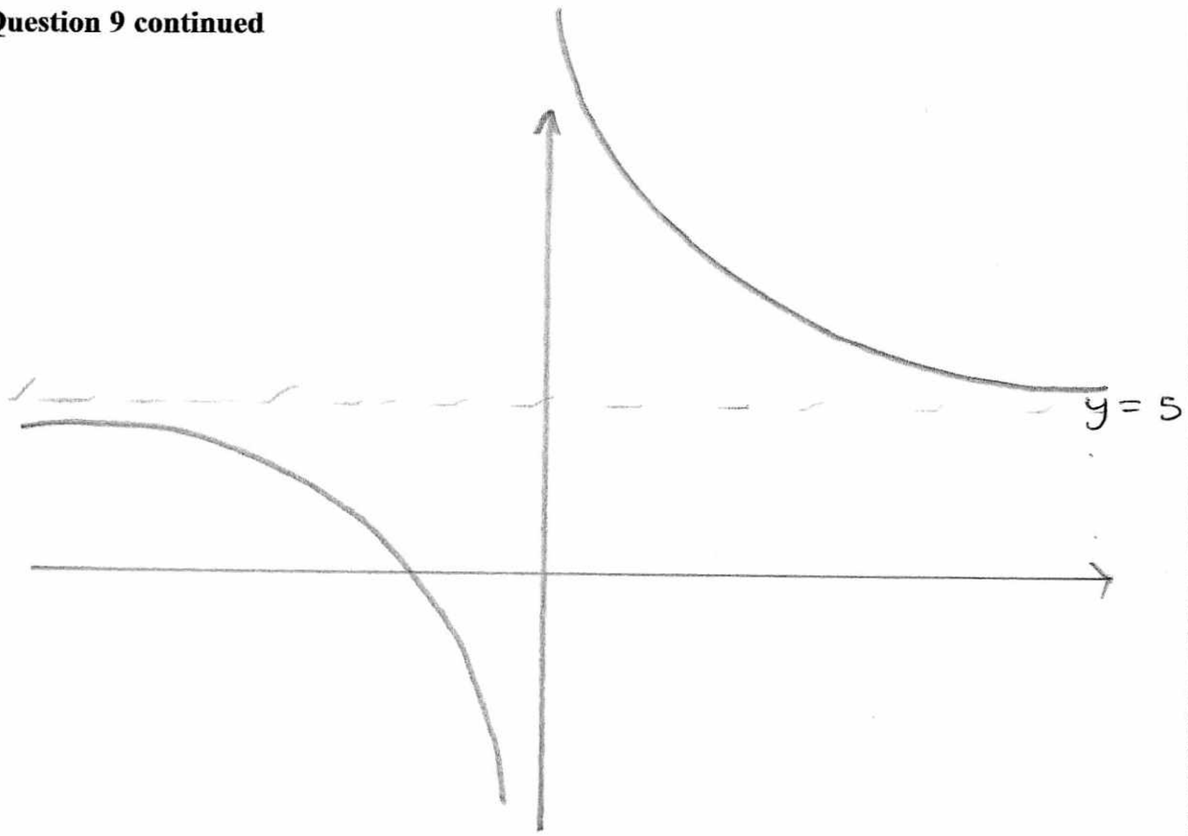
(3)

(c) Hence find the range of possible values for c.

(4)



Question 9 continued



$$-3x + c = \frac{1}{x} + 5$$

$$-3x^2 + cx = 1 + 5x$$

$$0 = 3x^2 + 5x \cancel{+} cx + 1$$

$$0 = 3x^2 + (5 - c)x + 1$$

two solutions $\therefore$ $b^2 - 4ac > 0$

$$(5-c)^2 - 4(3)(1) > 0$$

$$(5 - c)^2 - 12 > 0$$

$$(5 - c)^2 > 12$$

c/ $\quad 5 - c = \pm\sqrt{12}$

$$5 \pm \sqrt{12} = c \qquad \sqrt{12} = \sqrt{4}\sqrt{3}$$

$$c = 5 \pm 2\sqrt{3}$$

$0 < c < 5 - 2\sqrt{3}$ or $c > 5 + 2\sqrt{3}$

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10.

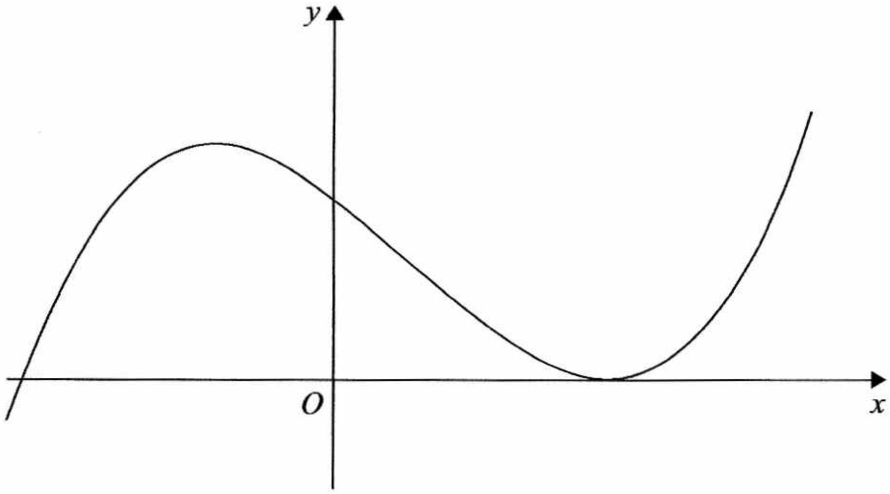

Figure 2

Figure 2 shows a sketch of part of the curve $y = \mathrm{f}(x)$, $x \in \mathbb{R}$, where

$$\mathrm{f}(x) = (2x - 5)^2(x + 3)$$

(a) Given that

(i) the curve with equation $y = \mathrm{f}(x) - k$, $x \in \mathbb{R}$, passes through the origin, find the value of the constant k,

(ii) the curve with equation $y = \mathrm{f}(x + c)$, $x \in \mathbb{R}$, has a minimum point at the origin, find the value of the constant c.

(3)

(b) Show that $\mathrm{f}'(x) = 12x^2 - 16x - 35$

(3)

Points A and B are distinct points that lie on the curve $y = \mathrm{f}(x)$.

The gradient of the curve at A is equal to the gradient of the curve at B.

Given that point A has x coordinate 3

(c) find the x coordinate of point B.

(5)

a) $f(x)$ crosses y when $x = 0$

$f(0) = (2(0) - 5)^2((0) + 3)$

$= (-5)^2(3)$

$= 75$

$k = 75$

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Question 10 continued

$f(x)$ crosses x when $y=0$

$0 = (2x-5)^2(x+3)$

$x = \frac{5}{2}$ $\quad x = -3$

$c = \frac{5}{2}$

b/ $f(x) = (2x-5)^2(x+3)$

$= (4x^2 - 10x - 10x + 25)(x+3)$

$= (4x^2 - 20x + 25)(x+3)$

$= 4x^3 - 20x^2 + 25x + 12x^2 - 60x + 75$

$= 4x^3 - 8x^2 - 35x + 75$

$f'(x) = 12x^2 - 16x - 35$

c/ $f'(3) = 12(3)^2 - 16(3) - 35$

$= 12(9) - 48 - 35$

$= 108 - 48 - 35$

$= 25$

B is where $f'(x) = 25$

$12x^2 - 16x - 35 = 25$

$12x^2 - 16x - \cancel{35}\,60 = 0$

$3x^2 - 4x - 15 = 0$

$(3x+5)(x-3) = 0$

$x = -\frac{5}{3}$ $\quad x = 3$

x coordinate of B is $-\frac{5}{3}$